The Macroeconomic Implications of Firms' Debt Maturity*

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Abstract

Long-term debt is the main source of firm-financing. However, what long-term debt implies for the transmission of monetary and financial shocks is not well understood. We show that longer debt maturity can amplify or dampen the response of investment depending on firms' costs of issuing equity. Our framework nests two benchmarks: When firms equity issuance is costless, longer maturity amplifies the investment response through a debt overhang channel. When firms cannot issue any equity, longer maturity dampens the response of investment as the lower value of debt results in a smaller drop in firms' net worth and the debt overhang channel allows firms to leverage up more. We estimate equity issuance costs using firm-level balance sheet data and find that in a calibrated medium scale DSGE model long-term debt *reduces* the sensitivity of investment and output to monetary policy and financial shocks. In an extended model, we introduce banks that intermediate credit to firms and face an occasionally binding financial constraint. In this case we find that longer debt maturity can amplify the investment response of firms for shocks that are large enough.

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Keywords: Long-term debt; Financial frictions; Debt overhang; Macroeconomic activity.

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1 Introduction

The share of long-term debt in total non-financial corporate debt is around seventy percent and has been rising since the 1980s. In this paper we study the role of firms' debt maturity in the transmission of shocks to the economy.

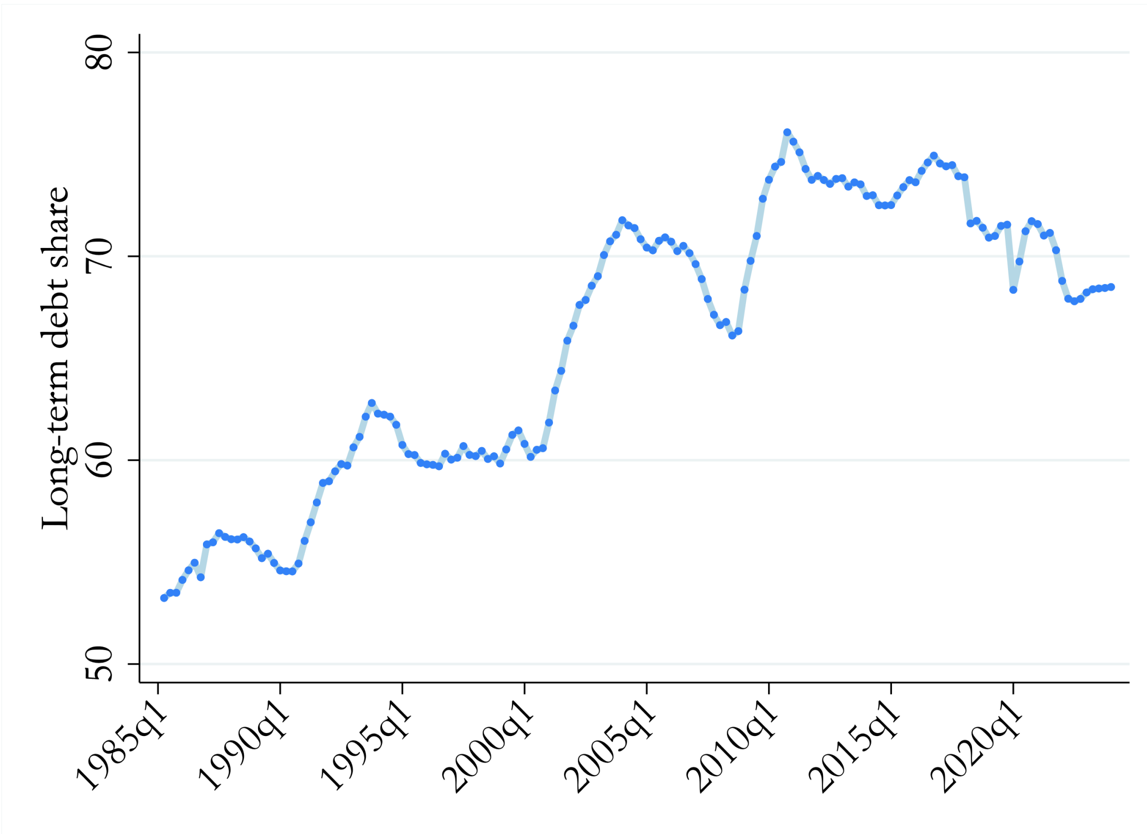

Figure 1: Long-term debt share

Note.- Figure shows the share of long-term debt as a percentage of total debt for non-financial corporate businesses. Source: US Financial Accounts.

