Debt Dilution, Debt Covenants, and Macroeconomic Fluctuations *

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February 26, 2024

preliminary and incomplete,
empirical analysis is in progress

Abstract

Debt covenants are pervasive in debt contracts. To prevent the dilution of existing debt, most creditors set covenants of a maximum debt-to-earnings ratio for borrowing firms. In this paper, to study its macroeconomic implications, we embed debt covenants into a workhorse real business cycle model with defaultable long-term debt. In our model, creditors penalize firms when covenants are violated. We show such a mechanism that covenants significantly reduce debt dilution and default over the business cycles. Furthermore, reduced debt dilution due to covenants also mitigates the debt overhang problem and thus boosts capital accumulation. Compared to counterfactual economies without covenants, the baseline economy with covenants experiences endogenous stabilization of macroeconomic shocks and higher levels of capital, output, and consumption.

Keywords: Debt Covenants; Debt Dilution; Financial Frictions; Business Cycle;
JEL Classification: E32, E44, G32

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*We thank Kim Ruhl and Charles Engel for their comments and suggestions. All errors are ours.
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1 Introduction

Financial frictions have long been recognized as an essential component of macroeconomic fluctuations. An extensive literature has studied how one friction, the inability to commit to debt repayment, affects firm and macroeconomic dynamics (Bernanke, Gertler, and Gilchrist, 1999; Cooley and Quadrini, 2001; Albuquerque and Hopenhayn, 2004; Arellano, Bai, and Kehoe, 2019; Jungherr and Schott, 2022).[1] Commitment is particularly important as most corporate debt is long-term. Conditional on not defaulting today, in many models firms are unable to commit to not diluting the value of existing debt through future borrowing. If creditors cannot intervene in such cases, severe macroeconomic consequences result, especially during economic downturns.

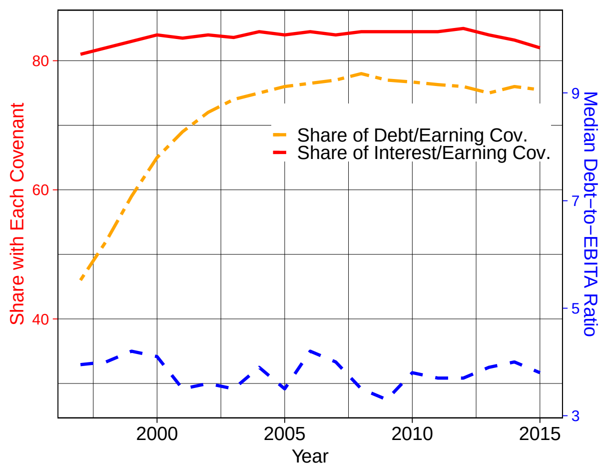
But in reality, creditors do act. To prevent dilution of existing debt, most creditors usually set covenants enforcing a maximum debt- or (interest)-to-earnings ratio for borrowing firms *(debt-to-earnings ratio, for short).* As documented by Lian and Ma (2021), 80% of U.S. corporate borrowing by non-financial firms (by value) utilizes debt covenants. These debt covenants have a long history spanning millennia. One of the earliest recorded instances of debt covenants is found in the Code of Hammurabi, a set of ancient Babylonian laws from around 1754 BCE.[2]

The purpose of debt covenants is **not to place an additional burden on the borrower but to align the interests of the lender and the borrower if necessary**, especially in difficult situations.[3] Figure 1 and Table 1 below provide some well-documented stylized facts concerning debt covenants from an emerging literature (Lian and Ma, 2021; Greenwald, 2019; Adler, 2020; Drechsel, 2023; Chodorow-Reich and Falato, 2022). Most covenants limit debt-to-earnings or interest-to-earnings ratios, which are quite stable over the business cycle (Figure 1). However, covenants are frequently violated, especially during the Great Financial Crisis (Table 1). Firms that violate covenants tend to be more financially distressed. This evidence reminds us that debt covenants are common tools in corporate financial markets and potentially play an essential role during economic downturns. Natural questions are whether debt covenants amplify or stabilize the business cycle during economic downturns, especially given the corresponding spikes in covenant violation rates, and whether debt covenants are in general beneficial or not.

[1]There is also an extensive literature, e.g., Bernanke and Gertler (1995) and Kiyotaki and Moore (1997) among many others, focusing on how collateral addresses shareholder expropriations of lenders' wealth and the business cycle consequences. We focus, however, on the defaultable debt framework.

[2]These laws contained provisions related to lending and borrowing, including the terms and conditions under which loans were made. While not referred to as "covenants" in the modern sense, these provisions served a similar purpose by outlining the obligations and consequences for borrowers who failed to meet them.

[3]The role of debt covenants in this paper is to mitigate commitment issues between shareholders and creditors when long-term debt contracts are signed. In other work, i.e., in Lian and Ma (2021) with earning-based borrowing constraints when debt dilution plays no role, earning-based borrowing constraints allow fast-growing firms to borrow more endogenously. All of these align the interests of the principal and the agent.



Panel A: Do Debt Covenants Bind?		
Annual Violation Rate	Data	Source
Threshold Reached	23%	Own Calculation
Text-Based SEC-filling	10%	Adler (2020)
During 08-09 GFC	33%	CR&F (2022)
Panel B: Selected Firm Characteristics from Adler (2020)		
Median Measure	Violation	Non-Violation
Investment Rate	3.3%	4.3%
Debt-to-EBITA	464.4%	182.9%
Leverage	46.0%	26.4%
EBITA/Asset	10.6%	16.4%
Cash Flow/Assets	5.3%	10.4%
Market-to-Book Value	117.2%	149.9%
Log Net Worth	5.9	6.4

Figure 1: Debt Covenant Conditions

Table 1: Debt Covenant Violations

Notes: Figure 1 and Table 1 combine various sources of motivating empirical evidence from the literature and the underlying Dealscan and Compustat datasets. Since we are not the first to document these facts, we refer to the literature documenting each below. Figure 1 displays the share of firms (left) with that particular covenant type among firms that Dealscan reports have at least one covenant from Greenwald (2019), and the median debt-to-EBITA ratio (right) as in Drechsel (2023). Table 1 shows statistics on covenant violations. CR&F stands for Chodorow-Reich and Falato (2022). Covenant violations are common; many firms reach the threshold annually, especially during financial crises. Also, according to Adler (2020), violating firms have lower investment, earnings, cash flows, market-to-book value, and wealth, along with more debt and much higher debt-to-earning ratios.

In this paper, we revisit the macroeconomic effects of debt in the presence of debt covenants. We build a business cycle model of production, long-term firm debt with covenants, and costly default based on the framework of Jungherr and Schott (2022). Holding financial conditions constant, we show that introducing debt covenants into a real business cycle model reduces business cycle volatility. Debt covenants also significantly ease the severity of debt overhang issues and increase potential capital accumulation, output, and consumption.

The key mechanism is that debt covenants significantly reduce debt dilution. Without debt covenants, firms have a strong incentive to dilute existing debt during economic downturns as shareholders are unwilling to reduce debt and correspondingly default risk since the benefits would mostly accrue to creditors. This worsens financial conditions and amplifies recessions. This debt dilution mechanism is well-documented in Jungherr and Schott (2022) for corporate debt and in Hatchondo, Martinez, and Sosa-Padilla (2016) for sovereign debt.

With debt covenants, an intermediate layer of covenant violation, or "technical default," is introduced between repaying and costly full default. Now, firms cannot easily dilute existing debt due to covenant penalties but are incentivized to remain financially healthy by keeping a relatively low debt-to-earnings ratio. Covenant violation still occurs frequently since "technical default" is much less costly than a true default. Less debt dilution improves financial conditions, reflected in higher debt prices and lower credit spreads, and mitigates recessions.

Furthermore, debt covenants boost capital accumulation and, therefore, the long-run levels of output and consumption. Without debt covenants, the debt dilution incentives facing shareholders create severe debt overhang problems since creditors ask for a higher credit spread, even when the firm is financially healthy. This dissuades profitable investments since earnings from such projects would largely go to debt holders. With debt covenants, reduced debt dilution mitigates the debt overhang problem. Creditors accept lower rates, especially for financially healthy firms, inducing investment as shareholders now keep most earnings from new projects. Therefore, holding financial constant, debt covenants increase long-run economic performance.

We quantitatively take our baseline business cycle model with both debt dilution and debt covenants to U.S. data and compare it to business cycle models with only debt dilution and with neither. For the model with only debt dilution, we recalibrate the model to the same leverage and credit spread moments for a fair comparison. For the model with neither, we assume a constrained efficiency model in which a social planner maximizes shareholder and creditor value. The constrained efficiency model completely resolves the debt dilution problem; consequently, debt covenants play no role. The constrained efficiency model has less leverage, lower credit spreads, and higher output, capital, and consumption.

Comparing all three models, we quantitatively show that debt covenants mitigate debt dilution by reducing desired leverage, increasing debt prices, and reducing credit spreads conditional on the outstanding debt level. More importantly, mitigation of debt dilution significantly reduces the countercyclical movements in leverage following a negative productivity shock. In response to a negative 2% TFP shock, the baseline model has a leverage spike of 4.2% instead of 7% in the model without debt covenants. Consequently, output and capital dropped by 8% and 6.8% instead of by 9.5% and 9.2% without debt covenants. In contrast, the constrained efficiency model has the smallest recessionary effects, as expected, absent any dilution problems. The presence of debt covenants helps stabilize the amplified recessions caused by debt dilution. We find that debt covenants also reduce business cycle asymmetry.

Finally, we show the long-run effects on economic performance with debt covenants. The model with debt covenants generates higher output, capital, and consumption than the model without debt covenants due to reduced financial frictions between firms and creditors. Although both models have the same mean leverage and credit spread, the model with debt covenants can maintain a 5% larger capital stock, 2% higher output, and 1.5% higher consumption.

