Optimal Time-Consistent Capital Controls as Dynamic Terms-of-Trade Manipulation*

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Abstract

This paper develops a simple theory of capital controls as dynamic terms-of-trade manipulation, when the policy maker cannot commit to future policies, and chooses current taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. In this situation, capital control not only affects the country's current terms-of-trade, but also affect its future terms-of-trade and future policy makers' optimal policies. We find that using capital control to manipulate term-of-trade not only reduces other countries welfare, i.e., it is a "beggar-thy-neighbor" policy, it could also be welfare reducing from the country's own perspective.

Keywords: Terms of Trade, Capital Control, Time-consistency

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1 Introduction

Most of the existing theoretical studies on capital controls focus on a small open economy (SOE). A small open economy do not have and substantial influence on the worldwide economy and take the worldwide interest rate (intertemporal terms of trade) as given. Hence we do not consider the international spillover effects caused by SOE's regulatory policies through terms of trade (TOT). However, when considering a large open economy (LOE), the setting is quite different. A LOE has the ability to influence the world interest rate and hence the regularity policies have non-negligible spillover effect, resulting in welfare gains or loss.

Theoretically, there are three main reasons that justify the capital control or capital management. The first is to manage the pecuniary externality ((Bianchi, 2011; Benigno et al., 2013, 2016; Bianchi and Mendoza, 2018; Korinek, 2018)), where the individual agents do not realize their borrowing decisions may erode the price of collateral, thereby causing a financial crisis; the second is to manage the demand externality (Schmitt-Grohé and Uribe (2016)), where the allocation during booms may have a negative impact on the allocation during recession due to some rigidities in the economy; our paper focus on the third reason, TOT manipulation, that is, a LOE manipulates the terms of trade in its favor by imposing regulation on capital flows ((Costinot et al., 2014; Davis and Devereux, 2022; Heathcote and Perri, 2016)). In our paper, TOT refers to both intertemporal TOT (interest rate of international borrowing and lending) and intratemporal TOT (relative price of different goods)

In this paper, we first investigate intertemporal TOT manipulation. We construct an infinite horizon, two country, representative agent model with symmetric stochastic endowments in the form of a single tradable good. We assume that the benevolent social planner manages the capital flow and hence manipulate the world interest through levying a tax on bond trading. Individual agents act as price takers, and do not internalize the impact of their choice on price.

From the perspective of a social planner, she realizes that the amount of borrowing would have substantial impact on the world interest rate, i.e., intertemporal TOT, thus managing international bond trading or the capital flows would be beneficial. However, individual agents do not internalize this benefit when allocating their resources, hence regularity policies are designed to correct this externality. This paper proposes another effect of borrowing, also not internalized by individual agent: borrowing today would not only affect today's interest rate (contemporaneous TOT) through contemporaneous fund supply, but also affect the position of fund supply function in next period, thus changing the corresponding future interest rate (future TOT) for every value of foreign bond holding. Suppose that domestic country is a saver today and choose to restrict the amount of savings. Due to this manipulation, domestic country receives a higher interest rate for its savings. However, lower domestic savings means a lower debt position for other countries. Therefore, in the next period, other countries will be in less need of domestic savings. In turn, if domestic still saves in the next period, the future interest rate would be lower. When optimizing the social welfare, a social planner face this trade-off between contemporaneous TOT and future TOT: manipulating interest rate in today's favor may have an negative impact on future's interest rate. This trade-off only appears in our infinite horizon model and thus changes the conventional view that only considers contemporaneous TOT.

To conduct our analysis, we assume the social planner cannot commit to future policies, instead, she conjectures the future policy rules and takes them as given. The social planner can only affect future allocation through choosing today's optimal amount of saving or borrowing. We study the optimal time consistent policy rules where the conjectured future policy rules coincide with today's optimal decision rules.

We derive the optimal taxation under unilateral manipulation where only one country manipulate the TOT and the other act passively. Our computational results indicate that the optimal tax in our infinite horizon model is more complicated due to the consideration of future TOT effect. We study the welfare effect of country scale where the endowment differs significantly. Our welfare analysis suggests that the welfare gain and the corresponding spillover effect of TOT manipulation depend crucially on the *relative* scale of countries. In our baseline case where both countries are the same scale, imposing tax is *scarcely* welfare-improving for country who manipulates the TOT comparing to the no-tax regime, also this regularity would bring welfare losses to other countries, i.e. it is a "beggar-thy-neighbor" policy. Taxation is only recommended for larger countries. Next we study the bilateral manipulation where both countries impose taxes on capital flow and hence a Nash equilibrium. When other country manipulates TOT, imposing a tax is better than acting passively for domestic country.

Next we extend our model to incorporate intratemporal TOT manipulation. There are two kinds of tradable goods and each country is endowed with only one kind of tradable good. However, consumption is defined on both tradable goods and thus countries have to trade. A social planner now has the incentive to manipulate both inter- and intra-temporal TOT. We find a richer mechanism emerging from the interaction between inter- and intra-temporal TOT: capital control not only affects the world interest rate but also the relative price. Welfare analysis indicates that the welfare gain from manipulation mainly comes from the improvement of intratemporal TOT rather than intertemporal TOT, further supports the conclusion that capital control is scarcely welfare-improving from the perspective of terms of trade manipulation.

Our paper closely related to the recent work on capital controls from the perspective of TOT manipulation. Davis and Devereux (2022) constructs a two country, two goods model to study the interaction of pecuniary externality and inter-temporal TOT manipulation. TOT manipulation impedes the efficiency of capital control as a tool to stabilize financial system. However, their work focus on a two-period model that only considers contemporaneous TOT manipulation. Costinot et al. (2014) proposes a theory to justify the necessities of capital control from the perspective of dynamic TOT manipulation, both inter-temporal and intra-temporal. In their model, the social planner actually can commit to future policies while social planner in our model settings cannot, thereby generating the concern of time inconsistency. Heathcote and Perri (2016) shows that country would be benefit from unilaterally imposing capital controls to obtain a favorable intra-temporal TOT, at the cost of other countries welfare loss.

Our paper also connects to the strand of literature that consider the non-cooperative capital control war and the cooperative capital control policy. Davis and Devereux (2022) suggest that when facing the trade off between improving terms of trade and easing financial constraints, an effective capital control policy demands international cooperation. **?** and Heathcote and Perri (2016) both find that imposing capital controls cooperatively can be welfare improving in an incomplete financial market due to the improvement of risk sharing, while capital control war leads to welfare losses for both countries. Costinot et al. (2014) also suggests that neither country benefit from the capital control war. **?** show that when domestic is in liquidity trap, non-cooperative terms of trade manipulation would hamper the role of capital flow in stabilizing aggregate demand. **?** suggests a cooperative capital control brings quite small welfare gains comparing to the Nash equilibrium. **?** focus on a few periphery countries and shows that capital control in order to mitigate the pecuniary externality but without international coordination leads to a welfare loss comparing to the laissez-faire, while coordinated capital control brings a sizable welfare gain. **?** also suggests a cooperative capital control policy in the context of most international prices are sticky in dollars.

Our paper is organized as follows. Section 2 describes the model settings when there is only intertemporal TOT manipulation, and proposes the optimal taxation in a single tradable good scenario. Section 3 computes the model and reports the results of unilateral manipulation and the Nash equilibrium. Section 4 introduces intratemporal TOT manipulation ans section 5 presents the quantitative results. Section 6 concludes.

2 The Basic Model

2.1 Model Environment

There are two countries and each country has its representative agent, A and B. Endowments are exogenous and stochastic, country A (B) is endowed with tradable goods in each period y_t^A (y_t^B). We normalize the total endowments in each period as one,

$$y_t^A + y_t^B = 1, \ \forall t.$$

y_t^A is assumed to follow a first order Markov process. Both country share the same utility function,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right).$$

In the asset market, there is only a one-period foreign bond, which has to be traded with the other country. We differentiate between individual and aggregate foreign bond holding, f_t and F_t. In equilibrium, aggregation implies $f_t^i = F_t^i$, $i = A, B$. Let r_t be the net world interest rate and $R_t \equiv 1 + r_t$ be the gross rate. Country A is allowed to manipulate the intertemporal terms of trade, i.e., the interest rate, by choosing the most favorable foreign bond holding, while B is assumed to act competitively all the time. This is referred to the *unilateral manipulation* case. In section 2.4, we extend this assumption, where both A and B are allowed to manipulate, and is referred to the *bilateral manipulation* case. Given the access to manipulation, however, individual agent in A take the interest rate and the tax rate as given, since she does not believe that the interest rate as a macro variable is under her control. Individual budget constraint are given by

$$c_t^i + \frac{f_{t+1}^i}{R_t} = y_t^i + f_t^i, \ i = A, B. \tag{1}$$

Bonds market clearing condition is

$$F_t^A + F_t^B = 0.$$

State variables of our model are $\left\{F_t^A, y_t^A\right\}$. For simplicity, we omit their superscripts. The competitive equilibrium is described by the traditional Euler equation, budget constraint and the market clearing condition. A social planner in A can fully realize her ability to manipulate the interest rate.

2.2 Social planner's Problem

One main setting in our paper is that the planner can not commit to future policies, but only takes the future policies (both home and abroad) as given. We use a superscript "m" to denote the unilateral manipulation case. In this case, the optimal choice of individual in B is described by the Euler equation as usual:

$$u'\left(c_t^B\right) = \beta R_t \mathbb{E}_t\left[u'\left(c_{t+1}^B\right)\right]. \tag{2}$$

Assume the future policy function of consumption as $C^{i,m}(F_t, y_t)$, $i = A, B$, and plug the budget constraint into 2, we obtain a relationship between interest rate and next period's bond holding:

$$u'\left(1 - y_t - F_t + \frac{F_{t+1}}{R_t}\right) = \beta R_t \mathbb{E}_t\left[u'\left(C^{B,m}(F_{t+1}, y_{t+1})\right)\right].$$

We call this relationship as the fund supply function, denoted as

$$R_t = R^{s,m}\left(F_{t+1}\middle| F_t, y_t\right). \tag{3}$$

Given F_t, the fund supply function is a downward-sloping curve, $\frac{\partial R^{s,m}\left(F_{t+1}\middle| F_t, y_t\right)}{\partial F_{t+1}} < 0$, where when the interest rate is higher, country B would save more (lower F_{t+1}) and when the interest rate is lower country B would save less (higher F_{t+1}^A). If F_t raises, which means country B has less resources in the beginning of time t, the interest rate R_t that agent in country B charges for lending to A would increase for $F_{t+1} < 0$, and the interest rate the agent in B is willing to pay for borrowing from A would also increase for for $F_{t+1} > 0$, hence $\frac{\partial R^{s,m}\left(F_{t+1}\middle| F_t, y_t\right)}{\partial F_t} > 0$. The effect of F_t on $R^{s,m}\left(F_{t+1}\middle| F_t, y_t\right)$ is central to our analysis.