Traditional models of firm financing, such as Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997) focus on how financial frictions can amplify monetary policy and other shocks. However, these models typically focus on short-term debt and do not allow firms to issue new equity. In a recent contribution, Gomes, Jermann, and Schmid (2016) incorporate nominal long-term debt and show that it amplifies aggregate fluctuations through a debt overhang channel: The presence of outstanding debt creates a commitment problem for the firm, which results in an additional incentive to increase leverage in order to dilute the value of debt. This leads to persistently elevated default rates, and a stronger contraction in investment. However, the model assumes no equity issuance costs.

We bridge the gap between these two strands of literature by developing a model in which firms use both long-term debt and costly equity issuance to finance investment. We find that equity adjustment costs are key in determining the role of long-term debt in transmitting shocks. We illustrate this point by nesting both an environment where firms cannot issue equity, as well as one where equity issuance is costless. In line with Gomes et al. (2016) we find that in a flexible price environment without equity issuance costs and with long-term debt, an unexpected one-time decrease in inflation leads to an amplified investment response. In the presence of long-term debt, an increase in the real value of debt increases firms' incentive to dilute preexisting creditors by increasing leverage. This debt overhang channel results in a higher default rate and lower investment. Importantly, however, we find that the presence of long-term debt *dampens* the same shock when equity adjustment costs are large (or infinity, as in Bernanke et al. (1999)). When equity issuance is very costly, the response of investment is determined by the evolution of firms' net worth as well as the elasticity of leverage to changes in net worth. Long-term debt can be dampening for two reasons. First, following the shock, a decline in firms' net worth leads to an increase in leverage, which lowers the price of long-term debt. This price decrease results in a smaller decline in net worth because it lowers the value of firms' outstanding debt obligations. Second, the elasticity of leverage to changes in net worth is higher in the presence of long-term debt. This effect results from the additional perceived benefit of debt which consists of diluting preexisting creditors. Due to this debt overhang channel, leverage is higher in the economy with long-term debt, which dampens the contraction in investment when firms are unable to issue equity. Interestingly, the same channel that is responsible for the drop in investment when there are no costs of adjusting equity, causes a smaller drop when it is costly to adjust equity.

We employ the model to derive an optimality condition of equity injections, which we estimate using balance sheet data on firms' dividend payouts and returns on equity from Compustat. The estimated value is used in a calibrated medium scale DSGE model to study the role of long-term debt in transmitting two key financial shocks: a monetary policy shock and the risk shock of Christiano, Motto, and Rostagno (2014), which causes an increase in the idiosyncratic risk of firms' investment returns. We find that at the estimated value of equity issuance costs, long-term debt *dampens* the response of investment and output to both of these shocks. The dampening effect of long-term debt is particularly large in response to a risk shock, causing the drop in investment and output with long-term debt to be about half as large as with short-term debt.

We then extend the model to include financial intermediaries. While long-term debt dampens the response of the economy to small shocks, if shocks are large enough to cause financial constraints on banks to become binding, long-term debt can become a source of amplification. In the extended model, banks issue deposits to households and hold firms' debt. Banks face an agency problem as in Gertler and Kiyotaki (2010) and the implied financial constraint is not binding in the steady state. The incentive constraint becomes binding if banks' net worth drops sufficiently. In an economy with short-term debt, financial amplification of shocks on banks' balance sheets is muted as there is no fluctuation in banks' asset prices. As a result banks' net worth is almost insensitive to shocks. On the other hand, when banks hold long-term debt, declines in debt prices cause declines in bank net worth. If these losses are large enough to trigger financial constraints on banks to be binding, the economy can enter a credit crunch. We illustrate this by feeding a sequence of contractionary monetary policy shocks that causes banks to enter the constrained region and show that, if these shocks are large enough, the economy with long-term debt has a larger drop in investment and output than the economy with short-term debt.

Related Literature By studying how the interaction of long-term corporate debt and financial frictions shapes the response to macroeconomic shocks in a medium-scale DSGE model, our work contributes to three strands of literature.

First, we contribute to the literature on the effects of defaultable long-term corporate debt. Gomes et al. (2016) show that unanticipated changes in inflation can have persistent effects on aggregate investment by affecting the real burden of long-term nominal debt. Relatedly, Jungherr and Schott (2022) show that risky long-term debt at the firm-level can lead to an amplified investment response to technology shocks and thereby rationalize the slow adjustment of aggregate leverage during recessions. Both of these papers highlight the distortionary effect of the debt overhang channel on firms' investment decisions but abstract from equity issuance costs. Our paper instead focuses on the interplay between long-term debt and firms' net worth.