Literature. Our paper contributes to two strands of literature. First, this paper is related to the broader literature of studies on financial frictions and their implications for the aggregate economy. In most work in this literature, debt dilution is not an issue since debt contracts are only short-term.[4] However, the literature shows that most corporate debt is long-term, which makes debt dilution a serious commitment issue between shareholders and creditors. As documented in Adrian, Colla, and Song Shin (2013), the average term to maturity is three to four years for bank loans and more than eight years for corporate bonds. Recent macroeconomic literature (Gomes, Jermann, and Schmid, 2016; Jungherr and Schott, 2021, 2022; Jungherr et al., 2022; Deng and Fang, 2022; Poeschl, 2023) shows that long-term debt commitment issues matter for macroeconomic dynamics and monetary policy.[5] The paper we build on, Jungherr and Schott (2022), shows that debt dilution generates countercyclical leverage, amplified output volatility, and prolonged recessions. We show that the existence of debt covenants partially resolves the issue of debt dilution, reduces the volatility of leverage and output, and shortens recessions. Meanwhile, reduced debt dilution due to debt covenants also mitigates the debt overhang problem and thus boosts capital accumulation, output, and consumption.

The second strand of literature investigates debt covenants in finance and macroeconomics. A large finance literature has empirically and clearly documented the existence of debt covenants and the effects of covenant violations on firm-level outcomes (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012; Roberts, 2015; Falato and Liang, 2016; Chodorow-Reich and Falato, 2022; Ersahin, Irani, and Le, 2021). More recently, macro-finance literature has started to focus on the micro and macro effects of debt covenants outside of covenant violations (earning-based borrowing constraints) both empirically and theoretically (Lian and Ma, 2021; Adler, 2020; Greenwald, 2019; Drechsel, 2023; Öztürk, 2022).[6] These five papers provide rich empirical evidence and quantitative implications, with Lian and Ma (2021) and Adler (2020) focusing more on empirical findings, and Drechsel (2023), Greenwald (2019), and Öztürk (2022) focusing more on quantitative implications.

These papers model debt covenants as earning-based borrowing constraints in which corporate debt is short-term, so there are no debt dilution or debt overhang issues for creditors. Lian and Ma (2021) shows that cash flows in the form of operating earnings can directly relax borrow-

[4]Including many influential papers such as Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Jermann and Quadrini (2012), Khan and Thomas (2013), Gilchrist and Zakrajšek (2012), and Arellano, Bai, and Kehoe (2019).

[5]There is a large sovereign default literature studying debt dilution and commitment, including Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo, Martinez, and Sosa-Padilla (2016), and Aguiar et al. (2019). The most closely related to us is Hatchondo, Martinez, and Sosa-Padilla (2016), which shows that if debt covenants existed for a sovereign, sovereign debt dilution could be largely resolved with corresponding welfare gains. Literature in corporate finance, e.g., Admati et al. (2018) and DeMarzo and He (2021), also considers such commitment issues. Literature in financial intermediation, e.g., Corhay and Tong (2021), examines commitment issues (profit shifting) between firms and banks due to inflation shocks.

[6]There are other important recent works on debt covenants, including Nikolov, Schmid, and Steri (2019), Xiang (2023), Gamba and Mao (2020), Davydenko, Elkamhi, and Salerno (2020), and Arnold and Westermann (2023).

ing constraints and consequently make firms less vulnerable to collateral prices. Adler (2020) focuses on the precautionary effect of covenants that reduce aggregate investment. Drechsel (2023) shows that earning-based borrowing constraints amplify the transmission of supply shocks to output. Greenwald (2019) shows that interest coverage covenants amplify monetary policy due to changes in coverage limits. In contrast, Öztürk (2022) focuses on changes in collateral value and shows that asset-based borrowers are more responsive than cash-flow-based borrowers to monetary policy. Our paper focuses on the essential role of debt covenants in preventing debt dilution and easing debt overhang, improving economic performance in terms of reduced business cycle volatility and increased long-run output level.

Roadmap. The rest of the paper is organized as follows. Section 2 exhibits the full model. Section 3 takes the model to the data. Section 4 illustrates the mechanism and properties of the model. Finally, Section 5 concludes.

2 The Model

We build a dynamic open economy business cycle model with firm production and financing following the long-term debt model in Jungherr and Schott (2022). We extend their framework in two ways. First, we add debt covenants. Second, we modify their capital quality shocks to match observed covenant violations and credit spread in the data. The international risk-free rate r is fixed as in Arellano, Bai, and Kehoe (2019).

2.1 Firm Setup

Production and Earning A firm i uses capital k_{it} and labor l_{it} to produce output according to $y_{it} = z_t(k_{it}^{\psi} l_{it}^{1-\psi})^{\zeta}$, where $\psi, \zeta \in (0,1)$. The natural logarithm of aggregate revenue productivity z_t follows an AR(1) process and is realized at the beginning of period t. The firm's earnings before interest, taxes, depreciation, and amortization (EBITDA) are then given by

$$\text{EBITDA}_{it} = y_{it} + \epsilon_{it}^{n} k_{it} - w_t l_{it} \tag{1}$$

where ϵ_{it}^{n} is the part of the capital quality shock realized in current cash flow as described below.

Capital Quality Shock The firm also faces an i.i.d. idiosyncratic capital quality shock ϵ_{it} in period t. To incorporate the fact that sometimes the capital quality shock is realized in the current period's earnings and sometimes is placed in future accounting recapitalization, we separate the capital quality shock into two parts: $\epsilon_{it} = \epsilon_{it}^{n} + \epsilon_{it}^{f}$, where the current ("now") component ϵ_{it}^{n}

follows a continuous normal probability distribution with σ_ϵ^n and the future component ϵ_{it}^t follows a continuous normal probability distribution with σ_ϵ^f. Without loss of generality, we assume half-half opportunities, that is, with 50% probability, the realized capital quality shock is reflected in the current period cash flow. We modify the capital quality shock to match additional moments concerning debt covenants. We denote the cumulative distribution by $\Phi(\epsilon_{it})$. Going forward, we refer to ϵ_{it}^n as cash flow shock volatility and ϵ_{it}^f as future capital quality shock volatility.

Firm Financing The firm can finance capital with equity and long-term debt. We model long-term debt using the computationally tractable specification from Leland (1994). A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment c and a fraction $\gamma \in (0, 1)$ of the principal in period t, implying total debt payments of $(c+\gamma)b_{it-1}$.

In period t, fraction $1-\gamma$ of the bond remains outstanding. Payments decay geometrically over time. The number of long-term bonds chosen by the firm in period t is b_{it}. The firm can issue equity freely at no cost but with a lower bound $e_{it} \geq -\underline{e}$ where $\underline{e} > 0$ means firms cannot finance through equity Ponzi games. The firm also has limited liability. Shareholders are free to default and hand over the firm's assets to creditors for liquidation. In this case, a fixed fraction ξ of firm assets is lost during the fire sale of assets or restructuring.

Debt Covenants Long-term debt has covenants. The creditor requires the firm to satisfy specific requirements if a certain level of earnings-to-debt ratio is breached. Specifically, we assume:

$$\frac{\text{EBITDA}_{it}}{b_{it}} \geq \frac{1}{\eta} \tag{2}$$

where $1/\eta$ is the lowest earnings-to-debt ratio that the creditor tolerates. When a debt is violated, the creditor punishes the firm with a financial penalty of χb_{it}, which is proportional to its current debt position. Such a reduced-form penalty summarizes various forms of debt covenant violation penalties that are usually proportional to the size of the debt, including renegotiation of higher interest rates, up-front fees in exchange for waiving the covenant violation, or immediate repayment of the debt.[7]

[7] As Greenwald (2019) *Background: Debt Covenants* describes: "In practice, lenders typically do not demand full repayment upon violation, but instead renegotiate the terms of the loan, often extracting some concession in the form of a higher interest rate or up-front fee in exchange for waiving the covenant violation." We also intentionally choose not to model the penalty as "repayment acceleration" which would shorten the maturity λ when covenants are violated since the shortening the maturity would mechanically reduce debt dilution issues. Jiang and Xu (2019) provides direct empirical evidence that paying covenant amendment fees after covenant violations is a common practice. They also show that there is a significantly positive real value added by creditors taking explicit actions intervening in the operation of borrowers in covenant violations.

2.2 Firms' Recursive Problem

Denote the aggregate state as S_t in period t, $n(z_{it}, k_{it}, b_{it}, \epsilon^n_{it}, \epsilon^f_{it})$ as the end-of-period net worth, and $V((1-\gamma)b_t, S_t)$ as the continuation value. The after-tax, interest, and depreciation end-of-period net worth formulas are as follows:

$$n_{it} = (1-\tau)[\underbrace{y_{it} + \epsilon^n_{it}k_{it} - w_t l_{it}}_{\text{EBITDA}_{it}}] + \underbrace{[1 + (1-\tau)(\epsilon^f_{it} - \delta)]k_{it}}_{\text{capital stock}} - \underbrace{[(1-\tau)c + \gamma + \chi \cdot \mathbf{1}_{CV}]b_{it}}_{\text{debt burden}} - f \tag{3}$$

where $\mathbf{1}_{CV} = 1$ indicates covenant violation; otherwise $\mathbf{1}_{CV} = 0$. f is a fixed cost of operation.