We assume the social planner cannot commit to future policies but take them as given. Social planner can only affect future allocation through the choice of F_{t+1}. Hence our analysis focuses on the optimal time-consistent decision rule. The planner's policies are time-consistent, if the current policy rules coincide with the future policy rules that are taken as given by the planner to solve for the current policy rules. Following Bianchi and Mendoza (2018), we give the definition of the time-consistent problem.

Definition 1. Denote the value function as $V^m(F, y)$, future policy rules as $C^{i,m}(F, y)$, $F^m(F, y)$, $R^m(F, y)$, and current policy rules as $\mathcal{C}^{i,m}(F, y)$, $\mathcal{F}^m(F, y)$, $\mathcal{R}^m(F, y)$. Current pol-

icy rules are time-consistent if and only if: 1), $\mathcal{C}^{i,m}(F,y)$, $\mathcal{F}^m(F,y)$, $\mathcal{R}^m(F,y)$ and $V^m(F,y)$ solve the planner's optimal allocation problem (4)*, taking future plans $C^{i,m}(F,y)$, $F^m(F,y)$, $R^m(F,y)$ and $V^m(F,y)$ as given; 2), current policy rules coincide with future policy rules, i.e., $C^{i,m}(F,y) = \mathcal{C}^{i,m}(F,y)$, $F^m(F,y) = \mathcal{F}^m(F,y)$, $R^m(F,y) = \mathcal{R}^m(F,y)$.*

Social planner in A anticipates the optimal decision of country B, 3, and chooses the most favorable combination $\{F_{t+1}, R_t\}$ in order to maximize the lifetime utility. Therefore the interest rate is manipulated in A's favor. The recursive form of country A's optimization problem is

$$V^m(F_t, y_t) = \max_{F_{t+1}} u\left(c_t^A\right) + \beta \mathbb{E}_t \left[V^m(F_{t+1}, y_{t+1})\right]. \tag{4}$$

subject to 1 and 3. Given state $\{F_t, y_t\}$, the first order condition is

$$u'\left(c_t^A\right)\left(-\frac{1}{R_t} + \frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial F_{t+1}}\right) + \beta \mathbb{E}_t \frac{\partial V^m(F_{t+1}, y_{t+1})}{\partial F_{t+1}} = 0. \tag{5}$$

The second term of 5 is given by the Envelope theorem:

$$\frac{\partial V^m(F_{t+1}, y_{t+1})}{\partial F_{t+1}} = u'\left[C^{A,m}(F_{t+1}, y_{t+1})\right]\left[1 + \Phi^m(F_{t+1}, y_{t+1})\right].$$

with

$$\Phi^m(F_{t+1}, y_{t+1}) \equiv \frac{F^m(F_{t+1}, y_{t+1})}{\left[R^m(F_{t+1}, y_{t+1})\right]^2} \frac{\partial R^{s,m}\left(F^m(F_{t+1}, y_{t+1}) \middle| F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}}.$$

where $F^m(\cdot)$, $R^m(\cdot)$ are future policy functions of bond holdings and interest rate. In (5), term $\left(F_{t+1}/R_t^2 \times \partial R_t / \partial F_{t+1}\right)$ reflects the *contemporaneous terms of trade effect* appearing in the two-period model (Davis and Devereux (2022)). This effect indicates that the social planner is choosing the most favorable combination $\{F_{t+1}, R_t\}$ along the fund supply curve $R^{s,m}\left(F_{t+1} \middle| F_t, y_t\right)$. Term $\Phi^m(F_{t+1}, y_{t+1})$ arises in our infinite horizon model because F_{t+1} also affects the position of the next period's fund supply curve $R^{s,m}\left(F_{t+2} \middle| F_{t+1}, y_{t+1}\right)$. This refers to the *future terms of trade effect*, a novel channel through which current choice of F_{t+1} affects the model dynamic. As we mentioned above, if a social planner reduces the foreign bond holding F_{t+1}, current interest rate would rise; however, this manipulation would also lead to a downward shift of the fund supply curve $R^{s,m}\left(F_{t+2} \middle| F_{t+1}, y_{t+1}\right)$ next period.

Combining (1), (3) and (5), a social planner would solve for the current optimal foreign bond holding for country A.

2.3 Decentralization

The social planner's equilibrium can be decentralized as a competitive regime by imposing a lump-sum capital flow tax τ_t on agent A's bond trading. With capital flow tax, the budget constraint of agent A becomes

$$c_t^A + (1+\tau_t)\frac{f_{t+1}}{R_t} = y_t + f_t + T_t.$$

Country A's government balances its budget by rebating the tax revenue to individual agent,

$$T_t = \tau_t \frac{F_{t+1}}{R_t}.$$

Proposition 1. The social planner's problem can be decentralized with a tax τ_t^{sp} in the form of

$$\tau_t^{sp} = \underbrace{-\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}}}_{contemporaneous\ TOT\ effect} \underbrace{-\beta R_t \frac{\mathbb{E}_t\left\{u'\left[C^{A,m}\left(F_{t+1},y_{t+1}\right)\right]\Phi^m\left(F_{t+1},y_{t+1}\right)\right\}}{u'\left(c_t^A\right)}}_{future\ TOT\ effect}. \tag{6}$$

with tax revenue rebated in the form of lump-sum transfer and R_t implied by the fund supply function $R^{s,m}\left(F_{t+1}\middle|F_t,y_t\right)$.

The first term reflects the decentralization of the contemporaneous terms of trade effect, and the second term reflects the decentralization of the future terms of trade effect.

2.4 Bilateral Manipulation

In this subsection, we allow both country A and B to manipulate the intertemporal terms of trade, i.e., we study a Nash equilibrium. We use a superscript "*ne*" to denote this bilateral manipulation case. In a nash equilibrium, both social planners in A and B take their opponent's choice as given and decide their optimal manipulation policies respectively. In section 2.3, we have proved that the unilateral case can be decentralized by imposing a tax on foreign bond trading. Under state $\{F_t, y_t\}$, given a tax τ_t^* imposed by country B , a social planner anticipates the fund supply by solving equation

$$u'\left(c_t^B\right) = \beta\frac{R_t}{1+\tau_t^*}\mathbb{E}_t\left[u'\left(C^{B,ne}\left(F_t,y_t\right)\right)\right].$$

It is worth noting that the foreign tax τ_t^* also affects the fund supply, which we denote as $R^{s,ne}\left(F_{t+1}\middle|\tau_t^*, F_t, y_t\right)$. If τ_t^* increases, the corresponding R_t would be raised, i.e., $\partial R^{s,ne}/\partial\tau_t^* > 0$.

As the unilateral case, first order condition of A is

$$u'\left(c_t^A\right)\left(-\frac{1}{R_t}+\frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial F_{t+1}}\right)+\beta\mathbb{E}_t\frac{\partial V^{ne}\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}}=0. \tag{7}$$

In time $t+1$, the recursive form of the social planner's problem is slightly different to the unilateral case, the fund supply in $t+1$ now includes the optimal taxation $\tau_{t+1}^*\left(F_{t+1}, y_{t+1}\right)$. Using Envelope theorem[1],

$$\frac{\partial V^{ne}\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}}=u'\left[C^{A,ne}\left(F_{t+1}, y_{t+1}\right)\right]\left[1+\Phi^{ne}\left(F_{t+1}, y_{t+1}\right)\right].$$

where

$$\Phi^{ne}\left(F_{t+1}, y_{t+1}\right)\equiv\frac{F^{ne}\left(F_{t+1}, y_{t+1}\right)}{\left[R^{ne}\left(F_{t+1}, y_{t+1}\right)\right]^2}\times$$
$$\left[\frac{\partial R^{s,ne}\left(F^{ne}\left(F_{t+1}, y_{t+1}\right)\middle|\tau_{t+1}^*\left(F_{t+1}, y_{t+1}\right), F_{t+1}, y_{t+1}\right)}{\partial\tau_{t+1}^*}\frac{\partial\tau_{t+1}^*\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}}+\frac{\partial R^{s,ne}\left(F^{ne}\left(F_{t+1}, y_{t+1}\right)\middle|\tau_{t+1}^*\right.}{\partial F_{t+1}}\right.$$

Term $\partial R^{s,ne}/\partial F_{t+1}$ appears for the same reason as the unilateral case. Comparing to Φ^m, there is another derivative in Φ^{ne}, $\partial R^{s,ne}/\partial\tau_{t+1}^*\times\partial\tau_{t+1}^*\left(F_{t+1}, y_{t+1}\right)/\partial F_{t+1}$, which stands for the effect that the endogenous state in $t+1$ will affect the optimal taxation in country B, thereby affecting fund supply and country A's optimal choice.

As subsection 2.3, we proceed on showing that the social planner's problem can still be decentralized by an optimal tax.

Proposition 2. Given a tax choice of country B, τ_t^*, the social planner's problem of country A can be decentralized with a tax τ_t in the form of

$$\tau_t=-\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}}-\beta R_t\frac{\mathbb{E}_t\left\{u'\left[C^{A,ne}\left(F_{t+1}, y_{t+1}\right)\right]\Phi^{ne}\left(F_{t+1}, y_{t+1}\right)\right\}}{u'\left(c_t^A\right)}. \tag{8}$$

with tax revenue rebated in the form of lump-sum transfer and R_t implied by the fund supply

[1] For detailed derivation, please refer to the appendix

function $R^{ne}\left(F_{t+1}\middle|\tau_t^*, F_t, y_t\right)$.

Equation 8 is the best response of τ_t^*, and implicitly determines a best response function of country A, denoted as

$$\tau_t = \tau\left(\tau_t^*, F_t, y_t\right). \tag{9}$$

Following the same procedure, we would also have the best response of country B, denoted as

$$\tau_t^* = \tau^*\left(\tau_t, F_t, y_t\right), \tag{10}$$

to the tax choice τ_t of A. The optimal taxation for A and B under state $\{F_t, y_t\}$ is obtained by combining 9 and 10.

3 Quantitative Analysis of Intertemporal TOT Manipulation

First we demonstrate how the economy behaves in a unilateral manipulation regime and then introduce the case of bilateral manipulation.

3.1 Parameter Setting

We assume the utility function is CRRA form: for $i = A, B$,

$$u\left(c_t^i\right) = \frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma}$$

and we choose $\sigma = 1.5$, $\beta = 0.99$.