Second, a large literature has studied the implications of firm debt for macroeconomic fluctuations. These models typically focus on one-period debt and thus abstract from any frictions related to debt with long maturity (e.g. Bernanke et al. (1999), Kiyotaki and Moore (1997), Christiano et al. (2014), Arellano, Bai, and Kehoe (2019), Ottonello and Winberry (2020)). Further, firms typically cannot raise equity in these models.[1] Papers in this literature include a financial accelerator mechanism which links firm investment to changes in net worth, but abstract from the interaction between the value of long-term debt and firms' balance sheets.[2]

A third strand of literature is related to macroeconomic models with financially constrained intermediaries. Gertler and Kiyotaki (2010) and Gertler et al. (2012) study how the balance sheet of financial intermediaries can amplify the effect of shocks. Gertler, Kiyotaki, and Prestipino (2020b) study the macroeconomic effects of bank run episodes. These frameworks typically assume that banks directly hold firm equity. A contribution of our paper is to present a more realistic model in which banks hold long-term loans on their balance sheet.[3]

2 Model

There is a continuum of households, each consisting of a continuum of members. At each point in time a proportion e of household members are entrepreneurs, a proportion b are bankers, and the remaining proportion $1 - e - b$ are workers. We describe the optimization problems of these three types of agents in turn.

[1] Notable exceptions include Jermann and Quadrini (2012), Gertler, Kiyotaki, and Queralto (2012), and Gertler, Kiyotaki, and Prestipino (2020a), which feature short-term debt and (costly) equity adjustments. Ferrante (2019) features long-term debt but assumes infinitely costly equity issuance.

[2] A related literature focuses on firm heterogeneity. Jungherr, Meier, Reinelt, and Schott (2022) show that the investment of firms with a larger amount of maturing debt is more responsive to monetary policy, and rationalize this finding with a heterogeneous firm model with defaultable long-term debt and costly equity adjustments.

[3] In related work, Ferrante (2019) presents a model in which banks lend to firms and households using long-term defaultable debt. Firms cannot adjust equity and the leverage constraint on banks is always binding. Corhay and Tong (2024) study how changes in inflation redistribute resources between financial intermediaries and non-financial firms issuing long-term debt. Firms can adjust equity freely and the model does not capture the occasionally binding nature of banks financial constraint.

2.1 Workers

Workers choose household consumption, C_t, a wage rate, w_{it}, and labor supply, h_i, as well as the amounts of savings in banks deposits, d_t and government bonds, b_t^g to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t - bC_{t-1}\right) - \frac{\psi}{1+\varphi} \int h_{it}^{1+\varphi} di \right]$$

Workers' budget constraint is

$$C_t + d_t + b_t^g = \int w_{it} h_{it} di + d_{t-1} \frac{\tilde{R}_{t-1}^d}{\pi_t} + b_{t-1} \frac{R_{t-1}^g}{\pi_t} + T_t, \tag{1}$$

where $\tilde{R}_{t-1}^d$ and R_{t-1}^g are the nominal rates of return on deposits and government bonds between time $t-1$ and time t, π_t is the rate of inflation, and T_t collects all transfers to the household from firms, entrepreneurs and bankers.

We follow Erceg, Henderson, and Levin (2000) to introduce nominal wage rigidities. In particular, in each quarter only a fraction $1-\theta_w$ of workers can reset their wage optimally. We denote the optimal real reset wage by w_t^o.

A labor agency aggregates individual labor supply into a composite labor input that it sells to firms using a constant elasticity of substitution technology:

$$H_t^{\frac{\varepsilon_w - 1}{\varepsilon_w}} = \int h_{it}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di$$

Optimal demand of labor h_{it} is given by:

$$h_{it} = \left(\frac{w_{it}}{w_t} \right)^{-\varepsilon_w} H_t \tag{2}$$

where w_t is the real wage paid by firms to purchase the labor composite H_t.