Firms will maximize shareholder value and discount cash flows at the international risk-free rate r. Conditional on not defaulting, shareholder value at the end of period $t-1$ can be written as the sum of net worth and a continuation value term that depends on future firm behavior: $n_{it-1} + V(b_{it-1}, S_{t-1})$. Because there are no equity adjustment costs, the net worth, n_{it-1}, does not affect optimal firm policy or the value $V(b_{it-1}, S_{t-1})$. We write the equity value at time t with three states:

$$\begin{cases} n(z_{it}, k_{it}, b_{it}, \epsilon^n_{it}, \epsilon^f_{it}) + V((1-\gamma)b_{it}, S_t) & \text{if } \epsilon^n_{it} \geq \epsilon^c_{it}, \left(\frac{\text{EBITDA}_{it}}{b_{it-1}} \geq \frac{1}{\eta}\right) \\ n(z_{it}, k_{it}, b_{it}, \epsilon^n_{it}, \epsilon^f_{it}) + V((1-\gamma)b_{it}, S_t) & \text{if } \epsilon^d_{it} - \epsilon^f_{it} < \epsilon^n_{it} < \epsilon^c_{it}, \text{(Covenant violation)} \\ 0 & \text{if } \epsilon_{it} \leq \epsilon^d_{it}, \text{(Default)} \end{cases} \tag{4}$$

where ϵ^c_{it} and ϵ^d_{it} are the capital quality shock cutoffs that trigger a debt covenant violation and default, respectively. Since the effect of the realizations of ϵ^n_{it} and ϵ^f_{it} is isomorphic to the net worth $n(z_{it}, k_{it}, b_{it}, \epsilon^n_{it}, \epsilon^f_{it})$ contain all necessary information, in the case of the default cutoff, we only need to care about the total capital quality shock ϵ_{it} and its corresponding net worth $n(z_{it}, k_{it}, b_{it}, \epsilon_{it})$. The capital quality shock cutoffs that trigger a debt covenant violation, ϵ^c_{it}, and default, ϵ^d_{it}, are:

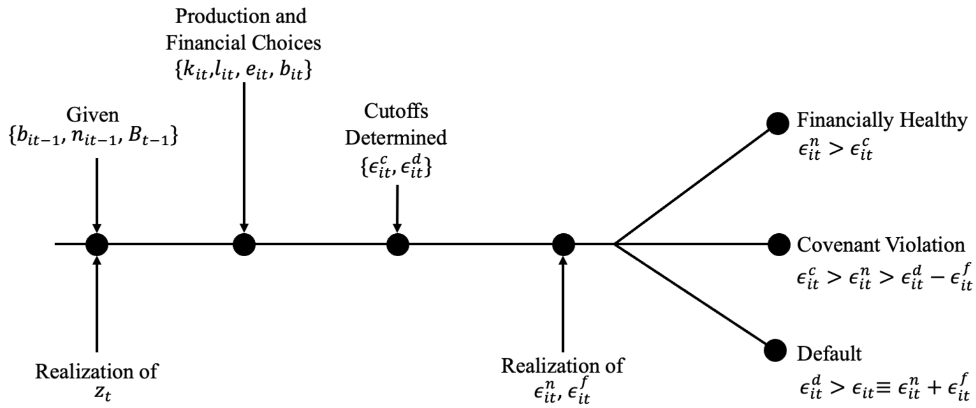
$$(y_{it} - \epsilon^c_{it}k_{it} - w_t l_{it}) \times \eta - b_{it} = 0 \tag{5}$$

$$n(z_{it}, k_{it}, b_{it}, \epsilon^d_{it}) + E_{S_t|S_{t-1}}\left[V((1-\gamma)b_{it}, S_t)\right] = 0 \tag{6}$$

where $\epsilon^d_{it} < \epsilon^c_{it}$ always holds since creditors will never ask for debt covenant conditions that only trigger after firm default. Note that both cutoffs $\epsilon^c(k_{it}, b_{it}, S_t)$ and $\epsilon^d(k_{it}, b_{it}, S_t)$ are determined by the firm's choice of capital k_{it}, and debt b_{it}, along with the aggregate state S_t.

The timing within period t is illustrated in Figure 2 and is as follows. At the beginning of period t, a firm carries an individual debt burden b_{it-1} in period $t-1$ and an individual net worth

Figure 2: The Timing of A Firm's Decision



n_{it-1}. The economy is characterized by aggregate firm debt B_{t-1} and an aggregate productivity z_t that arrives before the firms make choices. B_{t-1} and z_t forms the aggregate state $S_{t-1}(z_t, B_{t-1})$. The firm i then chooses labor l_{it}, capital k_{it}, equity issuance $e_{it} \geq -\underline{e}$, and new debt b_{it} before its individual capital quality shock ϵ_{it} realizes. Since the distribution of the capital quality shock is known to the firm, it also endogenously chooses the cutoffs ϵ^d_{it} and ϵ^c_{it}. After the realization of ϵ_{it}, the firm's end-of-period net worth n_{it} is determined. Meanwhile, the firm forms expectations of its continuation value $E_{S_t|S_{t-1}}[V((1-\gamma)b_{it}, S_t)]$. The firm will be in one of the three states: Financially Healthy, Covenant Violation, or Default. Given a realized net worth n_{it-1}, existing debt b_{it-1}, and the aggregate state of the economy S_{t-1}, the firm solves:

$$\begin{aligned}\left(n_{it-1} + V(b_{it-1}, S_{t-1})\right) = \max_{k_{it}, e_{it}, b_{it}, \epsilon^c_{it}, \epsilon^d_{it}} & -e_{it-1} \\ & + \frac{1}{1+r} E_{S_t|S_{t-1}}\left[\int_{\epsilon^d_{it}}^{\infty} \left(n(z_t, k_{it}, b_{it}, \epsilon^n_{it}, \epsilon^f_{it}) + V((1-\gamma)b_{it}, S_t)\right) d\Phi(\epsilon_{it})\right]\end{aligned} \tag{7}$$

subject to net worth equations (3), cutoff equations (5) and (6), and capital accumulation

$$k_{it} = n_{it-1} + e_{it} + (b_{it} - b_{it-1})Q_{it} \tag{8}$$

2.3 Creditor Problem

Competitive creditors break even in expectations. Like shareholders, they discount cash flow at the international risk-free rate r. The break-even price of debt depends on the probabilities and values of the three states. In case of default, the liquidation value of the firm's assets is

$$n_{it}^{d} = (1-\xi)\left(\underbrace{(1-\tau)[y_{it} - w_t l_{it}]}_{\text{Earning less capital loss}_{it}} + \underbrace{[1-(1-\tau)\delta]k_{it}}_{\text{capital value}} + \underbrace{(1-\tau)\epsilon_{it}^{d} k_{it}}_{\text{capital quality loss}}\right) \tag{9}$$

where ξ reflects the proportional liquidation cost of the firm as a whole. The current debt price depends on the future market value of long-term debt:

$$Q_{it} = \left[\underbrace{\int_{-\infty}^{\epsilon_{it}^{d}} \frac{n_{it}^{d}}{b_{it}} d\Phi(\epsilon_{it})}_{\text{default}} + \underbrace{\int_{\epsilon_{it}^{d}}^{\infty} (\gamma + c) + (1-\gamma)E_t\left[Q\left((1-\gamma)b_{it}, S_t\right)\right] d\Phi(\epsilon_{it})}_{\text{non-default}}\right] \cdot \frac{1}{1+r} \tag{10}$$

2.4 Firm Policy and Aggregation

In equilibrium, a firm maximizes shareholder value (7) subject to the equilibrium bond price functions (10), net worth accumulation equations (3), covenant violation equation (5), default cutoff equation (6), and capital accumulation equation (8). Because of the existence of debt covenants, a firm would like to commit to maintaining low levels of debt to avoid covenant violations in the future when selling long-term debt. This helps to mitigate the commitment issue and time-consistent policy as in Jungherr and Schott (2022) without debt covenants.

Firm Policy To solve for the equilibrium firm policy, we compute the continuation value term $V(b_{it-1}, S_{t-1})$ recursively. We first rearrange the equity value function (7) by moving n_{it-1} to the right-hand side and redefine it as a choice variable $\tilde{e}_{it} = (n_{it-1} + e_{it}) \geq -\tilde{e}$ where $\tilde{e}$ is the redefined net equity issuance limit. We then define a policy vector $\phi(b_{it-1}, S_{t-1}) = \{\tilde{e}_{it}, k_{it}, b_{it}, \epsilon_{it}^{c}, \epsilon_{it}^{d}\}$ which solves

$$\begin{aligned} V(b_{it-1}, S_{t-1}) = \max_{\phi(b_{it-1}, S_{t-1})} & -\tilde{e}_{it} \\ & + \frac{1}{1+r} E_{S_t|S_{t-1}}\left[\int_{\epsilon_{it}^{d}}^{\infty} \left(n(a_t, k_{it}, b_{it}, \epsilon_{it}^{n}, \epsilon_{it}^{f}) + V((1-\gamma)b_{it}, S_t)\right) d\Phi(\epsilon_{it})\right] \end{aligned} \tag{11}$$

subject to

$$n(z_{it}, k_{it}, b_{it-1}, \epsilon_{it}^{n}, \epsilon_{it}^{f}) = (1-\tau)[y_{it} + \epsilon_{it}^{n} k_{it} - w_t l_{it}] + [1 + (1-\tau)(\epsilon_{it}^{f} - \delta)]k_{it} - [(1-\tau)c + \gamma + \chi \cdot \mathbf{1}_{CV}]b_{it} - f$$

$$(y_{it} - \epsilon_{it}^{c} k_{it} - w_t l_{it}) \times \eta - b_{it} = 0$$

$$n(z_{it}, k_{it}, b_{it}, \epsilon_{it}^d) + E_{S_t|S_{t-1}}\Bigg[V((1-\gamma)b_{it}, S_t)\Bigg] = 0$$

$$k_{it} = \tilde{e}_{it} + (b_{it} - b_{it-1})Q_{it}$$

$$Q_{it} = \left[\int_{-\infty}^{\epsilon_{it}^d} \frac{n_{it}^d}{b_{it}} d\Phi(\epsilon_{it}) + \int_{\epsilon_{it}^d}^{\infty} (\gamma + c) + (1-\gamma)E_t\left[Q\left((1-\gamma)b_{it}, S_t\right)\right] d\Phi(\epsilon_{it})\right] \cdot \frac{1}{1+r}$$

Aggregation We ensure all firms in the economy are ex-ante identical with three assumptions. First, we assume a constant unit mass of firms. Second, we presume defaulting firms exit the economy and are replaced by precisely the same amount of new entrants. Third, entering firms pay an entry cost financed by long-term debt b that matches the debt of incumbent firms. These assumptions imply the model aggregates exactly such that the aggregate quantities equal firm-level quantities $\{b = B, l = L, k = K\}$.