Endowment is denominated in units of consumption. There are three possible value for the exogenous endowment in country A, a lower endowment $y^L = 0.4$, a medium endowment $y^M = 0.5$, and a higher endowment $y^H = 0.6$, with equal possibility of $\frac{1}{3}$:

$$P\left(y_t = y^L\right) = P\left(y_t = y^M\right) = P\left(y_t = y^H\right) = \frac{1}{3}$$

We denote this discrete distribution as $\delta\left(y\right)$.

We use the global method to solve for this model due to the highly nonlinear property of the model variables, as we demonstrate later. To implement the global solution, we use time iteration method introduced in Cao et al. (2020).

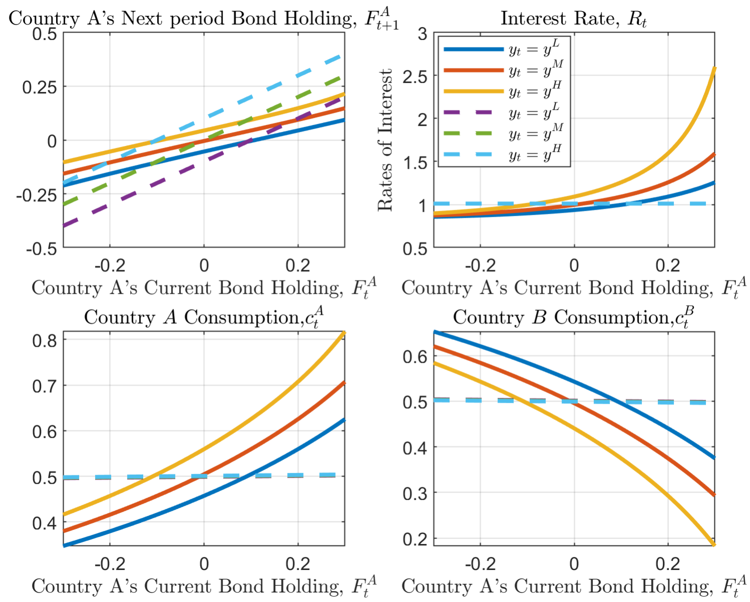

Figure 1: Policy Function. The solid line represents the manipulation case and the dashed line represents the competitive case. y^L, y^M and y^H represent the low, medium and high endowment shock of country A, respectively.

3.2 Unilateral Manipulation

3.2.1 Basic Dynamics

Figure 1 illustrates the policy function in both manipulation and competitive regime. With the capital flow taxation at hand, social planner in A would restrict both capital inflows and outflows for a more favorable world interest rate. Therefore the policy function of F^A_{t+1} is flatter in manipulation regime. Comsumption changes accordingly. When the current state F_t is high, to receive a higher rate of payment, the planner reduces the amount of saving. Therefore the consumption is higher in the manipulation regime. Also in time $t+1$, there is less disposable resource (lower F_{t+1}), causing a downward shift in the fund supply due to the future TOT effect. Since the summation of c^A_t and c^B_t is constant, the consumption dynamic of c^B_t is opposite to c^A_t.

To illustrate the future TOT effect, figure 2 shows the fund supply given medium endowment shock $y_t = y^M$ and three different initial foreign asset position, $F_t = -0.025$, $F_t = 0$, and $F_t = 0.025$. We would observe that as country A save more (borrow less) before period t, i.e., country A becomes wealthier, country B charges a universally higher interest rate for both country A's borrowing and saving. If country A decides to borrow from B ($F_{t+1} < 0$), it must be more attractive for B to lend as B get poorer, otherwise country B will find it more beneficial to consume rather than lending and receiving a relatively

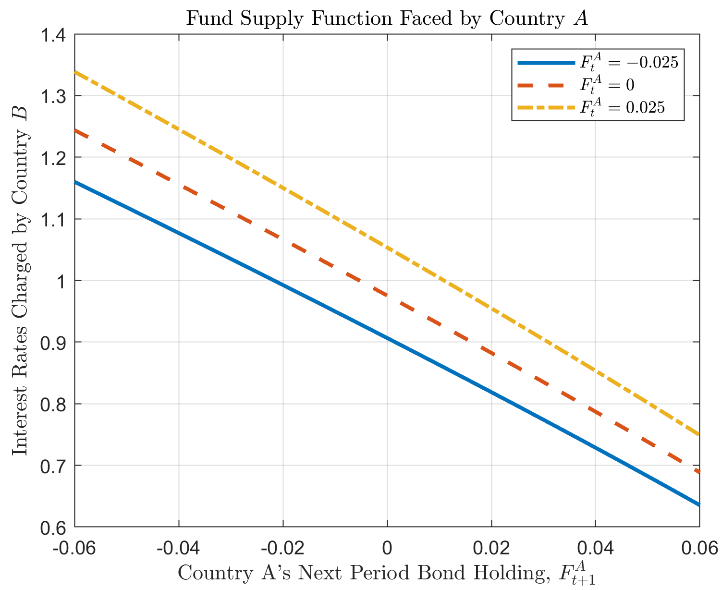

Figure 2: Fund Supply Function Given $y_t = 0.5$ And Different Initial Asset Position $F_t^A = -0.025$, $F_t^A = 0$, and $F_t^A = 0.025$

low payment. If country B decides to borrow from A ($F_{t+1} > 0$), then country B would be in more desperate need in country A's fund as B gets poorer and hence more willingly to pay a higher interest payment. When optimizing life-time utility, social planner in A faces a trade-off between contemporaneous TOT and future TOT effect: if she limits the amount of borrowing to receive a relatively lower interest from B, then B would be poorer next period, thereby raising the fund supply curve and would be less favorable if A still borrows in the next period. Similarly, reducing the amount of saving to receive a relatively higher interest from B leads to more initial resources of B next period, thereby shifting the fund supply curve downward and would be less favorable if A still saves in the next period.

The left panel of figure 3 shows the optimal taxation. The blue, orange and yellow curves are referring to low, medium and high endowment shock, respectively. As country A save more for the next period, the optimal tax is increasing correspondingly, suggesting an transition towards discouragement on capital outflow. However, unlike the conventional view that the optimal tax should be zero if today's *choice* of foreign bond holding next period is zero, the optimal tax τ_t' is positive instead, under all kinds of endowment shocks.

Using 6, we further decompose the optimal taxation as the contemporaneous TOT effect (solid line) and the non-negligible future TOT effect (dashed line), as figure 3 demonstrates. The contemporaneous TOT is the main consideration of capital flow taxes, aiming

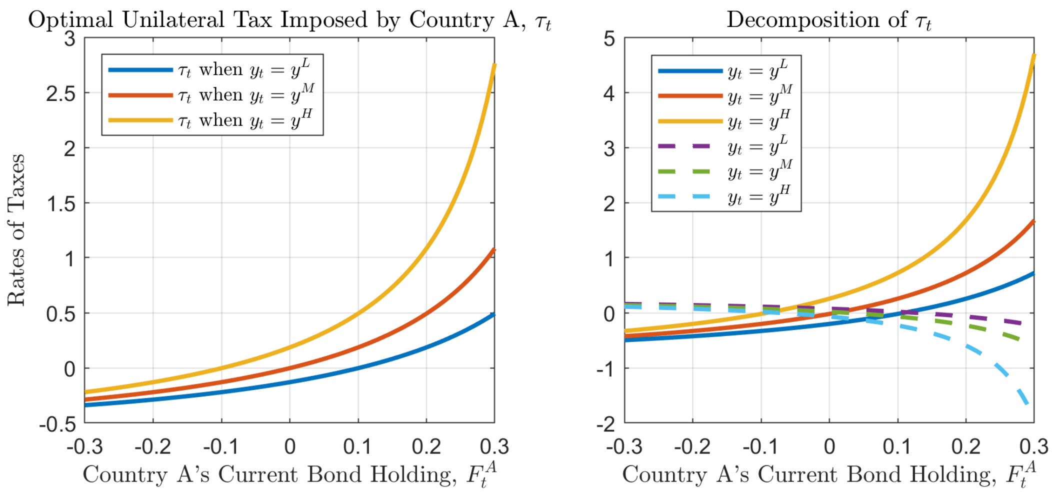

Figure 3: Optimal Taxation

at obtaining a favorable contemporaneous terms of trade. The future part of taxation represents the concern of the effect of manipulation at time t on the fund supply curve at time $t+1$. As long as the economy keeps involving in bond trading from time t to $t+1$, the sign of the contemporaneous part and the future part are exactly opposite, reflecting the policy trade-off we mentioned above.

3.2.2 Welfare

We first analyze the welfare effect of TOT manipulation qualitatively. Though country A has the option to manipulate, the fund supply function itself is determined by country B. Figure 4 compares the fund supply function (left panel) and the policy function of c_t^B (right panel) when there is no uncertainty in the future. First, standing at time $t+1$, for a given $F_{t+1} > 0$, social planner in A will restrict its capital outflow, leading to a lower c_{t+1}^B. This is the TOT manipulation we discussed above. Second, standing at time t and for the same given $F_{t+1} > 0$, in anticipation of A's future manipulation, i.e., of a lower future consumption, country B has an additional incentive to save. To offset the saving incentive, interest rate must fall, which is shown as the left column of figure 4. The analysis for a given $F_{t+1} < 0$ is analogous. From country A's perspective, restrictions on capital outflow (inflow) will actually bring a lower (higher) repayment, comparing to the competitive regime. Hence we may deduce that manipulation will actually lead to a welfare deterioration for those who imposes it.