Households optimal holdings of deposits and government bonds satisfy:

$$1 = E_t \Lambda_{t+1} \frac{\tilde{R}_t^d}{\pi_{t+1}} = E_t \Lambda_{t+1} \frac{R_t^g}{\pi_{t+1}} \tag{3}$$

where $\Lambda_{t+1} = \beta \frac{U_c(t+1)}{U_c(t)}$ is the household stochastic discount factor with the marginal utility of consumption given by

$$U_c\left(t\right) = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t}.$$

The optimal reset wage, w_t^o satisfies:

$$\sum_{s\geq t}^{\infty}\beta^{s-t}\left(1-\theta_w\right)^{s-t}\left[\varepsilon_w h_{is}^{1+\varphi}\frac{1}{w_t^o}+U_c\left(s\right)\left(1-\varepsilon_w\right)h_{is}\frac{\pi_{wt,s}^{nt}}{\pi_{t,s}}\right]=0 \tag{4}$$

where $\frac{\pi_{wt,s}^{nt}}{\pi_{t,s}}$ is the real wage increase between time t and time s if the worker does not optimize again before time s.

2.2 Entrepreneurs

Entrepreneurs are the agents that hold productive capital in the economy. To simplify the analysis we assume that entrepreneurs rent their capital to firms at a market rental rate z_t. An entrepreneur's return on capital investment is given by

$$\xi_t R_t^k = \xi_t \frac{z_t + (1-\delta)Q_t^k}{Q_{t-1}^k},$$

where ξ_t is an idiosyncratic shock to capital returns, Q_t^k is the price of capital, and δ is the depreciation rate of capital. We assume that $\log(\xi_t)$ is distributed as a Normal with mean 0 and standard deviation σ_{t-1}^l, and we allow for "risk shocks" causing exogenous fluctuations in σ_t^l as in Christiano et al. (2014).

Entrepreneurs finance capital purchases with own equity, x_t, and by issuing long-term nominal debt l_t to bankers. Their flow budget constraint at time t is:

$$Q_t^k k_t = x_t + Q_t^l l_t$$

where Q_t^l is the price of debt.[4]

Each unit of debt issued at $t-1$ pays back a coupon c_l at time t, while the remaining portion $1-\lambda_l$ remains outstanding. An entrepreneur that enters time t with capital k_{t-1} and debt l_{t-1}, has net worth n_t given by

$$n_t = \xi_t R_t^k Q_{t-1}^k k_{t-1} - \frac{\left(c_l + \left(1-\lambda_l\right)Q_t^l\right)}{\pi_t} l_{t-1}. \tag{5}$$

We assume that it is costly for entrepreneurs to adjust the amount of equity that they invest

[4]For ease of notation, we are omitting the dependence of the price of debt on the entrepreneur's financial decisions. We will be explicit about it when describing the entrepreneur's optimality conditions.

in the firm. In particular, entrepreneurs pay a proportion, $1-\omega_t$, of net worth back to the family as a dividend, and keep within the firm an amount of equity, $x_t = f(\omega_t)n_t$, where

$$f(\omega_t) = (1-\overline{\alpha})\omega_t - \frac{\alpha}{2}(\omega_t - \overline{\omega})^2,$$

with $\overline{\alpha} \geq 0$, $\alpha \geq 0$, and $1-\overline{\omega}$ a fixed target dividend payout rate.

This formulation of equity injection costs is a simple way to capture agency problems that limit entrepreneurs ability to substitute equity and debt finance.[5] The parameter $\bar{\alpha}$ represents a small fixed equity issuance cost which guarantees that, around the steady state of the model, entrepreneurs have an incentive to use debt to finance capital purchases.[6] With no equity issuance costs, i.e. $\bar{\alpha} = 0$ and $\alpha = 0$, entrepreneurs are able to efficiently substitute debt finance with equity finance in response to changing financial conditions. This is the assumption, for instance, of Gomes et al. (2016). At the other extreme, when it is infinitely costly to adjust equity issuance, i.e. $\alpha = \infty$, entrepreneurs always retain a fixed share of net worth, $\overline{\omega}$, as equity in the firm, and their only active margin of financial adjustment is through debt. This is the assumption in most papers with financial frictions following Bernanke et al. (1999). Here we allow for α to take on any positive value and estimate it in section 3.2.

Let $V_t(k_{t-1}, l_{t-1}, \xi_t)$ be the optimal value of a firm that enters time t with capital k_{t-1}, debt l_{t-1} and idiosyncratic shock ξ_t. This value is the optimized present discounted value of dividend payouts. It is given by:

$$V_t\left(k_{t-1}, l_{t-1}, \xi_t\right) = \max_{k_t, l_t, \omega_t, n_t} \left\{\left(1-\omega_t\right) n_t + E_t \Lambda_{t,t+1} \max\left\{0, V_{t+1}\left(k_t, l_t, \xi_{t+1}\right)\right\}\right\} \tag{6}$$

subject to (5) and a flow budget constraint

$$Q_t^k k_t = f(\omega_t) n_t + Q_t^l l_t. \tag{7}$$

Let $\eta_t = \frac{l_t}{k_t}$ denote the entrepreneur's leverage ratio, defined as the ratio of debt per unit of capital, and guess that

$$V_t(k_{t-1}, l_{t-1}, \xi_t) = \nu_t(\eta_{t-1}, \xi_t) k_{t-1}. \tag{8}$$

[5] See Jermann and Quadrini (2012) for a similar approach at modelling costly dividend payouts.