2.5 Households and Equilibrium

Households We close the general equilibrium model by introducing a representative domestic household. The household works, consumes, and invests its savings at the international risk-free rate r. Government revenue from taxation is paid out to the household as a lump-sum transfer. We assume GHH preferences over consumption C and labor L. Period utility is, therefore, $u(C_t - \frac{L_t^{1+\theta}}{1+\theta})$, which yields a labor supply curve $w_t = L_t^{\theta}$. Period consumption C_t equals the total output Y_t minus capital depreciation (aggregate investment).

Equilibrium We define the equilibrium of a dynamic open economy business cycle model by a given international risk-free rate r and an endogenous wage w. The aggregate state, S, of the economy is sufficiently summarized by the aggregate productivity z' and the aggregate stock of existing debt B: $S = (z', B)$.

Definition 1 (Recursive Competitive Equilibrium)**.** *The recursive competitive equilibrium consists of (1) a policy vector $\phi(b, S) = \{\tilde{e}, k, b, \epsilon^c, \epsilon^d\}$, bond price Q, and a value function $V(b, S)$; (2) a wage function $w = L^{\theta}$; (3) a stochastic aggregate law of motion $S' = F(S)$; and (4) aggregate quantities equalling firm-level quantities $\{b = B, l = L, k = K\}$ such that*

1. *$\phi(b, S)$, Q, and $V(b, S)$ solve the firm problem (11).*
2. *The labor market clears $L = l(b, S)$.*
3. *Total output $Y = y - f - \frac{\xi}{1-\xi}\int_{-\infty}^{\epsilon^d} \frac{n^d}{b} d\Phi(\epsilon') - \chi \int_{\epsilon_d}^{\epsilon^c} b d\Phi(\epsilon')$.*

2.6 Constrained Efficiency

The baseline model is inefficient because of firms' inability to commit to future actions. They excessively dilute debt as part of the costs is placed externally on the existing creditors, who bear increased default risk from new debt issuance. Though debt covenants do help align incentives and keep default risks low, firms still often violate covenants. To compare efficiency between models, we construct a constrained efficiency model as a benchmark.

We assume a social planner maximizes the total value of individual firms, which is the sum of all equity and all outstanding debt, both existing and newly issued. The planner is still subject to the same lack of commitment and faces the same set of constraints. The planner solves:

$$\begin{aligned} W(b_{it-1}, S_{t-1}) = \max_{\phi(b_{it-1}, S_{t-1})} & - \tilde{e}_{it-1} + b_{it-1} Q_{it-1} \\ & + \frac{1}{1+r} E_{S_t|S_{t-1}} \left[\int_{\epsilon_{it}^d}^{\infty} \left(n(a_t, k_{it}, b_{it}, \epsilon_{it}^n, \epsilon_{it}^f) + V((1-\gamma) b_{it}, S_t) \right) d\Phi(\epsilon_{it}) \right] \end{aligned} \tag{12}$$

subject to the same set of constraints as in the firm problem (11).

2.7 Solution Method

We follow Jungherr and Schott (2022) and Hatchondo, Martinez, and Sosa-Padilla (2016) to solve our model. We solve for the equilibrium of a finite-horizon economy with sufficient periods to ensure the value functions and bond prices for the first and second periods are very close. We then use the first-period equilibrium functions as the infinite-horizon economy equilibrium functions.

3 Parameterization

3.1 Calibration of the Baseline Model

Our parameterization of the baseline model proceeds in two steps. First, we externally fix a set of parameters to match standard macroeconomic targets in the open economy business cycle models (Arellano, Bai, and Kehoe, 2019; Jungherr and Schott, 2022). Second, given these fixed parameter values, we choose the remaining fitted parameters to match moments in the US data.

Table 2: Externally Fixed Parameters

Parameter	Description	Value	Source/Targets
	(a).General Environment		
β	Discount factor	0.97	Annual frequency
r	International risk-free rate	0.0309	$r = 1/\beta - 1$
θ	Inverted Frisch elasticity	0.25	King and Rebelo (1999)
τ	Corporate tax rate	0.40	Gomes, Jermann, and Schmid (2016)
	(b).Production Technology		
ψ	Capital share	0.33	Standard as in Bloom et al. (2018)
ζ	Decreasing returns-to-scale	0.75	Standard as in Bloom et al. (2018)
δ	Depreciation rate	0.10	Annual rate of 10%
ρ_z	Persistence	0.909	Standard as in Khan and Thomas (2008)
	(c).Financial Market		
γ	Debt repayment rate	0.1284	Maturity $1/\gamma = 6.47$ years
τ	Debt coupon	0.0309	Debt coupon= r
η	Debt-to-earnings ratio threshold	0.25	Market threshold as in Lian and Ma (2021)
ξ	Default cost	0.469	Liquidation cost as in Kermani and Ma (2023)

Fixed Parameters Table 2 lists the externally fixed parameters. We first pick parameters for the general environment faced by firms. The model is set at a yearly frequency, so we choose a discount factor of 0.97. Correspondingly, the international risk-free rate equals $1/\beta - 1 = 0.0309$. The inverted Frisch elasticity is set to 0.25 as in King and Rebelo (1999), and the corporate tax rate is taken to be 40% as in Gomes, Jermann, and Schmid (2016), which suggests that τ should be capturing all benefits of using debt rather than equity. We then choose the production technology parameters. Firms face a capital share ψ of 0.33, a decreasing returns-to-scale parameter ζ of 0.75, and an annual capital depreciation of 0.1. We choose the persistence of the aggregate TFP shock ρ_z to be 0.0909 as in Khan and Thomas (2008) and would later fit the volatility σ_z to match the business cycle output volatility in the U.S. GDP data.

Finally, we choose the parameters capturing the financial market. The debt repayment rate γ is set to 0.1284 to match an average debt maturity of 6.47 years as suggested by Gilchrist and Zakrajšek (2012). The debt coupon c is chosen to be the same as the risk-free rate r. Finally, we choose the debt-to-earnings ratio threshold η to be 0.22 to match a standard characteristic of U.S.

debt covenants as documented for the majority of U.S. debt documented in Lian and Ma (2021) and the default cost ξ to be 0.469 to match the bankruptcy liquidation cost estimated in Kermani and Ma (2023).

Table 3: Internally Fitted Parameters and Model Fit

Param.	Description	Value	Targets	Data	Model
f	Fixed operation cost	0.151	Leverage ratio	33%	33%
χ	Covenant violation cost	0.015	Covenant violation rate	23%	23%
σ_z	Productivity shock vol.	0.006	Volatility of U.S. GDP	2.8%	2.8%
σ_ϵ^n	Cash flow shock vol.	0.326	Frequency of negative EBIT	18%	18%
σ_ϵ^f	Future capital quality shock vol.	0.795	Credit spread	2.0%	2.0%

Notes: This table lists the parameters we internally fitted and the corresponding matched moments in the data. Though the fitted parameters are jointly determined, they are closely tied to specific moments. We first choose the productivity shock volatility $\sigma_z = 0.0006$ to generate a relative GDP volatility of 3.2%. We then choose the capital quality shock volatility $\sigma_\epsilon^n = 0.293$ and the fixed operation cost $f = 0.169$ to match the average leverage of 34% roughly and the frequency of negative EBITDA of 15%. We target a default rate of about 3.2% with the future capital quality shock volatility $\sigma_\epsilon^f = 0.782$.[8] Finally, we match an annual covenant violation rate of 18% by choosing the covenant violation cost $\chi = 0.015$.

Fitted Parameters Table 5 lists the parameters that we internally fitted and the corresponding moments they matched in the data. Though the fitted parameters are jointly determined, they are closely tied to specific moments. We first choose the productivity shock volatility $\sigma_z = 0.0006$ to generate GDP volatility of 2.8%. We then choose the capital quality shock volatility $\sigma_\epsilon^n = 0.293$, and the fixed operation cost $f = 0.169$ to match the average leverage of about 34% roughly and a 15% frequency of negative EBITDA. We target a default rate of about 3.2% with the future capital quality shock volatility $\sigma_\epsilon^f = 0.782$.[9] Finally, we match an annual covenant violation rate of 18% by choosing the covenant violation cost $\chi = 0.015$.

3.2 Calibration of Alternative Models

To illustrate the properties of the baseline model, we compare our baseline model with covenants, the COV model, to the model without covenants, the NoCOV model, and the constrained efficiency model, the CEE model. The fitted parameters and model fits are displayed in Table 4 below. In the CEE model, since the social planner maximizes the total value of the firm, creditors do not punish firms with covenant violation costs. The debt dilution issue is also fully resolved by the social planner. We observe lower leverage ratios, GDP volatility, and credit spreads relative to the baseline COV model with debt dilution.

[9]The mean annual default rate of 3.2% is taken from the survey by Dun and Bradstreet (www.dnb.com).