We examine the welfare gains or costs of imposing the optimal tax, or equivalently, the welfare change of manipulating the world interest rate, comparing to the competitive regime without tax. For country i ($i = A, B$), define the welfare change given a particular

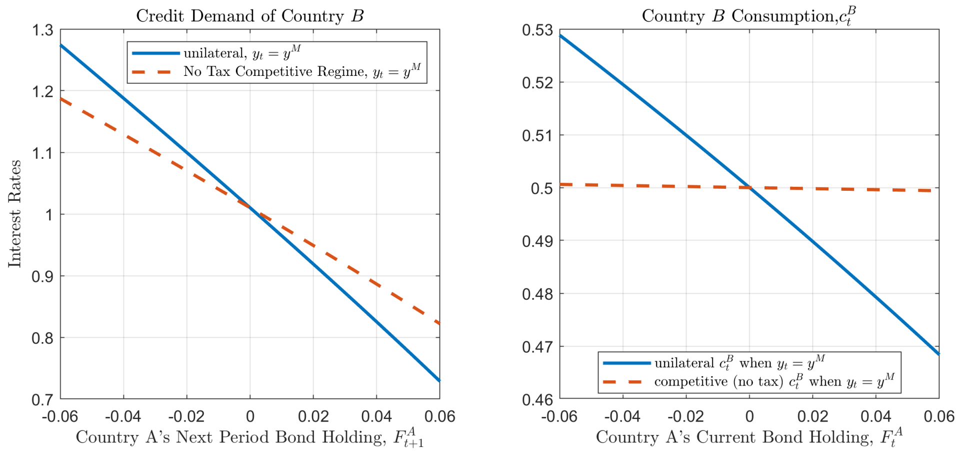

Figure 4: Comparison between manipulation and competitive regime when there is no uncertainty in the future. The red dashed line is correpsonds to the competitive regime without capital flow tax and the solid blue line corresponds to the manipulation regime with optimal capital flow taxation. The left panel is obtained assuming $F_t = 0$.

state $\{F_t, y_t\}$, denoted $\gamma^i(F_t, y_t)$, as the percentage change in the lifetime welfare under competitive regime without tax that makes an individual agent live as well off as living under manipulation regime. Specifically, $\gamma^i(F_t, y_t)$ is defined by

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}\beta^j U\left[\tilde{c}^i_{t+j}\left(1+\frac{\gamma^i(F_t,y_t)}{100}\right)\right]\middle|(F_t,y_t)\right\} = \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s U\left(c^i_{t+s}\right)\middle|(F_t,y_t)\right\}$$

where $\tilde{c}^i_{t+j}$, c^i_{t+s} denotes the equilibrium consumption of country i under competitive regime and manipulation regime, respectively. Under the model specification, given a particular state s_t, the equation above becomes [2]

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{\left[\widetilde{c}^i_{t+j} \left(1 + \frac{\gamma^i(F_t,y_t)}{100} \right) \right]^{1-\sigma}}{1-\sigma} \middle| (F_t, y_t) \right\} = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\left(c^i_{t+s}\right)^{1-\sigma}}{1-\sigma} \middle| (F_t, y_t) \right\}$$

Furthermore, we evaluate the expectation of welfare gains across the ergodic distribution of state F_t, $\Omega(F_t)$, through the following formula:

$$\gamma^i = \sum_{y_t} \left[\sum_{F_t \in \Omega(F_t)} \gamma^i (F_t, y_t) \, \Omega(F_t) \right] \delta(y_t) \tag{12}$$

The expectation of welfare change in country A is $\gamma^A = -0.001\%$, and in country B is $\gamma^B = -1.368\%$, which proves our conclusion that manipulation does not bring any substantial benefit, and moreover, has a negative spillover effect.

Why we focus on the time consistent policy without commitment. (Bianchi and Mendoza (2018))

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One might be tempted to believe that it would be better not to manipulate the intertemporal TOT from country A's perspective. What if the social planner in country A promises to set zero tax permanently, starting from period t? Actually, social planner in country A would always tend to manipulate the world interest regardless of the behavior of country B. Agent in country B anticipates this tendency and would never believe this promise, keeping the expectation that country A will manipulate the world interest. Hence the fund supply function in period t would not change. Given this premise,

[2] Solving for $\gamma^i\left(F^A_t, Y_t\right)$ yields

$$\gamma^i \left(F^A_t, Y_t\right) = 100 \left(\left(\frac{V_{manip}\left(F^A_t, Y_t\right)}{V_{com}\left(F^A_t, Y_t\right)} \right)^{1/(1-\sigma)} - 1 \right) \tag{11}$$

where

$$V_{manip}\left(F^A_t, Y_t\right) = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{\left(c^i_{t+s}\right)^{1-\sigma} - 1}{1-\sigma} \middle| \left(F^A_t, Y_t\right) \right\}$$

$$V_{com}\left(F^A_t, Y_t\right) = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\left(\widetilde{c}^i_{t+j}\right)^{1-\sigma}}{1-\sigma} \middle| \left(F^A_t, Y_t\right) \right\}$$

Welfare Changes (%)	A in SS	A in MS	A in LS
Country A	-0.154%	-0.001%	0.300%
Country B	-0.545%	-1.368%	-3.379%

Table 1: Expected Welfare Gains For Each Country Under Different Country Scales

country A still has the incentive to manipulate in period t, thus the economy stays in the manipulation regime, and the promise of country A has no effect. Therefore, we would never find setting tax to zero is optimal when country A has the power to manipulate the world interest, unless there exists some method to credibly prohibit country A from manipulation, for example, a worldwide social planner would find setting zero tax be more desirable when maximizing the total welfare of country A and country B.

3.2.3 Country Scale

In this subsection we extend our model assumption to further study the effect of country scale. We assume that the scale of country A could have three types: small scale (SS), medium scale (MS) and large scale (LS), which correspond to three types of endowment shock. The total endowment is always kept at 1. If country A is small, agent in country A receive $x^L = 0.32$, $x^M = 0.4$ and $x^H = 0.48$ unit (20 percent deviation from x^M) of consumption good under low, medium and high endowment shock with equal probability of 1/3. Similarly, if country A is large, agent in country A receive $z^L = 0.48$, $z^M = 0.6$ and $z^H = 0.72$ unit (20 percent deviation from z^M) of consumption good under low, medium and high endowment shock with equal probability of 1/3. Finally if country A is in medium scale, the endowment shock coincides with our baseline settings.

Under our settings, table 1 shows the expected welfare change when country A is in different scale. When country A is in small scale, manipulation regime implies a welfare loss to the extent of $\gamma^{A,SS} = -0.154\%$ for country A and a welfare gain of $\gamma^{B,SS} = -0.545\%$ for country B, all comparing to the no tax competitive regime; when country A is in large scale, our results indicates the otherwise: a welfare improvement of country A by $\gamma^{A,LS} = 0.300\%$ and a welfare deterioration of country B by $\gamma^{B,LS} = -3.379\%$. Hence we conclude that country A would benefit from the increase in its endowment. The intuition is that a larger country scale of A implies a stronger power to intervene in the foreign bond market and thus manipulate the world interest, bringing more benefit for country A and being more harmful for country B; and a smaller country scale of A implies a stronger power of country B to defend itself from being manipulated hence country A benefit less from manipulation.

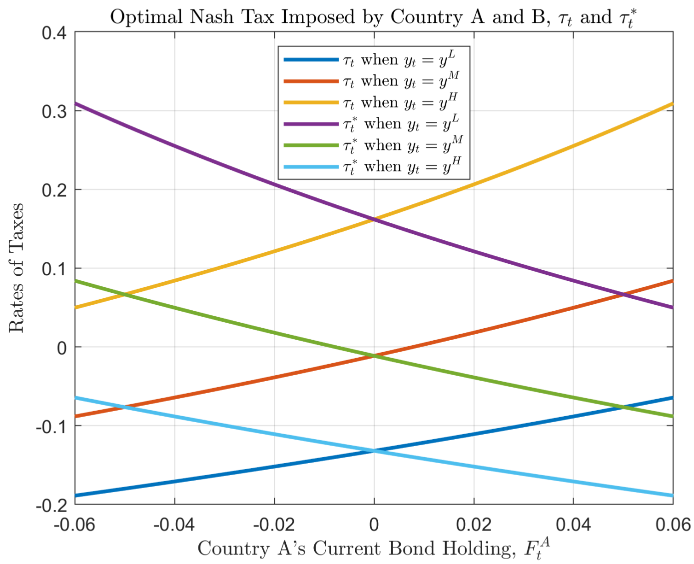

Figure 5: Optimal Nash Taxes

The policy enlightenment of our welfare analysis is that the welfare gain and the corresponding spillover effect of TOT manipulation depend crucially on the *relative* scale of countries. When two countries are roughly the same scale, manipulation may not bring any substantial welfare gain for country who manipulates, moreover, country which is manipulated would suffer an non-negligible welfare loss, i.e., manipulation always has a negative spillover effect. Hence from the perspective of global welfare (welfare summation of both countries), it may be unwise to impose taxation. A relatively large country who manipulates TOT would receive an welfare gain from manipulation, hence the taxation is favorable for a large country though the negative spillover effect is more severe than the roughly-same-scale case. For a relatively small country manipulation is not recommended since she would not benefit from the manipulation.

3.3 Bilateral Manipulation

In this subsection, we present our result of Nash equilibrium. Figure 5 plots the optimal taxation in Nash equilibrium. Due to the same scale and possibility of exogenous shocks of country A and B, we would observe the optimal taxation for both countries are exactly symmetric.

The expected welfare changes in our baseline model computed by 12 are $\gamma^{A,ne} = \gamma^{B,ne} = -1.096\%$. The incentive for both countries to manipulate the world interest rate would result in a welfare loss for both countries equally. In other words, it would be

Welfare Changes (%)	A in SS	A in MS	A in LS
Country A	-1.290%	-1.096%	-0.792%
Country B	-0.344%	-1.096%	-3.011%

Table 2: Expected Welfare Gains For Each Country Under Different Country Scales

welfare-improving for both countries not to impose any taxes on international borrowing or lending. However, this conclusion is on the premise of an identical country scale. Intuitively, when the country scale is not symmetric, larger country may benefit more or suffer an milder welfare loss from manipulation, because of a dominating power to bargain, while small country would be worse-off. Table 2 compares the welfare gains conditional on the small, medium and large scale of country A.

As expected, country scale do have an significant impact on the welfare changes. As expected, when A is relatively larger, she suffers an milder welfare loss of -0.792% while the welfare loss of B is -3.011%, significantly higher than the identical scale baseline. Conclusion is reversed when A is in smaller scale. Comparing to the results in table 1, we find that by imposing taxes as a "countermeasure", planner of country B could mitigate her welfare loss under all country scales comparing to the case when country A manipulates the TOT unilaterally. Due to this countermeasure, country A in large scale no longer receives a positive welfare gain and the welfare loss increases significantly when A is in smaller scale. Hence, though it is welfare-reducing globally when both countries impose taxes, from the perspective of the social planner in country B (or A as well[3]), when her counterpart manipulates TOT, imposing a tax is better than acting passively.

4 Introducing Intratemporal Terms of Trade

4.1 Model Setting

In this section we introduce intratemporal terms of trade into our model. There are two goods in our two-country model, denote as good a and b. Country A is only endowed with good a, y_t^a, and country is only endowed with b, y_t^b. The utility function of A and B

[3]Under our model settings, if country B unilaterally manipulates the world interest, the welfare gains of both countries are listed as below:

Welfare Changes (%)	A in SS	A in MS	A in LS
Country A	-1.464%	-1.368%	-1.247%
Country B	0.124%	-0.001%	-0.353%

are the same, $u(\cdot)$, and the utility is defined on the aggregate consumption $c_t^i\ (i = A, B)$, which follows an Armington-type CES aggregation:

$$c_t^i = \left[\omega^i \left(c_t^{i,a}\right)^{-\eta} + \left(1-\omega^i\right)\left(c_t^{i,b}\right)^{-\eta}\right]^{-\frac{1}{\eta}}$$

where $c_t^{i,j}$ denotes the type $j\ (j = a, b)$ of good consumed by country i. The elasticity of substitution is $1/\left(1+\eta\right)$. We use good b as a numeraire, and the relative price of good a is p_t. We assume that only country A has the ability to manipulate the interest rate and the relative price. The rest of our model specification is the same as section 2, except that the foreign bond is denominated in units of good b. Let $\mathbf{y}_t = \left[y_t^a, y_t^b\right]$ be the vector of endowments.