[6] This cost plays a role similar to a tax advantage subsidy for debt, used, for example, in Gomes et al. (2016).

Using (5), (7) and (8) to substitute for n_t, k_t and V_t in (6), we get

$$v_t\left(\eta_{t-1},\xi_t\right)=\max_{\eta_t,\omega_t}\mu_t\left(\eta_t;\eta_{t-1},\xi_t\right)\left\{\left(1-\omega_t\right)+\frac{f\left(\omega_t\right)}{\left(Q_t^k-Q_t^l\eta_t\right)}E_t\Lambda_{t,t+1}\max\left\{0,v_{t+1}\left(\eta_t,\xi_{t+1}\right)\right\}\right\} \tag{9}$$

where $\mu_t\left(\eta_t;\eta_{t-1},\xi_t\right)$ is the amount of entrepreneur's net worth per unit of capital, given by

$$\mu_t\left(\eta_t;\eta_{t-1},\xi_t\right)=\frac{n_t}{k_{t-1}}=\xi_t R_t^k Q_{t-1}^k-\frac{\left(c_l+\left(1-\lambda_l\right)Q_t^l\right)}{\pi_t}\eta_{t-1}. \tag{10}$$

Notice that μ_t depends on η_t because the price of the entrepreneur's debt, Q_t^l, will vary depending on the individual entrepreneur's choice of leverage, η_t.

As the value to the entrepreneur is increasing in the idiosyncratic shock to the returns on capital, ξ_{t+1}, there is a threshold value for this shock, $\overline{\xi}_{t+1}$, such that the firm defaults when $\xi_{t+1}<\overline{\xi}_{t+1}$. The threshold value satisfies

$$v_{t+1}\left(\eta_t,\overline{\xi}_{t+1}\right)=0, \tag{11}$$

and it is the value that sets the entrepreneur's equity to zero:

$$\bar{\xi}_{t+1}R_{t+1}^k Q_t^k=\frac{\left(c_l+\left(1-\lambda_l\right)Q_{t+1}^l\right)}{\pi_{t+1}}\eta_t. \tag{12}$$

Equity injections The optimality condition for equity injections is given by the derivative of (9) with respect to ω_t:

$$\begin{aligned}1&=f'\left(\omega_t\right)\frac{E_t\Lambda_{t,t+1}\max\left\{0,v_{t+1}\left(\eta_t,\xi_{t+1}\right)\right\}}{\left(Q_t^k-Q_t^l\eta_t\right)}\\&=f'\left(\omega_t\right)\frac{E_t\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}v_{t+1}\left(\eta_t,\xi_{t+1}\right)dF_t\left(\xi_{t+1}\right)}{\left(Q_t^k-Q_t^l\eta_t\right)}\end{aligned} \tag{13}$$

where F_t is the distribution of idiosyncratic shocks at $t+1$, and the second equality follows from (11). The optimality condition for equity injections requires that the marginal cost of equity injections, which is unity, equals the marginal benefits, given by the marginal increase in entrepreneur's equity, $f'(\omega_t)$, times the expected discounted return on equity injections, which we denote by γ_t:

$$\gamma_t=\frac{E_t\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}v_{t+1}\left(\eta_t,\xi_{t+1}\right)dF_t\left(\xi_{t+1}\right)}{\left(Q_t^k-Q_t^l\eta_t\right)} \tag{14}$$

To characterize γ_t it is useful to introduce the entrepreneur's Tobin's Q on net worth, that is the ratio between the value to the entrepreneur of a unit of net worth and the book value of a unit of net worth, $\varphi_t = \frac{\nu_t}{\mu_t}$. Using (9) and (12) we can write φ_t as:

$$\varphi_t = (1-\omega_t) + f(\omega_t)\frac{Q_t^k}{\left(Q_t^k - Q_t^l \eta_t^l\right)} E_t \Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right) R_{t+1}^k dF_t\left(\xi_{t+1}\right) \tag{15}$$