Table 4: Alternative Models: Fitted Parameters and Model Fit

Description Calibration	Param.	COV (baseline)	NoCOV (same mom.)	NoCOV (no recali.)	CEE (no recali.)
Fixed operation cost	f	0.151	0.164	0.151	0.151
Covenant violation cost	χ	0.015	n.a.	n.a.	n.a.
Productivity shock vol.	σ_z	0.006	0.006	0.006	0.006
Cash flow shock vol.	σ_ϵ^n	0.326	0.652	0.326	0.326
Future capital quality shock vol.	σ_ϵ^f	0.795	0.652	0.795	0.795
Moments	**Data**				
Leverage ratio	33%	33%	33%	37%	27%
Covenant violation rate	23%	23%	n.a.	n.a.	n.a.
U.S. volatility of GDP	2.8%	2.8%	3.1%	2.8%	2.6%
Frequency of negative EBITA	18%	18%	n.a.	n.a.	n.a.
Credit Spread	2.0%	2.0%	2.0%	2.2%	1.8%

Notes: This table shows all our alternative models' fitted parameters and model fit, demonstrating the role of debt covenants. We need to recalibrate the model without debt covenants because we need to keep the same average leverage ratio and credit spread to make the alternative model comparable to our baseline model. We change as few parameters as possible. For the constrained efficiency model, we do not recalibrate it since it is not meant to be an equivalent comparison but rather an ideal.

For the NoCOV models, we focus on the recalibrated one. Alternative models are only comparable to the baseline COV model when both economies have similar indebtedness and credit risks. Therefore, we recalibrated a NoCOV model to match the baseline COV model's leverage ratio and credit spread. Without debt covenants, the NoCOV economy suffers from more volatile output. Removing debt covenants also reveals properties of the baseline COV model. To this end, we also show a NoCOV model that removes debt covenants without recalibration. Such an economy has higher leverage and credit spreads than the baseline COV model, indicating the role of covenants in equilibrium financial conditions. These secondary results are located in the appendix.

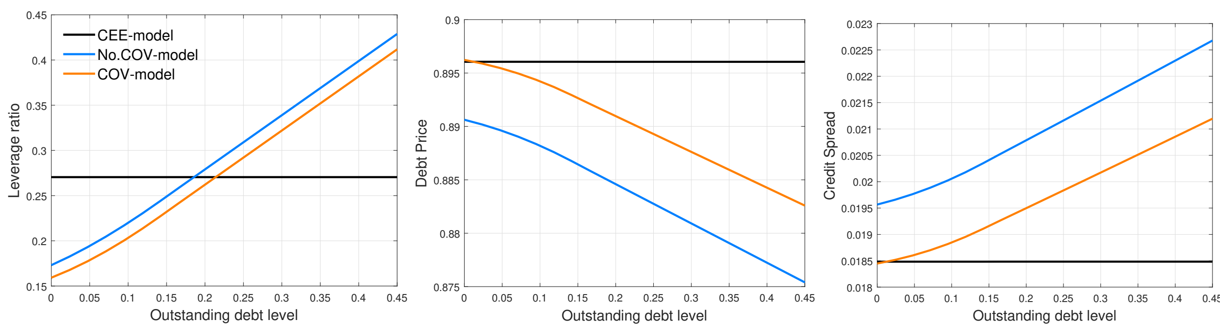
4 Quantitative Results

We now quantitatively simulate the model and examine the key roles of debt covenants: 1) debt dilution mitigation, 2) business cycle stabilization, 3) reduction of business cycle asymmetry, and 4) improved long-run economic outcomes.

4.1 Debt Dilution Mitigation

We first show how introducing debt covenants reduces debt dilution. Without debt covenants, shareholders reoptimize debt every period conditional on their existing debt. Taking on any new debt dilutes the value of existing debt by reducing the claim incumbent borrowers have on earnings and assets. More indebted shareholders are further incentivized to dilute as this implies a larger value transfer from existing creditors to shareholders.

Figure 3: Debt Dilution Mitigation with Debt Covenants



Notes: These figures show whether the level of existing debt affects the leverage policy, debt prices, or credit spreads in three alternative models. The *CEE model* indicates the constrained efficiency model in which a social planner maximizes the total firm value. The *NoCOV model* indicates an alternative model without debt covenants. The *COV model* indicates our baseline model with debt covenants. The calibration and model fit of all three models is presented in Table 5.

Figure 3 shows how the model with debt covenants prevents debt dilution in equilibrium. To better display the mechanism, we compare the baseline COV model with alternative models: the constrained efficiency CEE model and the NoCOV model without debt covenants. Figure 3(a) shows the leverage policy (b'/k') conditional on existing debt (b). In the CEE model, leverage policy is independent of the existing debt since the social planner optimizes the total value of shareholders and creditors, eradicating debt dilution. In both the NoCOV and COV models, firms choose more leverage when carrying existing debt as shareholders are incentivized to issue new debt to dilute existing debt. The COV model partially mitigates such incentives through covenant violation penalties.

Similar patterns are observed in Figure 3(b) and 3(c) for debt prices and credit spreads, respectively. In the CEE model, the debt price and credit spread are unaffected by the level of existing debt since the social planner optimizes the total value of shareholders and creditors. In both the NoCOV and COV models, more debt increases default risk and, therefore, decreases debt prices and increases credit spreads. Due to the existence of covenant violation penalties, which mitigate the debt dilution incentives of shareholders in the COV model, creditors are willing to provide a better debt price – equivalently, a lower credit spread – given the same stock of debt.

4.2 Business Cycle Stabilization

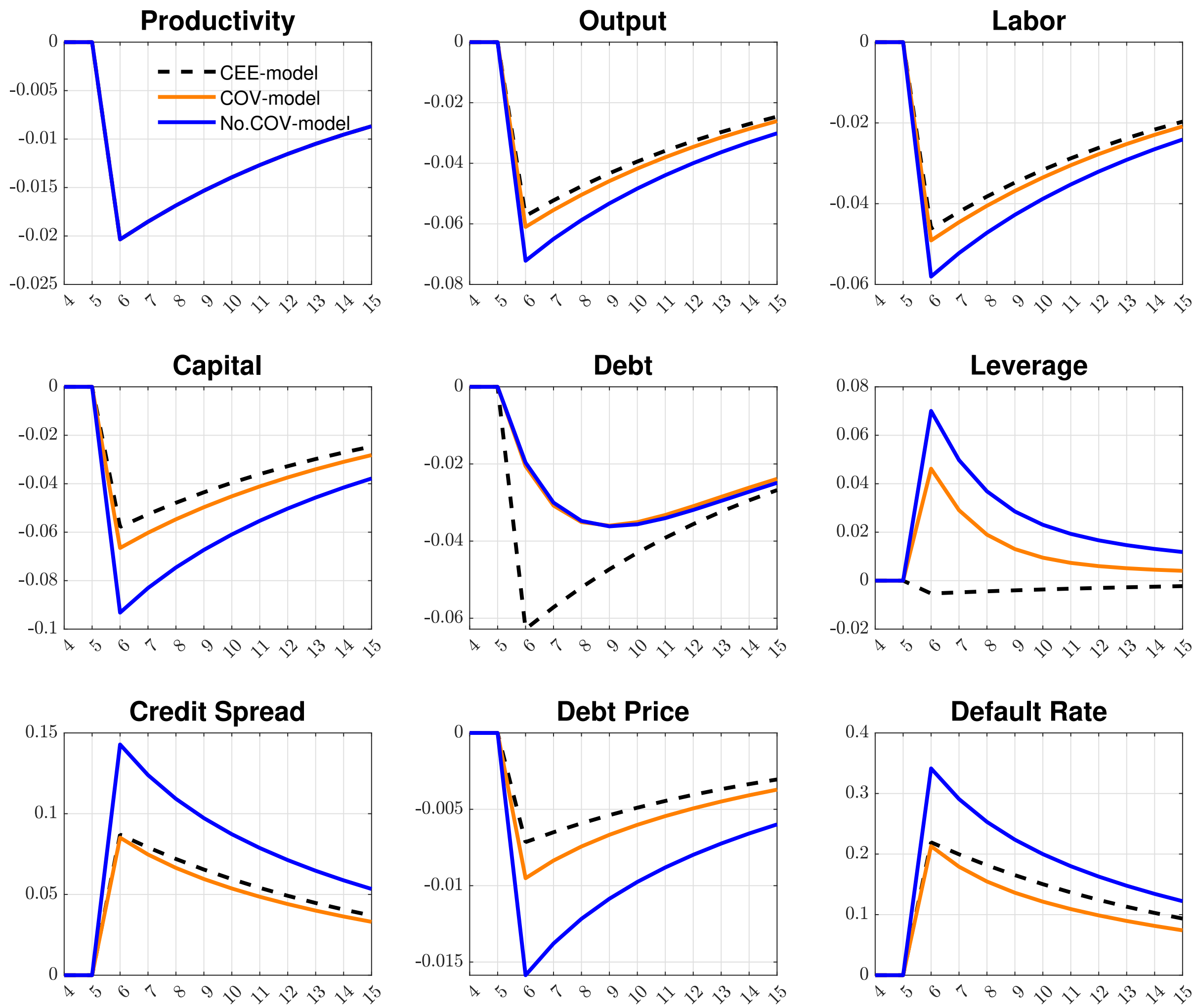
We now show how debt covenants partially mitigate the amplified and prolonged responses of output and capital due to debt dilution following a negative productivity shock in Figure 4. We again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

Without Debt Dilution We first demonstrate the effects of a negative 2% productivity shock Z_t in the CEE model, i.e., without debt dilution incentives. The social planner maximizes the combined value of shareholders and creditors and reacts to a negative productivity shock by reducing investment and labor demand, which results in declining output, labor, and capital (sub-figures 2 to 4). As shown above, the social planner internalizes potential default costs, which accrue to the holders of existing debt, leading to reductions in outstanding debt to reduce default risks. This results in a sharp decline in debt and even a counterfactual procyclical leverage response (sub-figures 5 and 6). An increased credit spread, decreased debt price, and increased default risk remain, which are beyond the control of the social planner due to the bad shock.

With Debt Dilution/Without Debt Covenants Given how the model without debt dilution performs, we now demonstrate the effects of strong debt dilution incentives in the NoCOV model compared to the CEE model. Such effects are well-documented and thoroughly discussed in Jungherr and Schott (2022) as *slow debt, deep recession, and slow recovery*. In the NoCOV model, firms again react to a negative productivity shock by reducing investment and labor demand, which results in the initial decline of output, labor, and capital. The key difference from the CEE model is the financial decisions as now shareholders do not internalize the increased default risks from carrying debt.