The budget constraints for A and B, denominated in units of b, are

$$p_t c_t^{A,a} + c_t^{A,b} + \frac{f_{t+1}^A}{R_t} = p_t y_t^a + f_t^A \tag{13}$$

$$p_t c_t^{B,a} + c_t^{B,b} + \frac{f_{t+1}^B}{R_t} = y_t^b + f_t^B \tag{14}$$

Market clearing conditions are

$$F_{t+1}^A + F_{t+1}^B = 0, \tag{15}$$

$$c_t^{A,a} + c_t^{B,a} = y_t^a, \tag{16}$$

$$c_t^{A,b} + c_t^{B,b} = y_t^b. \tag{17}$$

4.2 A Social planner's Problem

In this model setting, we also focus on the time-consistent policy, where the future policy rules coincide with the current policy rule. A benevolent social planner determines the optimal level of bond holdings and consumption on behalf of individual households. This can be achieved by credit operating and consumption quota. The first order conditions for B are

$$u'\left(c_t^B\right)\frac{\partial c_t^B}{\partial c_t^{B,b}} = \beta R_t \mathbb{E}_t\left[u'\left(c_{t+1}^B\right)\frac{\partial c_{t+1}^B}{\partial c_{t+1}^{B,b}}\right] \tag{18}$$

$$p_t = \frac{\omega^B}{(1-\omega^B)} \left(\frac{c_t^{B,b}}{c_t^{B,a}} \right)^{\eta+1} \tag{19}$$

State variables are $\{F_t^A, \mathbf{y}_t\}$. We omit the superscript F_t^A in our following expression. For individual agent in B, denote the future policy function of her consumption as $C^{B,a}(\cdot)$ and $C^{B,b}(\cdot)$, then $c_{t+1}^{B,a} = C^{B,a}(F_{t+1}, \mathbf{y}_{t+1})$, $c_{t+1}^{B,b} = C^{B,b}(F_{t+1}, \mathbf{y}_{t+1})$. Given state $\{F_t, \mathbf{y}_t\}$, the relative price p_t and the interest rate R_t, individual agent in B would have all she need to determine her allocation. Therefore, from 14, 18 and 19, we could solve for the optimal consumption decision $c_t^{B,j}$ $(j = a, b)$, and the foreign bond holding $f_{t+1}^B = F_{t+1}^B$, as functions of $\{R_t, p_t, F_t, \mathbf{y}_t\}$. Equivalently, we have relative price p_t and the interest rate R_t as function of $\left\{F_{t+1}^B, c_t^{B,a}, F_t, \mathbf{y}_t\right\}$ or $\left\{F_{t+1}^B, c_t^{B,b}, F_t, \mathbf{y}_t\right\}$.

If we use the good market clearing condition and the bond market clearing condition, we have p_t as function of $\left\{F_{t+1}, c_t^{A,a}, F_t, \mathbf{y}_t\right\}$ or $\left\{F_{t+1}, c_t^{A,b}, F_t, \mathbf{y}_t\right\}$ denoted as

$$p_t \equiv p^{A,a}\left(F_{t+1}, c_t^{A,a} \middle| F_t, \mathbf{y}_t\right), \tag{20}$$

or

$$p_t \equiv p^{A,b}\left(F_{t+1}, c_t^{A,b} \middle| F_t, \mathbf{y}_t\right), \tag{21}$$

respectively. Expression 20 and 21 are the *goods supply functions*. Note that any two of the equations 19, 20 and 21 implies the third. The good supply function states that the relative price across the world is affected not only by country A's consumption choice (we refer this as the *relative price effect* of consumption) *but also* foreign bond holding choice. The intuition is that, choices of foreign bond F_{t+1} will affect the interest rate R_t and furthermore the consumption-saving decision of B, and finally affect the unit of good a demanded by B and b supplied by B, leading to a corresponding change in the relative price p_t. We refer this effect of foreign bond holding on the relative price as the *intratemporal terms of trade effect*.

Also we have the interest rate R_t as function of $\left\{F_{t+1}, c_t^{A,a}, F_t, \mathbf{y}_t\right\}$ or $\left\{F_{t+1}, c_t^{A,b}, F_t, \mathbf{y}_t\right\}$, denoted as

$$R_t \equiv R^{s,a}\left(F_{t+1}, c_t^{A,a} \middle| F_t, \mathbf{y}_t\right). \tag{22}$$

or

$$R_t \equiv R^{s,b}\left(F_{t+1}, c_t^{A,b} \middle| F_t, \mathbf{y}_t\right). \tag{23}$$

Expression 22 and 23 are the *fund supply functions*. Note that any two of the equations 19, 22 and 23 implies the third. The inclusion of consumption leads to a difference from the fund supply in section 2. The intuition is that, changes in $c_t^{A,a}$ or $c_t^{A,b}$ will affect country B's consumption through market clearing condition, and finally affect the interest rate through the Euler equation of B. We refer this effect of consumption on the interest rate as the *interest rate effect* of consumption.

A social planner in A realizes her ability to manipulate both intertemporal and intratemporal terms of trade, R_t and p_t, i.e., she anticipates the equation 19, the good supply 20 and 21, and the fund supply 22 and 23, which are affected by her choice of $c_t^{B,j}$ $(j = a, b)$ and F_{t+1}, and maximizes the welfare of A.

We use the superscript "tt" to denote the case where country A unilaterally manipulates interest rate and the relative price, which is done by choosing the optimal F_{t+1} and $c_t^{A,a}$. Due to 19, this manipulation can also be implemented by choosing F_{t+1} and $c_t^{A,b}$. The recursive form of the social planner's problem is given by

$$V^{tt}\left(F_t, \mathbf{y}_t\right) = \max_{c_t^{A,a}, c_t^{A,b}, F_{t+1}} u\left(c_t^A\right) + \beta \mathbb{E}_t\left[V^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right] \tag{24}$$

because 19, constraints can be simplified as 13, 20 and 22. The first order condition is

$$c_t^{A,a}: \; u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,a}} + \lambda_t\left[\left(y_t^a - c_t^{A,a}\right)\frac{\partial p_t}{\partial c_t^{A,a}} - p_t + \frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial c_t^{A,a}}\right] = 0 \tag{25}$$

$$c_t^{A,b}: \; u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,b}} = \lambda_t \tag{26}$$

$$F_{t+1}: \; \lambda_t\left[-\frac{1}{R_t} + \frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial F_{t+1}} + \frac{\partial p_t}{\partial F_{t+1}}\left(y_t^a - c_t^{A,a}\right)\right] + \beta\frac{\partial \mathbb{E}_t\left[V^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]}{\partial F_{t+1}} = 0 \tag{27}$$

where λ_t is the Lagrange multiplier of 13. Notice that a novel term $\partial p_t / \partial F_{t+1} \times (y_t^a - c_t^a)$ represents the effect of F_{t+1} on *today's* relative price p_t, through the intratemporal terms of trade channel we mentioned before. Using Envelope theorem,

$$\frac{\partial \mathbb{E}_t V^{tt}\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}} = \mathbb{E}_t\left\{\Lambda^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\left[1 + \Omega^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right) + \Phi_t^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]\right\} \tag{28}$$

where

$$\Omega^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right) \equiv \frac{\partial p^{A,a}\left(F^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right), C^{A,a,tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right) \middle| F_{t+1}, \mathbf{y}_{t+1}\right)}{\partial F_{t+1}}\left(y_{t+1}^a - C^{A,a,tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right) \tag{29}$$

$$\Phi^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)\equiv\frac{F^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)}{\left[R^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)\right]^{2}}\frac{\partial R^{s,a}\left(F^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right),C^{A,a,tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)\middle|F_{t+1},\mathbf{y}_{t+1}\right)}{\partial F_{t+1}} \tag{30}$$

Term $\Phi^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)$ is the future intertemporal TOT effect as introduced in section 2. There is another novel term in 27, which is the *future intratemporal TOT effect* of today's foreign bond holding, $\Omega^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)$: changes in F_{t+1} now also affects the position of good supply curve in $t+1$, $\Omega^{tt}\left(F_{t+1},\mathbf{y}_{t+1}\right)$ represents the benefit from manipulating the future relative price times the future unit of goods exported to B.

We will show that the social planner's optimal allocation can be decentralized with lump-sum taxes in a competitive regime. To achieve the intertemporal and intratemporal manipulation, we need two type of taxes, one is the capital flow tax, the other is the consumption tax levied on consumption of a (or b). In Benigno et al. (2016), consumption tax is used to support the relative price aiming at alleviating the financial crisis caused by the binding borrowing constraint (see Mendoza (2010), Bianchi (2011), Bianchi and Mendoza (2018), Korinek). In our paper, consumption tax is used to manipulate the relative price in domestic favor, as many literature emphasized when studying the optimal tariff in international trade (see).

Specifically, suppose the planner has the policy combination of capital flow tax τ_t^{tt} and good a consumption tax τ_t^a. All tax revenue is rebated to individual. The individual agent's budget constraint is given by

$$p_t\left(1+\tau_t^a\right)c_t^{A,a}+c_t^{A,b}+\left(1+\tau_t^{tt}\right)\frac{f_{t+1}}{R_t}=p_t y_t^a+f_t+T_t^a+T_t^{tt} \tag{31}$$

with $T_t^a=p_t\tau_t^a c_t^{A,a}$ and $T_t^{tt}=\tau_t^{tt}\frac{F_{t+1}}{R_t}$.

Proposition 3a. The allocation of a social planner who manipulates both intertemporal and intratemporal terms of trade can be decentralized with a combination of capital flow tax τ_t^{tt} and consumption tax τ_t^a on good a in the form of

$$\tau_t^a=\underbrace{-\frac{c_t^{B,a}}{p_t}\frac{\partial p_t}{\partial c_t^{A,a}}}_{\textit{Relative price channel}}\underbrace{-\frac{F_{t+1}}{p_t R_t^2}\frac{\partial R_t}{\partial c_t^{A,a}}}_{\textit{Interest rate channel}} \tag{32}$$

$$
\begin{aligned}
\tau_t^{tt} = & \underbrace{-\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}}}_{Inter-TOT\,effect} \underbrace{-\frac{\partial p_t}{\partial F_{t+1}} R_t \left(y_t^a - c_t^{A,a}\right)}_{Intra-TOT\,effect} \\
& \underbrace{-\frac{\beta R_t \mathbb{E}_t \left\{\Lambda^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\Omega^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right\}}{\lambda_t}}_{future\,Intra-TOT\,effect} \underbrace{-\frac{\beta R_t \mathbb{E}_t \left\{\Lambda^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\Phi_t^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right\}}{\lambda_t}}_{future\,Inter-TOT\,effect}
\end{aligned} \tag{33}
$$

with tax revenue rebated to individual agent.