Using $\nu_{t+1} = \varphi_{t+1}\mu_{t+1}$ together with the definition of μ_{t+1} from (10) to substitute for ν_{t+1} in (14), we can express the expected discounted returns on equity investment as:

$$\gamma_t = \frac{Q_t^k}{\left(Q_t^k - Q_t^l \eta_t\right)} E_t \Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right) R_{t+1}^k dF_t\left(\xi_{t+1}\right), \tag{16}$$

that says that one unit of equity gets leveraged into $\frac{Q_t^k}{\left(Q_t^k - Q_t^l \eta_t\right)}$ units of capital, which repay a return, $\xi_{t+1}R_{t+1}^k$, net of the debt repayment, $\bar{\xi}_{t+1}R_{t+1}^k$, whenever the idiosyncratic shock is larger than the threshold.

The optimality condition for equity injection then becomes

$$1 = f'(\omega_t)\frac{Q_t^k}{\left(Q_t^k - Q_t^l \eta_t\right)} E_t \Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right) R_{t+1}^k dF_t\left(\xi_{t+1}\right). \tag{17}$$

Leverage The optimality condition for leverage is given by:

$$\frac{\partial \mu_t}{\partial \eta_t}\varphi_t + \frac{f(\omega_t)\mu_t}{\left(Q_t^k - Q_t^l \eta_t\right)}\left[\left(Q_t^l + \frac{dQ_t^l}{d\eta_t}\eta_t\right)\gamma_t - E\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\frac{\partial v_{t+1}}{\partial \eta_t} dF_t\left(\xi_{t+1}\right)\right] = 0, \tag{18}$$

where

$$\frac{\partial \mu_t}{\partial \eta_t} = -\frac{(1-\lambda_l)\,\eta_{t-1}}{\pi_t}\frac{dQ_t^l}{d\eta_t}. \tag{19}$$

The first term in (18), $\frac{\partial \mu_t}{\partial \eta_t}\varphi_t$, captures the benefit of increasing leverage through the associated decline in the price of outstanding debt, $\frac{dQ_t^l}{d\eta_t} < 0$. When entrepreneurs have long-term debt, i.e. $\lambda_l < 1$, they have an additional incentive to increase leverage in order to lower the price of debt of preexisting creditors, and hence increasing the net worth of the entrepreneur.

The second term, $\frac{f(\omega_t)\mu_t}{\left(Q_t^k - Q_t^l \eta_t^l\right)}\left(Q_t^l + \frac{dQ_t^l}{d\eta_t}\eta_t^l\right)\gamma_t$, is the marginal benefit of increasing leverage associated with higher returns on capital.

The last term, $-\frac{f(\omega_t)\mu_t}{\left(Q_t^k - Q_t^l \eta_t^l\right)}E\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\frac{\partial v_{t+1}}{\partial \eta_t}dF_t\left(\xi_{t+1}\right)$, captures the marginal cost of leverage.

Using the envelope condition we get:

$$\frac{dv_t}{d\eta_{t-1}} = -\frac{\left(c_l + (1-\lambda_l)\, Q_t^l\right)}{\pi_t}\varphi_t. \tag{20}$$

So the marginal cost of leverage is given by the cost of repaying the loans in case of non default:

$$E_t\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\frac{\partial v_{t+1}}{\partial \eta_t}dF_t\left(\xi_{t+1}\right) = E_t\Lambda_{t,t+1}\frac{\left(c_l + (1-\lambda_l)\, Q_{t+1}^l\right)}{\pi_{t+1}}\varphi_{t+1}\left(1 - F_t\left(\bar{\xi}_{t+1}\right)\right).$$

Using (19) and (20) in (18) we can rewrite the optimality condition for entrepreneur's leverage as:

$$\begin{gathered}\frac{(1-\lambda_l)\eta_{t-1}}{\pi_t}\frac{dQ_t^l}{d\eta_t}\varphi_t + \frac{f(\omega_t)\mu_t}{\left(Q_t^k - Q_t^l\eta_t^l\right)}\left(Q_t^l + \frac{dQ_t^l}{d\eta_t}\eta_t^l\right)\frac{Q_t^k}{\left(Q_t^k - Q_t^l\eta_t^l\right)}E_t\Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1} - \bar{\xi}_{t+1}\right)R_{t+1}^k dF_t\left(\xi_{t+1}\right) \\ = \\ \frac{f(\omega_t)\mu_t}{\left(Q_t^k - Q_t^l\eta_t^l\right)}E_t\Lambda_{t,t+1}\varphi_{t+1}\frac{\left(c_l + (1-\lambda_l)Q_{t+1}^l\right)}{\pi_{t+1}}\left(1 - F_t\left(\bar{\xi}_{t+1}\right)\right)\end{gathered} \tag{21}$$

Equations (12), (17) and (21), together with the definition of μ_t and φ_t in (10) and (15), are the optimality conditions for the entrepreneurs problem. We now turn to describing aggregation of the entrepreneurs' policy fucntions.