Firms choose to dilute existing debt by embracing more leverage and higher default risk. Their optimal leverage policy leads to debt falling more slowly than capital (*slow debt*). The slow deleveraging process leads to higher default risk, which in turn increases the cost of capital

Figure 4: Impulse Response Functions to a -2% TFP Shock



Notes: CEE model: Black dashed lines show impulse response functions in the constrained efficiency model without debt dilution. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants.

through higher credit spreads and further discourages investment and capital formation (and further increases leverage and default risk). Such an accelerating feedback loop between default risk and capital leads to deeper negative responses in capital and output compared to the CEE model without debt dilution (*deep recession*). Finally, the slow-moving debt choices create an extended period of high credit spreads relative to the CEE model without debt dilution. Output and capital remain further from their unconditional means for longer compared to the CEE model (*slow recovery*). These results show that debt dilution amplifies and prolongs a productivity recession.

With Debt Dilution and Debt Covenants Understanding how debt dilution amplifies and prolongs recessions caused by negative productivity shocks, we demonstrate the effects of debt covenants in the COV model compared to both the NoCOV model and the CEE model. As in both models above, COV model firms also react to a negative productivity shock by reducing investment and labor demand, again initially decreasing output, labor, and capital. Firms in the COV model also want to dilute existing debt by taking on more leverage and higher default risk. Ideally, they would choose enough leverage to maximize current tax benefits but transfer the potential losses due to default costs to debt holders. However, creditors are aware of shareholders' incentives and debt covenants are formalized in lending contracts. Shareholder optimization now internalizes the potential penalties of an increased debt-to-earnings ratio hitting the covenant violation threshold. Such debt covenants lead to a less aggressive optimal leverage policy response to the initial capital drop following the negative productivity shock by breaking the accelerating feedback loop between default risk and capital.

To avoid debt covenant violation costs, firms adopt relatively lower leverage ratios even though the potential gain via tax benefits from diluting existing debt is high. Meanwhile, since debt covenants are always triggered before default, the motivation to maintain a lower leverage ratio and, simultaneously, a lower debt-to-earnings ratio, also lowers default risk. This decreases the credit spread and increases debt prices relative to the NoCOV model. The slow debt, deep recession, and slow recovery effects are less severe in the COV model. Output and capital decline less compared to the NoCOV model and recovery from the recession is faster. However, the COV model economy still suffers from some debt dilution since debt covenants are only a partial solution, especially when firms are far from the violation threshold. Therefore, all debt dilution effects still exist but are mitigated.

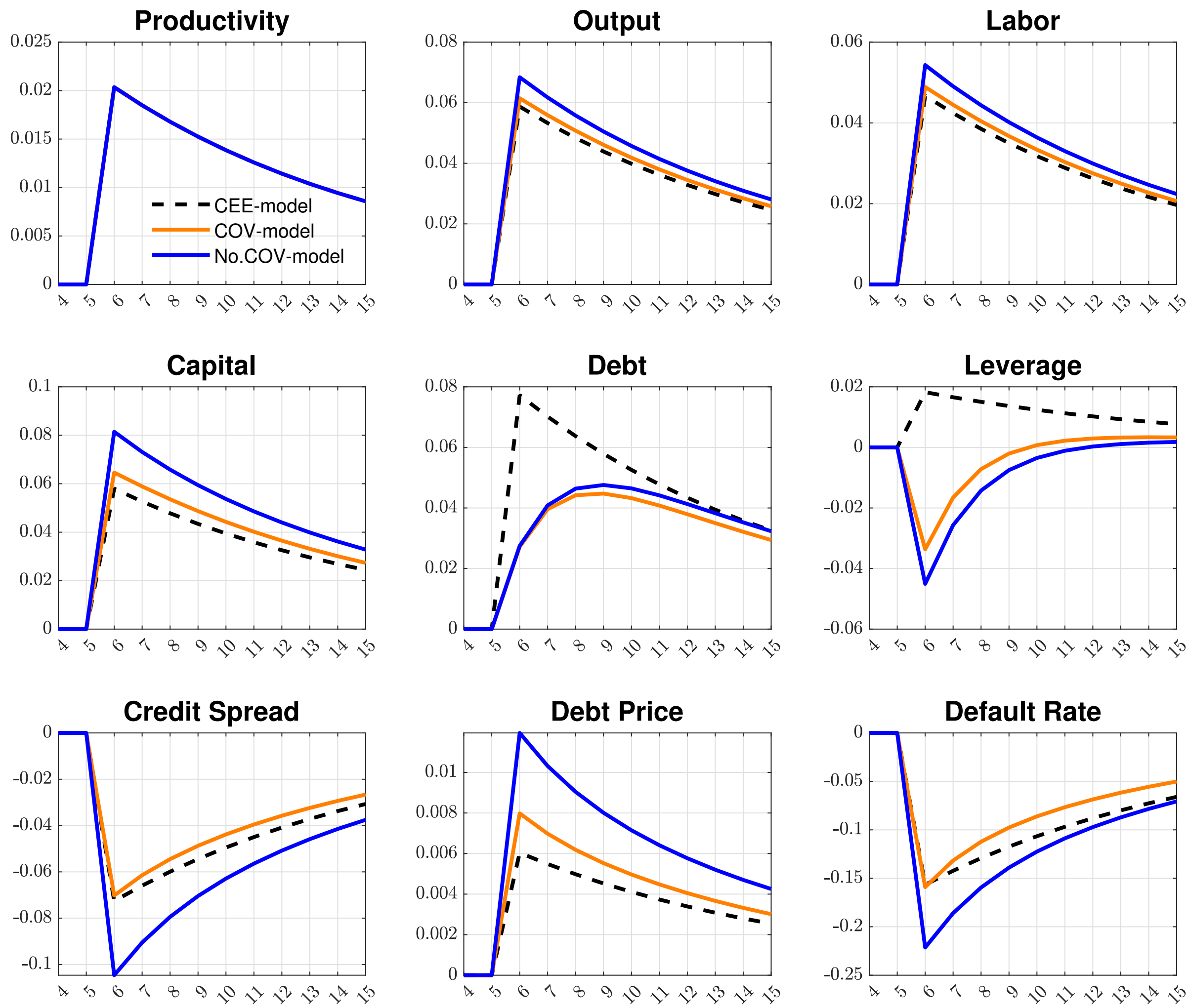
As a result, the effects are quantitatively significant. In response to a negative 2% TFP shock, the baseline model has a leverage spike of 4.2% instead of 7% in the model without debt covenants. Consequently, output and capital dropped by 8% and 6.8% instead of 9.5% and 9.2% without debt covenants. In contrast, the constrained efficiency model has the smallest recessionary effects.

4.3 Business Cycle Asymmetry Reduction

This section compares the peaks from a +2% TFP shock to the troughs from a -2% TFP shock. We show that the existence of debt covenants also reduces business cycle asymmetry. To better display the mechanism, we again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

We plot the impulse responses to a 2% positive TFP shock in Figure 5. To provide a more intuitive comparison, we calculate the difference in the peak responses between the absolute

Figure 5: Impulse Response Functions to a +2% TFP Shock



Notes: CEE model: Black dashed lines show impulse response functions in the constrained efficiency model without debt dilution. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants.

value of peak responses and present these in Table 5. For example, the peak difference of the NoCOV model output during the boom is +9.15%, and while the bust is -9.80%, the asymmetry is $\frac{9.80\%}{9.15\%} - 100\% = 7\%$. We calculate asymmetry across all three models for real and financial variables.

In the CEE model, both output and capital responses are symmetric since the social planner maximizes the total value of both shareholders and creditors. Hence, there is no debt dilution to amplify bad shocks. The business cycle remains asymmetric in financial variables since default risk remains high during the bust. The NoCOV model shows the largest asymmetry since the amplification effects during the bust are much stronger than during the boom. The COV model reduces asymmetries between the bust and the boom.

Table 5: Asymmetry in the Peak Responses to TFP Shocks: *(Recession/Boom-100%)*

Model	Output	Capital	Leverage	Credit Spread	Debt Price	Default Rate
CEE.	0%	0%	-84%	14%	20%	46%
Cov.	0%	4%	40%	14%	20%	46%
No.Cov.	7%	14%	56%	36%	29%	55%

Notes: This table calculates the asymmetries in the peak responses to TFP shocks across three models. The peak responses are displayed in Figures 4 and 5. CEE model: the constrained efficiency model without debt dilution. NoCOV model: the model with debt dilution but without debt covenants. COV model: the model with debt dilution and debt covenants.

4.4 Long-run Level Effects

Finally, we discuss the long-run effects of debt dilution and debt covenants on the macroeconomy. We again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

Table 6: Long-run Effects of Debt Covenants

Model	**Output**		**Capital**		**Consumption**	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
CEE model	0.659	0.026	1.115	0.042	0.548	0.024
COV model	0.653	0.028	1.089	0.050	0.544	0.027
NoCOV model	0.639	0.031	1.036	0.062	0.536	0.031

Notes: This table calculates the mean and standard deviation of output, capital, and consumption in each model. CEE model: the constrained efficiency model without debt dilution. NoCOV model: the model with debt dilution but without debt covenants. COV model: the model with debt dilution and debt covenants.

Without debt covenants, debt dilution creates severe debt overhang problems since creditors ask for a higher credit spread even when firms are financially healthy. This dissuades firms from profitable investments since earnings from new projects largely accrue to debt holders. Debt covenants reduce debt dilution and mitigate the debt overhang problem. Creditors accept a lower credit spread, especially for financially healthy firms. These firms then undertake more profitable investments because the shareholders capture more of the earnings from new projects. Therefore, holding financial conditions (leverage and credit spread) constant, debt covenants increase economic performance in the long run.