When the available policy combination is a capital flow tax τ_t^{tt} and a consumption tax imposed on good b consumption τ_t^b, the budget constraint of an individual is

$$p_t c_t^{A,a} + \left(1+\tau_t^b\right) c_t^{A,b} + \left(1+\tau_t^{tt}\right)\frac{f_{t+1}}{R_t} = p_t y_t^a + f_t + T_t^b + T_t^{tt} \tag{34}$$

Proposition 3b. The allocation of a social planner who manipulates both intertemporal and intratemporal terms of trade can be decentralized with a combination of capital flow tax τ_t^{tt} and consumption tax τ_t^b on good b in the form of

$$\tau_t^b = \underbrace{-c_t^{B,a}\frac{\partial p_t}{\partial c_t^{A,b}}}_{Relative\ price\ channel} \underbrace{-\frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial c_t^{A,b}}}_{Interest\ rate\ channel} \tag{35}$$

and 33, with tax revenue rebated to individual agent.

Proposition 3 also shows that the social planner's allocation cannot be decentralized with single policy instrument. Switching from tax imposed on good a consumption to good b do not affect the optimal capital flow tax.

We further investigate the welfare gain if there is only one single policy instrument at hand. Assume that the social planner only runs credit operation and let the good markets clear competitively. In this case, the social planner only has the capital flow tax as the single policy instrument. Denote the value function as $V^F\left(F_t, y_t\right)$, the planning problem would be

$$V^F\left(F_t, y_t\right) = \max_{c_t^{A,a}, c_t^{A,b}, F_{t+1}} u\left(c_t^A\right) + \beta \mathbb{E}_t\left[V^{tt}\left(F_{t+1}, y_{t+1}\right)\right]$$

subject to

$$p_t = \frac{\omega^A}{\left(1-\omega^A\right)}\left(\frac{c_t^{A,b}}{c_t^{A,a}}\right)^{\eta+1} \tag{36}$$

as well as 13, 20 and 22.

The first order conditions with respect to $c_t^{A,a}$ and $c_t^{A,b}$ are

$$c_t^{A,a}: \; u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,a}} + \lambda_t y_t^a \frac{\partial p_t}{\partial c_t^{A,a}} - \lambda_t \frac{\partial p_t}{\partial c_t^{A,a}} c_t^{A,a} - \lambda_t p_t + \lambda_t \frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial c_t^{A,a}} + \theta_t\left[\frac{\partial p_t}{\partial c_t^{A,a}} - \frac{\omega^A(1+\eta)}{(1-\omega^A)}\left(\frac{c_t^{A,b}}{c_t^{A,a}}\right)^{\eta}\frac{-c_t^{A,b}}{\left(c_t^{A,a}\right)^2}\right] = 0 \tag{37}$$

$$c_t^{A,b}: \; u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,b}} - \lambda_t - \theta_t \frac{\omega^A(1+\eta)}{(1-\omega^A)}\left(\frac{c_t^{A,b}}{c_t^{A,a}}\right)^{\eta}\frac{1}{c_t^{A,a}} = 0 \tag{38}$$

where θ_t is the Lagrangian multiplier of condition 36. The first order condition with respect to F_{t+1} is

$$F_{t+1}: \; \lambda_t\left(\frac{\partial p_t}{\partial F_{t+1}}\left(y_t^a - c_t^{A,a}\right) - \frac{1}{R_t} + \frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial F_{t+1}}\right) + \theta_t \frac{\partial p_t}{\partial F_{t+1}} + \beta \frac{\partial \mathbb{E}_t\left[V^F\left(F_{t+1}, y_{t+1}\right)\right]}{\partial F_{t+1}} = 0 \tag{39}$$

Envelope theorem implies

$$\frac{\partial \mathbb{E}_t V^F\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}} = \mathbb{E}_t\left\{\Lambda^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)\left[1 + \Omega^F\left(F_{t+1}, \mathbf{y}_{t+1}\right) + \Phi_t^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]\right\} + \mathbb{E}_t\left[\Theta\left(F_{t+1}, \mathbf{y}_{t+1}\right)\Psi\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right] \tag{40}$$

where the expressions of $\Omega^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)$ and $\Phi^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)$ are analogous to 29 and 30. Comparing to the Euler equation 27 and 28, there are two novel terms, $\theta_t \frac{\partial p_t}{\partial F_{t+1}}$ and $\Theta\Psi$, where Ψ is simply defined as

$$\Psi\left(F_{t+1}, \mathbf{y}_{t+1}\right) = \frac{\partial p^{A,a}\left(F^F\left(F_{t+1}, \mathbf{y}_{t+1}\right), C^{A,a,F}\left(F_{t+1}, \mathbf{y}_{t+1}\right) \middle| F_{t+1}, \mathbf{y}_{t+1}\right)}{\partial F_{t+1}} \tag{41}$$

These two terms, together with terms involving θ_t in 37 and 38, arise because the social planner now loses the control of domestic consumption and has to consider the effect of manipulation on domestic relative price. When social planner controls the domestic consumption or has the access to consumption tax, domestic and foreign relative price can be different. During manipulation, social planner only cares about the effect of manipulation on the foreign relative price p_t but not domestic, because the consumption tax will alter the domestic relative price to $p_t\left(1+\tau_t^a\right)$, which ensures the domestic individuals will choose the desired level of consumption for the purpose of manipulation. However,

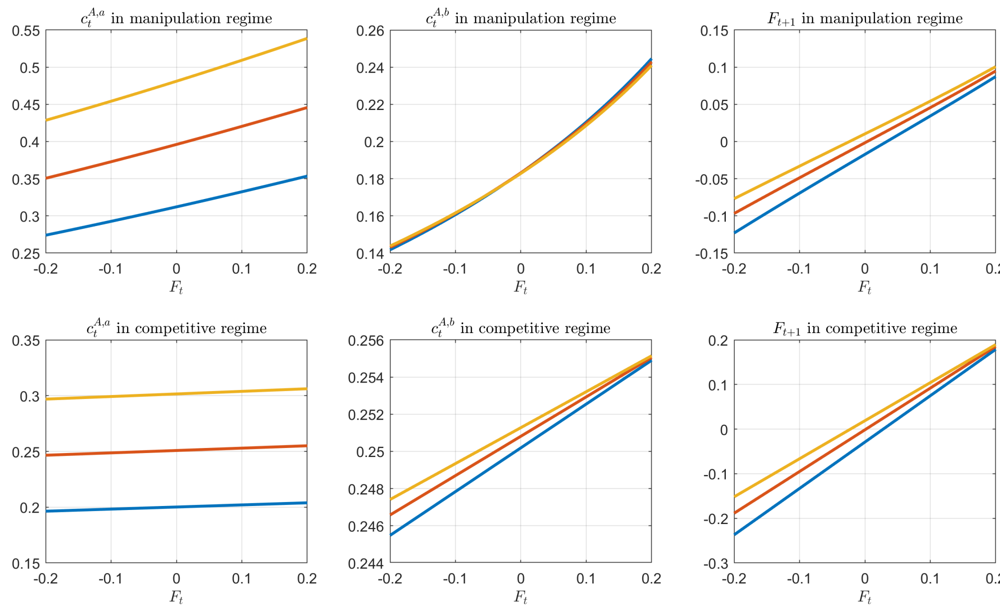

Figure 6: Policy Functions of Consumption and Bond Holding

when the consumption tax is absent, social planner has to consider the effect of manipulation on domestic relative price, since both domestic and foreign relative price must be equal. Proposition 4a describes the decentralization of this social planner's problem.

Proposition 4a. If there is only capital flow tax at hand, then the optimal policy rule is given by

$$\tau_t^F = -\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}} - \frac{\partial p_t}{\partial F_{t+1}} R_t \left(y_t^a - c_t^{A,a} \right) - \frac{\theta_t R_t \frac{\partial p_t}{\partial F_{t+1}} + \beta R_t \mathbb{E}_t \left\{ \Lambda^F \left[\Omega^F + \Phi_t^F \right] + \Theta \Psi \right\}}{\lambda_t} \tag{42}$$

where λ_t and θ_t are determined by 37 and 38.

5 Quantitative Analysis of Inter- and Intra-temporal TOT Manipulation

5.1 Double Policy Rules

Figure 6 demonstrates the policy function of consumption and bond holding, and figure 7 demonstrates the dynamics of relative price and interest rate, in both manipulation and no-tax competitive regime. Consumption in country $A(B)$ becomes higher(lower) in manipulation regime (the first row) for any pair of states $(F_t, \mathbf{y}_t)$. Foreign bond holding F_{t+1} drops in absolute value across states $(F_t, \mathbf{y}_t)$ because of the restriction on both capital

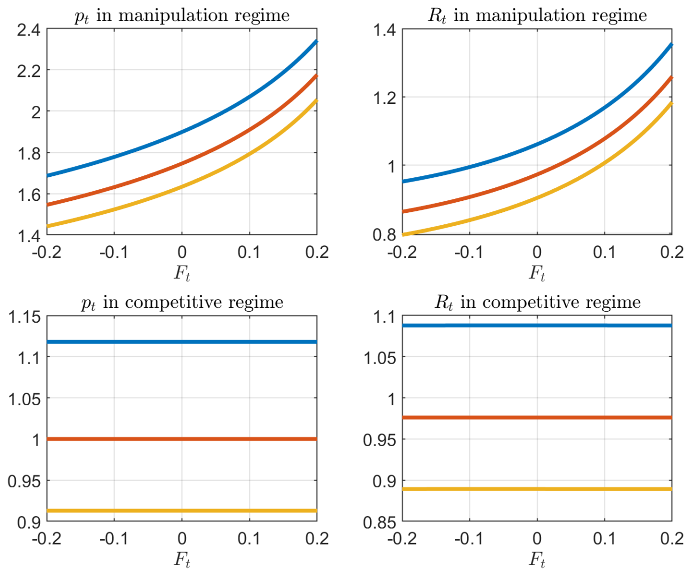

Figure 7: Policy Functions of Relative Price and Interest Rate

inflows and outflows.

Figure 8 shows the optimal consumption tax and capital flow tax, and their decomposition.