2.3 Aggregation of entrepreneurs choices

At the beginning of each period, all entrepreneurs return their net worth to the family. Defaulting entrepreneurs exit and become workers and are replaced by an equal number of new entrepreneurs.

New entrants receive a transfer T_t^e and use these resources to purchase capital

$$Q_t^k k_t^e = T_t^e + Q_t^l l_t^e$$

To preserve aggregation, we make two assumptions. First we assume that new entrants debt, l_t^e, is adjusted so that new entrants' leverage is the same as existing entrepreneurs, that is

$$l_t^e = \eta_t k_t^e$$

Second, we assume that non defaulting entrepreneurs are all given the same amount of capital,

$\tilde{\xi}_t K_{t-1} = \frac{1}{1-F_{t-1}(\bar{\xi}_t)} \int_{\bar{\xi}_t} \xi_t dF_{t-1}(\xi_t) K_{t-1}$ from the family before making their choices at time t. This ensures that the μ_t in (21) is constant across non defaulting entrepreneurs and hence the leverage choice is constant across all entrepreneurs active at time t.

Aggregate equity of entrepreneurs at time t is then given by:

$$X_t = f(\omega_t) \left[\left(\tilde{\xi}_t - \bar{\xi}_t \right) R_t^k Q_{t-1}^k \right] K_{t-1} \left(1 - F\left(\bar{\xi}_t^l\right)\right) + T_t^e$$

which sums the equity of entrepreneurs that do not default at time t and the transfer to new entrepreneurs. Aggregate capital demand is then given by

$$K_t = \frac{X_t}{\left(Q_t^k - \eta_t Q_t^l\right)},$$

with equations (17) and (21) that determine equity injections, ω_t, and leverage η_t.

2.4 Financial Intermediaries

At each point in time, a fraction b of household members are bankers managing a financial intermediary. At the beginning of each period, a fraction $(1 - \sigma_b)$ of bankers returns to the family and are replaced by an equal number of new bankers, so that the relative proportion of bankers and non-bankers within the family remains constant. New bankers enter with some startup funds from the family, as we discuss below. This approach is similar to the one used in Gertler and Karadi (2011). However, compared to Gertler and Karadi (2011), we assume that banks hold debt rather than equity in non-financial corporations. In addition to improving the realism of the model, this assumption allows us to study the implications of non-financial debt maturity for banks' balance sheets.

Each bank j, obtains deposits $d_{j,t}$ and invests in a continuum of long-term defaultable corporate loans, $l_{j,t}$, issued at price Q_t^l. Potentially each of these loans could have a different probability of default and a different price. However, as shown in the previous section, all entrepreneurs choose the same leverage, implying that bank loans will be priced with the same Q_t^l by every banker.[7]

[7] In addition, the loan price could also be bank-specific, but we show below that, in our model, the pricing equation will be the same for every banker

A financial intermediary uses its own net worth, n_{jt}^{b}, and deposits to invest in loans:

$$Q_t^l l_{j,t} = n_{jt}^b + d_{j,t}. \tag{22}$$

A banker's net worth is equal to the difference between the return on assets and the return on liabilities:

$$n_{t+1}^b = R_{t+1}^{l,b} Q_t^l l_{j,t} - R_{t+1}^d d_{j,t} \tag{23}$$

where $R_t^d = \frac{\tilde{R}_t^d}{\pi_t}$ is the real return on deposits and the variable $R_{t+1}^{l,b}$ is the return on firms' loans, which takes into account the possibility of default according to

$$R_{t+1}^{l,b} = [1 - F_t(\bar{\xi}_{t+1})] R_{t+1}^l + \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t Q_t^l} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t(\xi_{t+1}). \tag{24}$$

The first term in (24) represents the real return on loans to non-defaulting firms

$$R_{t+1}^l = \frac{1}{\pi_t} \frac{\left(c_l + (1 - \lambda_l) Q_{t+1}^l\right)}{Q_t^l}$$

whereas the second term represents the recovery value on defaulted loans, which is equal to a fraction γ_l of the capital of defaulting firms. Because each bank lends to a continuum of entrepreneurs, $R_{t+1}^{l,b}$ also represents the realized return on each bank's loan portfolio.