Beyond low volatility, the CEE model has the highest output, capital, and consumption since the model economy suffers the least from debt frictions when the social planner takes control of firms. The NoCOV model, on the other hand, has the lowest output, capital, and consumption because it suffers the most from the debt friction between firm owners and creditors. To avoid diluting existing debt, creditors reduce lending to firms, resulting in a low capital equilibrium m, implying low output and consumption. As usual, the levels of all real variables in the COV model are between those of the CEE and NoCOV models. Quantitatively, the model with debt covenants can maintain a 5% larger capital stock, 2% higher output, and 1.5% higher consumption. This indicates that debt covenants reduce business cycle fluctuations and improve long-run economic performance.

5 Conclusion

Corporate debt data suggests that debt covenants enforce a maximum debt(interest)-to-earnings ratio that creditors widely adopt for firms to prevent debt dilution. We have shown that introducing debt covenants into a standard business cycle model of production, firm financing, and costly default significantly reduces business cycle volatility. The key to such an equilibrium outcome is that debt covenants weaken the debt dilution motive of firm owners, which reduces the aggressive use of leverage and, thus, the responses of credit spreads and default risk following productivity shocks. Debt covenants also reduce business cycle asymmetry and improve long-run economic performance in terms of output, capital, and consumption. Debt covenants not only help creditors maintain the value of their debt but also help stabilize the business cycle and increase economic performance.

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A Appendix

A.1 Additional Quantitative Results

This section demonstrates additional quantitative results focusing on the non-recalibrated NoCOV model without debt covenants. As we have emphasized in Section 3.2, comparing the baseline COV model and the non-recalibrated NoCOV model without debt covenants is not economically meaningful. The NoCOV model would only be comparable to the baseline COV model when both economies' indebtedness and credit risk are similar and, more importantly, match the data. Still, we hope this comparison illustrates the mechanism of the baseline COV model. As we have already shown in Table 4, the non-recalibrated NoCOV model without debt covenants features a higher leverage ratio (37% vs. 33%) and a higher credit spread (2.2% vs. 2.0%) compared to the baseline COV model. Therefore, directly removing debt covenants yields a counterfactual overborrowing equilibrium and significantly increases debt and credit risk in the economy.

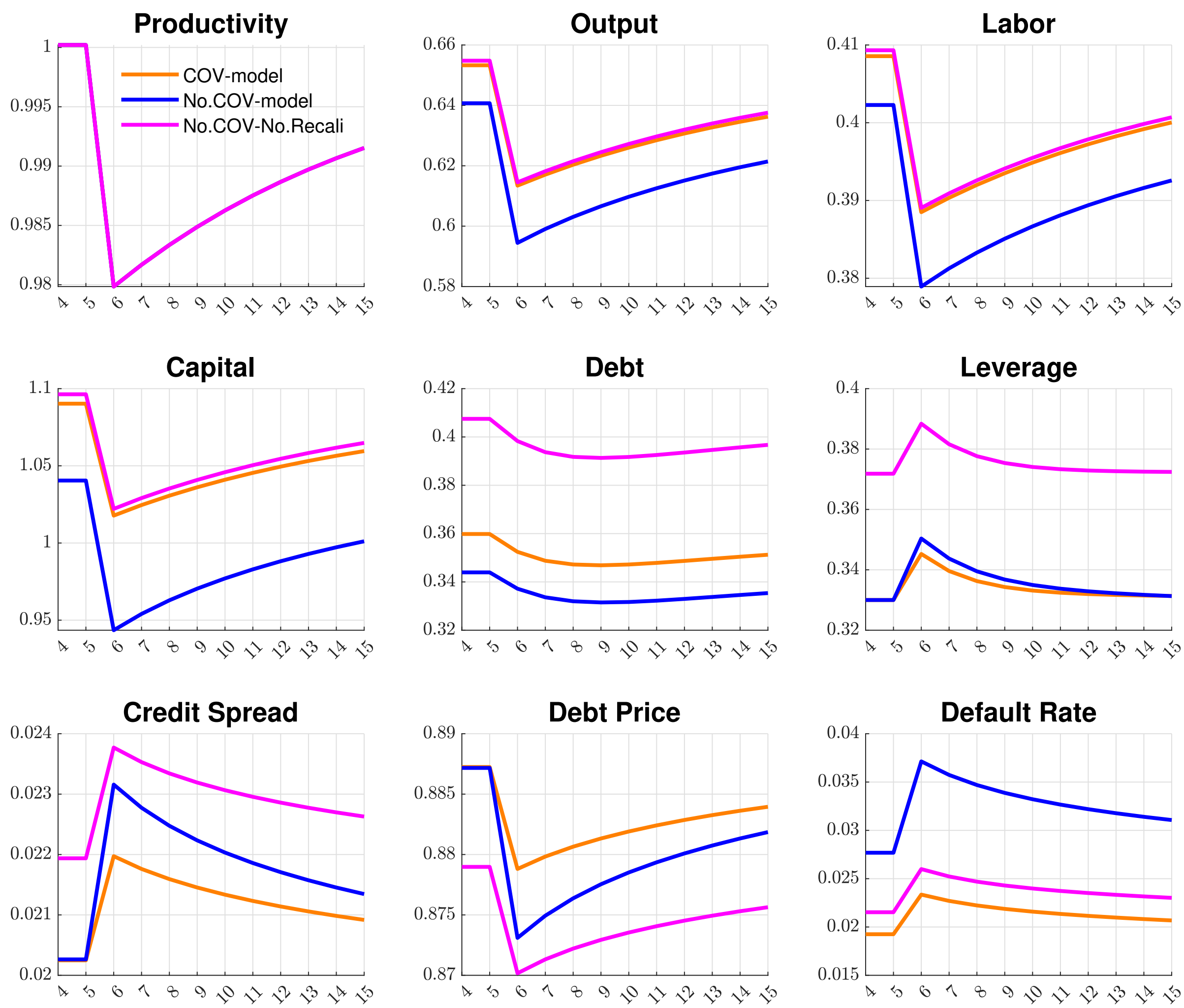
Business Cycle Stabilization Figures 6 and 7 show the impulse responses to a negative 2% TFP shock in levels and percentages, respectively. The non-recalibrated NoCOV model without debt covenants shows similar responses in real variables but has more debt and leverage, plus higher spreads and default rates. In terms of percentages, the debt changes are larger.

Business Cycle Asymmetry Reduction Figures 8 and 9 show the impulse responses to a positive 2% TFP shock in levels and percentages, respectively. The non-recalibrated NoCOV model without debt covenants shows similar responses in real variables but has higher levels of debt, leverage, spread, and default rate. In terms of percentage, the reactions in debt changes are deeper.

Long-run Level Effects Table 7 contrasts the long-term effects of debt covenants including the non-recalibrated NoCOV model without debt covenants. The equilibrium output, capital, and consumption of the non-recalibrated NoCOV model are almost identical to the baseline COV model with debt covenants.

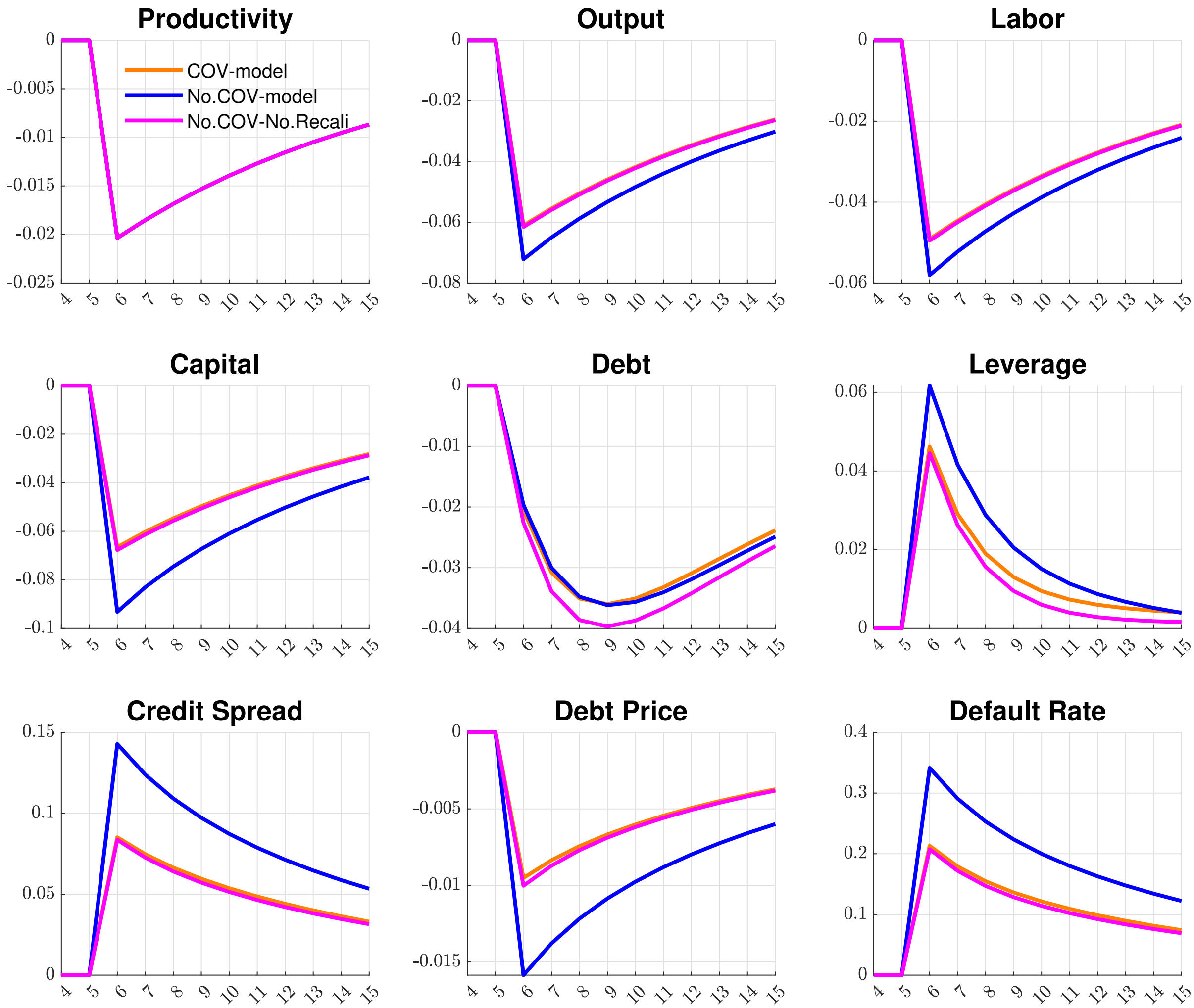
Overall Evaluation The counterfactual non-recalibrated NoCOV model exhibits similar levels and volatility of real economic variables, including output, capital, and consumption, to our baseline COV model. However, it features counterfactually high leverage (debt), credit spreads, and default risk. Considering that default is costly, the counterfactual non-recalibrated NoCOV model is also worse than the baseline COV model. Again, a fair comparison to the counterfactual non-recalibrated NoCOV model would be recalibrating the baseline COV model to the same counterfactual financial conditions (same higher level of leverage and credit spread). In such a case, we would exactly reproduce the same findings in the main text.