For consumption taxation τ_t^a, manipulating the relative price is the main consideration. Due to the market clearing condition, encouraging domestic demand of good a restricts the foreign consumption of good a,which will help boost its relative price, and country A can trade for more units of good b. The other component of τ_t^a is the concern about interest rate. Assume country A chooses $F_{t+1} > 0$ today, then from the perspective of interest rate manipulation, the social planner wants to raise the interest rate. This is done by encouraging domestic consumption of good a, depressing the consumption of country B due to the market clearing condition. From equation (18), country B would charge a higher interest rate in response to today's lower consumption.

For capital control tax τ_t^{tt}, manipulating contemporaneous interest rate is the main consideration. Capital control is also used to manipulate the contemporaneous and future intratemporal TOT, but with exactly opposite motivations. To manipulate contemporaneous intratemporal TOT, the social planner restricts capital inflow, leading to a lower interest rate, to which country B would increase consumption of $c_t^{B,b}$ in response (equation (18)). From (19), relative price rises. To manipulate future intratemporal TOT, the social planner restricts capital outflow, and country B thus have more resources tomorrow, followed by the downward shift of future fund supply curve. A lower interest rate would encourage country B's consumption of good b, again, from (19), relative price rises.

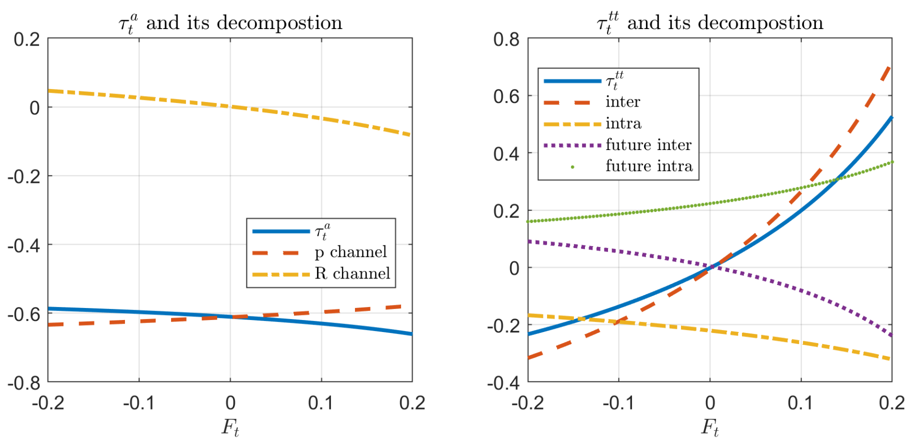

Figure 8: Optimal Taxes and Their Decomposition

Welfare Gain (%)	A in SS	A in MS	A in LS
Country A	11.37%	11.48%	11.52%
Country B	−19.54%	−21.60%	−23.34%

Table 3: Expected Welfare Gains For Each Country Under Different Country Scales

Table 3 presents the welfare gain/loss. Manipulating both inter- and intra-temporal terms of trade do have an significant welfare gain for country A (above 11% percent), at the cost of a huge welfare loss of country B (around 20% percent). Furthermore, country A only receives a mild welfare improvement from the increase in its country scale.

5.2 Single Policy Rule

Here we presents the numerical results when the social planner only has access to capital flow tax.As shown in table 4, comparing to the double policy case (table 3), the welfare gain of country A now becomes insignificant, suggesting the welfare gain in the double policy case mainly comes from the manipulation of intratemporal terms of trade rather than intertemporal. Again, consistent with the conclusion in section 3.2, intertemporal terms of trade manipulation brings little benefit but have non-negligible negative spillover effects.

Welfare Gain (%)	A in SS	A in MS	A in LS
Country A	−0.0093%	0.00036%	0.0058%
Country B	−0.1501%	−0.1700%	−0.1892%

Table 4: Expected Welfare Gains For Each Country Under Different Country Scales

6 Introducing the Borrowing Constraint

In this section, we begin to study the interaction of pecuniary externality and terms of trade manipulation, both of which are reasons that justify the control of capital flows (see). As many literature did, we impose an endogenous borrowing constraint on country A.

7 Conclusion

In this paper, we develop a simple theory of capital controls as dynamic terms-of-trade manipulation, when the policy maker cannot commit to future policies, and chooses current taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. In this situation, capital control not only affects the country's current terms-of-trade, but also affect its future terms-of-trade and future policy makers' optimal policies, and hence the optimal taxation is more complicated than the conventional view. Welfare analysis suggest that using capital control to manipulate term-of-trade not only reduces other countries welfare, i.e., it is a "beggar-thy-neighbor" policy, it is scarcely welfare-improving from the country's own perspective.

References

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2013). Financial crises and macro-prudential policies. *Journal of International Economics 89*(2), 453–470.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2016). Optimal capital controls and real exchange rate policies: A pecuniary externality perspective. *Journal of Monetary Economics 84*, 147–165.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. and E. G. Mendoza (2018). Optimal time-consistent macroprudential policy. *Journal of Political Economy 126*(2), 588–634.

Cao, D., W. Luo, and G. Nie (2020). Global dsge models. SSRN 3569013.

Costinot, A., G. Lorenzoni, and I. Werning (2014). A theory of capital controls as dynamic terms-of-trade manipulation. *Journal of Political Economy 122*(1), 77–128.

Davis, J. S. and M. B. Devereux (2022). Capital controls for crisis management policy in a global economy. *American Economic Journal: Macroeconomics 14*(1), 60–82.

Heathcote, J. and F. Perri (2016). On the desirability of capital controls. *IMF Economic Review 64*(1), 75–102.

Korinek, A. (2018). Regulating capital flows to emerging markets: An externality view. *Journal of International Economics 111*, 61–80.

Mendoza, E. (2010). Sudden stops, financial crises and leverage. *American Economic Reviews 100*(5), 1941–1966.

Schmitt-Grohé, S. and M. Uribe (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy 124*(5), 1466–1514.

8 Appendix

8.1 Proof of Proposition 1

Proof. Let superscript "c" denote the competitive regime. In the decentralized equilibrium, fund supply is determined by

$$u'\left(y_t + \frac{F_{t+1}}{R_t} - F_t\right) = \beta R_t \mathbb{E}_t\left[u'\left(C^{B,c}\left(F_{t+1}, y_{t+1}\right)\right)\right].$$

and the fund demand is determined by

$$u'\left(y_t - \frac{F_{t+1}}{R_t} + F_t\right) = \beta \frac{R_t}{1+\tau_t} \mathbb{E}_t\left[u'\left(C^{A,c}\left(F_{t+1}, y_{t+1}\right)\right)\right]. \tag{43}$$

If there exists a tax that decentralizes the social planner's equilibrium, i.e., a tax such that the policy rule of the competitive regime and the manipulation regime coincides, by comparing the Euler equation 43 5 and in both regime we must have

$$\tau_t^{sp} = -\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}} - \beta R_t \frac{\mathbb{E}_t\left\{u'\left[C^{A,m}\left(F_{t+1}, y_{t+1}\right)\right]\Phi^m\left(F_{t+1}, y_{t+1}\right)\right\}}{u'\left(c_t^A\right)}.$$

8.2 Time $t+1$ problem in Bilateral Manipulation

In time $t+1$, the recursive form of country A's social planner is

$$V^{ne}\left(F_{t+1}, y_{t+1}\right) = \max u'\left(c_{t+1}^A\right) + \beta V^{ne}\left(F_{t+2}, y_{t+2}\right).$$

$$c_{t+1}^A + \frac{F_{t+2}}{R_{t+1}} = y_{t+1} + F_{t+1}.$$

$$R_{t+1} = R^{s,ne}\left(F_{t+2}\middle|\tau_{t+1}^*\left(F_{t+1}, y_{t+1}\right), F_{t+1}, y_{t+1}\right).$$

Using Envelope theorem,

$$\frac{\partial V^{ne}\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}} = u'\left[C^{A,ne}\left(F_{t+1}, y_{t+1}\right)\right]\left[1 + \Phi^{ne}\left(F_{t+1}, y_{t+1}\right)\right].$$

where

$$\Phi^{ne}\left(F_{t+1}, y_{t+1}\right)=\frac{F^{ne}\left(F_{t+1}, y_{t+1}\right)}{\left[R^{ne}\left(F_{t+1}, y_{t+1}\right)\right]^{2}} \times$$
$$\left[\frac{\partial R^{s,ne}\left(F^{ne}\left(F_{t+1}, y_{t+1}\right) \middle| \tau_{t+1}^{*}\left(F_{t+1}, y_{t+1}\right), F_{t+1}, y_{t+1}\right)}{\partial \tau_{t+1}^{*}} \frac{\partial \tau_{t+1}^{*}\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}}+\frac{\partial R^{s,ne}\left(F^{ne}\left(F_{t+1}, y_{t+1}\right) \middle| \tau_{t+1}^{*}\right.}{\partial F_{t+1}}\right.$$

8.3 Proof of Proposition 3

First we prove proposition 3a. In a competitive regime, the first order conditions for A are

$$u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,a}}-\lambda_t p_t\left(1+\tau_t^a\right)=0$$

$$u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,b}}=\lambda_t$$

$$\lambda_t\left(1+\tau_t^{tt}\right)=\beta R_t \mathbb{E}_t\left[\Lambda^c\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]$$

Comparing with equation (25), (26), (27), if there exists a combination of taxation $\left\{\tau_t^{tt}, \tau_t^a\right\}$ such that the policy functions in the competitive regime coincide with those in the social planner's policy rule, then we must have

$$\tau_t^a=\frac{-\frac{\partial p_t}{\partial c_t^{A,a}}c_t^{B,a}-\frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial c_t^{A,a}}}{p_t}$$

$$\tau_t^{tt}=-\frac{F_{t+1}}{R_t}\frac{\partial R_t}{\partial F_{t+1}}-\frac{\partial p_t}{\partial F_{t+1}}R_t\left(y_t^a-c_t^{A,a}\right)-\frac{\beta R_t \mathbb{E}_t\left\{\Lambda^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\left[\Omega^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)+\Phi_t^{tt}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]\right\}}{u'\left(c_t^A\right)\frac{\partial c_t^A}{\partial c_t^{A,b}}}$$

For proposition 3b, proof is analogous.