Figure 6: Impulse Response Functions to a -2% TFP Shock (In Levels)



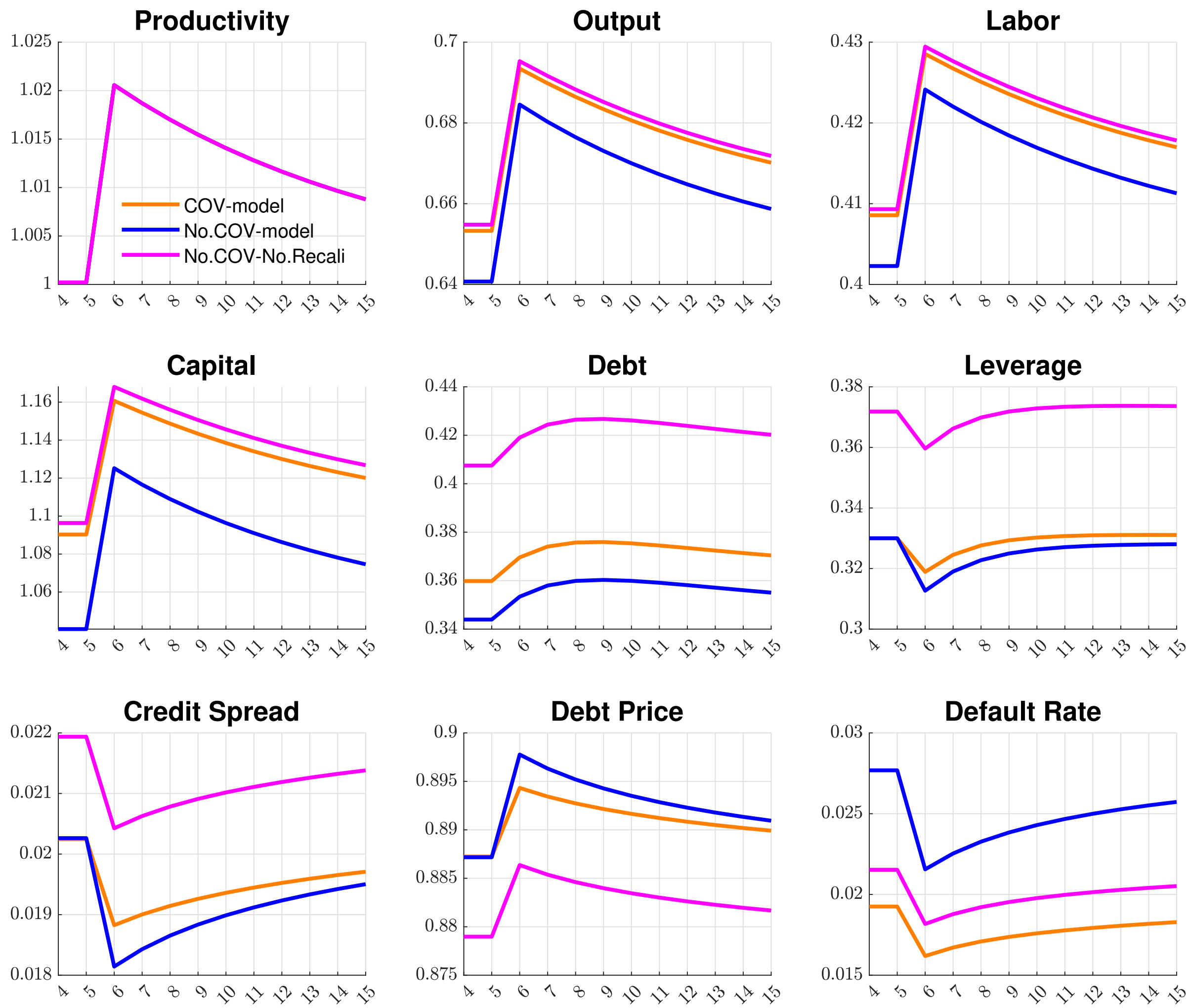
Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

Figure 7: Impulse Response Functions to a -2% TFP Shock (In Percentage)



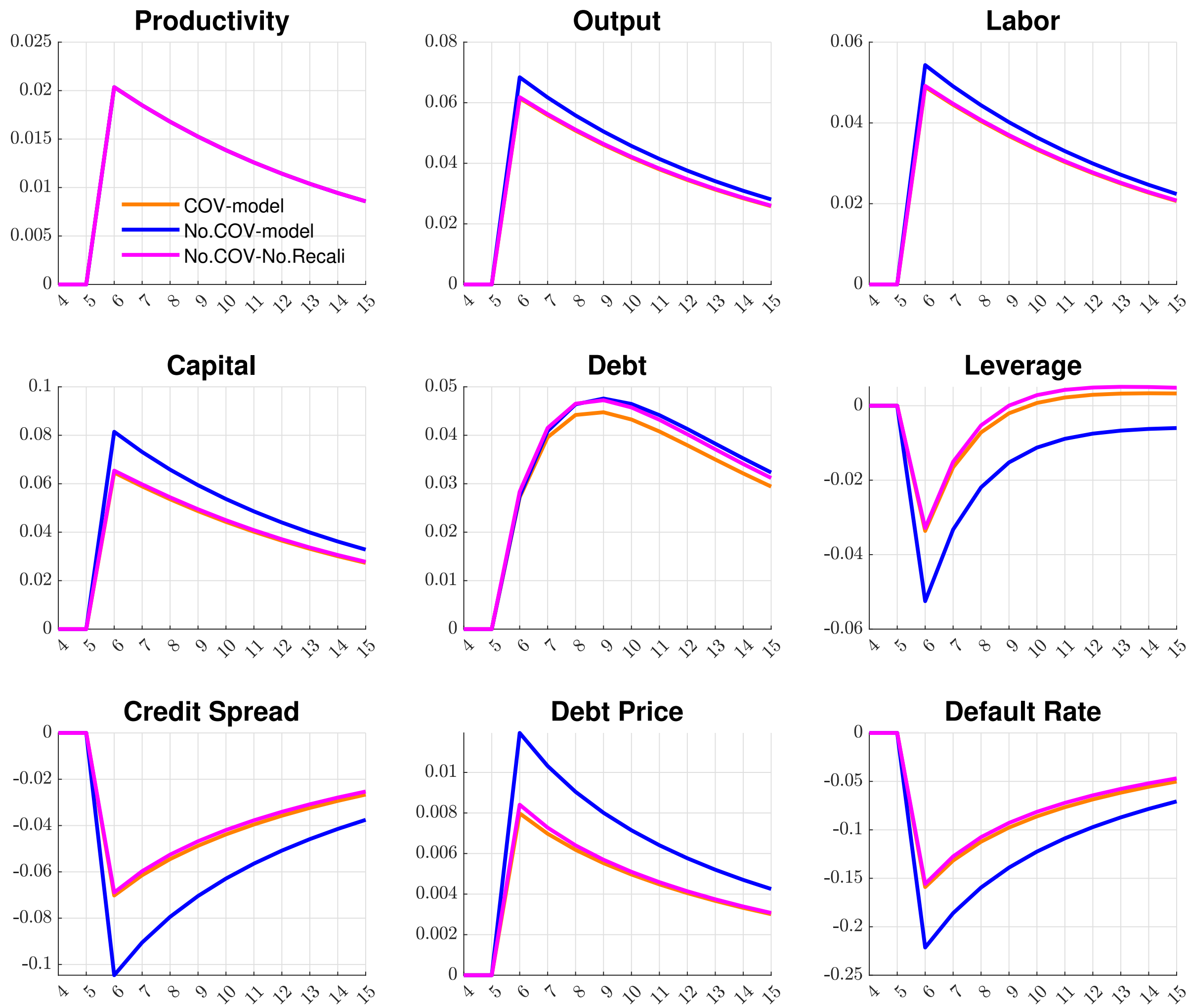
Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

Figure 8: Impulse Response Functions to a +2% TFP Shock (In Levels)



Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

Figure 9: Impulse Response Functions to a +2% TFP Shock (In Percentage)



Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

Table 7: Long-run Effects of Debt Covenants

Model	**Output**		**Capital**		**Consumption**	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
CEE model	0.659	0.026	1.115	0.042	0.548	0.024
COV model	0.653	0.028	1.089	0.050	0.544	0.027
NoCOV model	0.639	0.031	1.036	0.062	0.536	0.031
NoCOV-NoRecali model	0.654	0.028	1.095	0.051	0.545	0.027