8.4 Proof of Proposition 4

First we prove proposition 4a. When the capital flow tax be the single policy rule, FOC w.r.t F_{t+1} is

$$\beta\frac{\partial \mathbb{E}_t\left[V^{tt}\left(F_{t+1}, y_{t+1}\right)\right]}{\partial F_{t+1}}+\lambda_t\left(\frac{\partial p_t}{\partial F_{t+1}}\left(y_t^a-c_t^{A,a}\right)-\frac{1}{R_t}+\frac{F_{t+1}}{R_t^2}\frac{\partial R_t}{\partial F_{t+1}}\right)+\theta_t\frac{\partial p_t}{\partial F_{t+1}}=0$$

$$\frac{\partial \mathbb{E}_t V^F\left(F_{t+1}, y_{t+1}\right)}{\partial F_{t+1}} = \mathbb{E}_t \left\{ \Lambda^F\left(F_{t+1}, \mathbf{y}_{t+1}\right) \left[1 + \Omega^F\left(F_{t+1}, \mathbf{y}_{t+1}\right) + \Phi_t^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right] + \Theta\left(F_{t+1}, \mathbf{y}_{t+1}\right) \Psi\left(F_{t+\right.$$

where λ_t and θ_t are determined by 37 and 38. Comparing with the Euler equation in competitive regime,

$$\lambda_t \left(1 + \tau_t^F\right) = \beta R_t \mathbb{E}_t \left[\Lambda^F\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right]$$

we have the optimal taxation

$$\tau_t^F = -\frac{F_{t+1}}{R_t} \frac{\partial R_t}{\partial F_{t+1}} - \frac{\partial p_t}{\partial F_{t+1}} R_t \left(y_t^a - c_t^{A,a}\right) - \frac{\theta_t R_t \frac{\partial p_t}{\partial F_{t+1}} + \beta R_t \mathbb{E}_t \left\{\Lambda^F\left[\Omega^F + \Phi_t^F\right] + \Theta \Psi\right\}}{\lambda_t}$$

8.5 Computation Method

8.5.1 Model with Single Good

8.5.2 Model with Two Goods and Double Policy Instruments

For model with two goods, deriving for the derivatives of goods supply function and fund supply function, $p_t \equiv p^{A,a}\left(F_{t+1}, c_t^{A,a} \middle| F_t, \mathbf{y}_t\right)$ and $R_t \equiv R^{s,a}\left(F_{t+1}, c_t^{A,a} \middle| F_t, \mathbf{y}_t\right)$, is still the key to solve our model. The goods supply function and fund supply function are determined by

$$G_t = 0$$

$$H_t = 0$$

where G_t and H_t are obtained by plugging the policy rules $c_{t+1}^{B,a} = C^{B,a}\left(F_{t+1}, \mathbf{y}_{t+1}\right)$ and $c_{t+1}^{B,b} = C^{B,b}\left(F_{t+1}, \mathbf{y}_{t+1}\right)$ into equation (18), (19), and (14) and (16):

$$G_t \equiv \left(c_t^B\right)^{1+\eta-\sigma} \left(1 - \omega^B\right) \left(c_t^{B,b}\right)^{-\eta-1} - \beta R_t \mathbb{E}_t \left[\left(C_{t+1}^B\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right)^{1+\eta-\sigma} \left(1 - \omega^B\right) \left(c_{t+1}^{B,b}\left(F_{t+1}, \mathbf{y}_{t+1}\right)\right)^{-\eta-1}\right]$$

$$H_t \equiv p_t \left(y_t^a - c_t^{A,a}\right) + \left(y_t^a - c_t^{A,a}\right) \left[\frac{p_t\left(1 - \omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}} - \frac{F_{t+1}}{R_t} - y_t^b + F_t = 0$$

Define $\Pi\left(F_t, \mathbf{y}_t\right) \equiv \left(C_t^B\left(F_t, \mathbf{y}_t\right)\right)^{1+\eta-\sigma} \left(1 - \omega^B\right) \left(c_t^{B,b}\left(F_t, \mathbf{y}_t\right)\right)^{-\eta-1}$. The determinant of Jacobian matrix is

$$\mathcal{J} = \frac{\partial \left(G_t, H_t\right)}{\partial \left(p_t, R_t\right)} = \begin{vmatrix} \frac{\partial G_t}{\partial p_t} & \frac{\partial G_t}{\partial R_t} \\ \frac{\partial H_t}{\partial p_t} & \frac{\partial H_t}{\partial R_t} \end{vmatrix} \neq 0$$

Hence we have the following derivatives. For variable x $\left(x = F_{t+1}, c_t^{A,a}, F_t\right)$,

$$\frac{\partial p_t}{\partial x} = -\frac{1}{\mathcal{J}} \frac{\partial \left(G_t, H_t\right)}{\partial \left(x, R_t\right)}$$

$$\frac{\partial R_t}{\partial x} = -\frac{1}{\mathcal{J}} \frac{\partial \left(G_t, H_t\right)}{\partial \left(p_t, x\right)}$$

with

$$\frac{\partial G_t}{\partial F_{t+1}} = -\beta R_t \mathbb{E}_t \left[\frac{\partial \Pi \left(F_{t+1}, \mathbf{y}_{t+1}\right)}{\partial F_{t+1}}\right], \ \frac{\partial H_t}{\partial F_{t+1}} = -\frac{1}{R_t}$$

$$\begin{aligned} \frac{\partial G_t}{\partial c_t^{A,a}} = \left(-\frac{1+\eta-\sigma}{\eta}\right) \left(c_t^B\right)^{1+2\eta-\sigma} \left[\omega^B \eta \left(y_t^a - c_t^{A,a}\right)^{-\eta-1} - \eta \left(1-\omega^B\right) \left(c_t^{B,b}\right)^{-\eta-1} \frac{\partial c_t^{B,b}}{\partial c_t^{A,a}}\right] \left(1-\omega^B\right) \\ + \left(c_t^B\right)^{1+\eta-\sigma} \left(1-\omega^B\right) \left(-\eta-1\right) \left(c_t^{B,b}\right)^{-\eta-2} \frac{\partial c_t^{B,b}}{\partial c_t^{A,a}} \end{aligned}$$

$$\frac{\partial c_t^{B,b}}{\partial c_t^{A,a}} = -\left[\frac{p_t \left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}}$$

$$\frac{\partial H_t}{\partial c_t^{A,a}} = -p_t - \left[\frac{p_t \left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}}$$

$$\frac{\partial G_t}{\partial F_t} = 0, \ \frac{\partial H_t}{\partial F_t} = 1$$

Our iteration starts from a last period problem where there is no bond trading, hence social planner in A only has an incentive to manipulate the intratemporal TOT. Country A solves

$$\max u\left(c^A\right)$$

where

$$c^A = \left[\omega^A \left(c_t^{A,a}\right)^{-\eta} + \left(1-\omega^A\right) \left(c_t^{A,b}\right)^{-\eta}\right]^{-\frac{1}{\eta}},$$

subject to the budget

$$pc^{A,a} + c^{A,b} = py^a + F \tag{44}$$

and goods supply function $p = p^{A,a}\left(c_t^{A,a} \middle| F_t, \mathbf{y}_t\right)$, which is obtained by combining (46), (45) and (47):

$$p = \frac{\omega^B}{(1-\omega^B)}\left(\frac{c^{B,b}}{c^{B,a}}\right)^{\eta+1} \tag{45}$$

$$pc^{B,a} + c^{B,b} = y^b - F \tag{46}$$

$$c^{A,a} + c^{B,a} = y^a \tag{47}$$

The explicit form of the goods supply is given by:

$$p\left(y^a - c^{A,a}\right) + \left(y^a - c^{A,a}\right)\left[\frac{p\left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}} = y^b - F$$

First order conditions are given by:

$$\left(c^A\right)^{1+\eta-\sigma}\omega^A\left(c^{A,a}\right)^{-\eta-1} + \gamma\left(p + \left[\frac{p\left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}}\right) - \lambda p = 0$$

$$\left(c^A\right)^{1+\eta-\sigma}\left(1-\omega^A\right)\left(c^{A,b}\right)^{-\eta-1} - \lambda = 0$$

$$\gamma\left[1 + \left[\frac{\left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}}\frac{1}{1+\eta}p^{\frac{1}{1+\eta}-1}\right] = \lambda$$

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8.5.3 Model with Two Goods and Single Policy Instrument

Starting from a competitive regime. Good supply function $p = p^{A,a}\left(c^{A,a} \middle| F, \mathbf{y}\right)$ is written implicitly as

$$p\left(y^a - c^{A,a}\right) + \left(y^a - c^{A,a}\right)\left[\frac{p\left(1-\omega^B\right)}{\omega^B}\right]^{\frac{1}{1+\eta}} = y^b - F$$

Hence we have an initial value of $\Psi\left(F,\mathbf{y}\right)$:

$$\frac{\partial p}{\partial c^{A,a}}=-\frac{-p-\left[\frac{p\left(1-\omega^{B}\right)}{\omega^{B}}\right]^{\frac{1}{1+\eta}}}{\left(y^{a}-c^{A,a}\right)+\left(y^{a}-c^{A,a}\right)\left[\frac{\left(1-\omega^{B}\right)}{\omega^{B}}\right]^{\frac{1}{1+\eta}}\frac{1}{1+\eta}\left(p\right)^{\frac{1}{1+\eta}-1}}$$

Also we can have an initial value of $\Omega\left(F,\mathbf{y}\right)$:

$$\Omega\left(F,\mathbf{y}\right)=\frac{\partial p}{\partial F}\left(y^{a}-c^{A,a}\right)=\frac{1}{-1-\left[\frac{\left(1-\omega^{B}\right)}{\omega^{B}}\right]^{\frac{1}{1+\eta}}\frac{1}{1+\eta}p^{\frac{1}{1+\eta}-1}}$$

$$\Psi\left(F,\mathbf{y}\right)=\frac{\partial p}{\partial F}$$

And we backout the initial value of λ and θ according to (after setting $F_{t+1}=0$):

$$u'\left(c_{t}^{A}\right)\frac{\partial c_{t}^{A}}{\partial c_{t}^{A,a}}+\lambda_{t}y_{t}^{a}\frac{\partial p_{t}}{\partial c_{t}^{A,a}}-\lambda_{t}\frac{\partial p_{t}}{\partial c_{t}^{A,a}}c_{t}^{A,a}-\lambda_{t}p_{t}+\theta_{t}\left[\frac{\partial p_{t}}{\partial c_{t}^{A,a}}-\frac{\omega^{A}\left(1+\eta\right)}{\left(1-\omega^{A}\right)}\left(\frac{c_{t}^{A,b}}{c_{t}^{A,a}}\right)^{\eta}\frac{-c_{t}^{A,b}}{\left(c_{t}^{A,a}\right)^{2}}\right]=0$$

$$u'\left(c_{t}^{A}\right)\frac{\partial c_{t}^{A}}{\partial c_{t}^{A,b}}-\lambda_{t}-\theta_{t}\frac{\omega^{A}\left(1+\eta\right)}{\left(1-\omega^{A}\right)}\left(\frac{c_{t}^{A,b}}{c_{t}^{A,a}}\right)^{\eta}\frac{1}{c_{t}^{A,a}}=0$$