The banker's objective is to maximize the payouts to the representative family upon exit, which can be written recursively as

$$V_{jt}^b = E_t \Lambda_{t+1} \left[(1 - \sigma_b) n_{jt+1}^b + \sigma_b V_{jt+1}^b \right] \tag{25}$$

where Λ_{t+1} is the stochastic discount factor of the representative household between time t and t+1.

We follow Gertler and Karadi (2011) to introduce an agency problem between bankers and depositors. In particular, we assume that bankers can divert a portion θ^l of their loans and stop operating their financial subsidiary. As a result, depositors will limit the amount they lend to bankers to ensure that bankers do not have an incentive to divert funds:

$$V_{jt}^b \geq \theta^l Q_t^l l_{j,t}. \tag{26}$$

Given these assumptions, the problem of the banker will be to choose $l_{j,t}, d_{j,t}$ and n_{jt}^b in order to optimize (25) subject to (22), (23) and (26). Given the linearity of the problem it can be shown that the value function satisfies $V_t^b = \psi_t n_{jt}^b$ where ψ_t represents the bank's franchise value, that is the ratio between the marginal value of wealth inside the bank and the marginal value of wealth to the household. We will show that this variable does not depend on bank specific characteristic.

If we define bank leverage as $\phi_{jt}^b = Q_t^l l_{j,t}/n_{jt}^b$ we can then rewrite the bank problem as

$$\max_{\phi_{jt}^b} E_t \Lambda_{t+1} \left[(1-\sigma_b) + \sigma_b \psi_{t+1}\right] \left[\phi_{jt}^b \left(R_{t+1}^{l,b} - R_{t+1}^d\right) + R_{t+1}^d\right] n_{jt}^b \tag{27}$$

$$s.t.$$

$$\psi_t \geq \theta^l \phi_{jt}^b \tag{28}$$

The first order condition for leverage implies

$$\mu_t^l = E_t \Omega_{t+1} \left(R_{t+1}^{l,b} - R_{t+1}^d\right) = \theta^l \varepsilon_t^l \tag{29}$$

where μ_t^l represents the expected discounted excess return on loans, $\Omega_{t+1} = \Lambda_{t+1}\left[(1-\sigma_b) + \sigma_b \psi_{t+1}\right]$ is the banker's stochastic discount factor, and ε_t^l is the multiplier on the incentive constraint. We can think of the spread $E_t \left(R_{t+1}^{l,b} - R_{t+1}^d\right)$ as a liquidity premium, as defined in Bocola (2016) or Ferrante (2019), which, together with the default premium $E_t \left(R_{t+1}^{l} - R_{t+1}^{l,b}\right)$, affects the total cost of credit for entrepreneurs. In addition, ψ_t satisfies

$$\psi_t = E_t \Lambda_{t+1} \left[(1-\sigma_b) + \sigma_b \psi_{t+1}\right] \left[\phi_{jt}^b \left(R_{t+1}^{l,b} - R_{t+1}^d\right) + R_{t+1}^d\right] \tag{30}$$

To characterize the solution for the banker's problem, we have to consider two possible cases. If the incentive constraint in (28) does not bind, then excess returns are zero and the banker is indifferent between any leverage choice:

$$E_t \Omega_{t+1} \left(R_{t+1}^{l,b} - R_{t+1}^d\right) = 0 \tag{31}$$

If instead the incentive constraint binds, then $\mu_t^l > 0$ and the optimality condition for leverage is given by the incentive constraint at equality:

$$\psi_t = E_t \Omega_{t+1} \left[\phi_{jt}^b \left(R_{t+1}^{l,b} - R_{t+1}^d\right) + R_{t+1}^d\right] = \theta^l \phi_{jt}^b. \tag{32}$$

The two optimality conditions for leverage, (31) and (32), do not depend on bank specific characteristics so they imply that all banks will choose the same leverage $\phi_{jt}^{b}=\phi_{t}^{b}$ and will have the same ψ_t.

Equation (32) represents an endogenous leverage constraint for the financial intermediary. When bank net worth declines enough for the constraint to bind, in order for (32) to hold banks have to reduce their holdings of loans until lending spreads $E_t\left(R_{t+1}^{l,b}-R_{t+1}^{d}\right)$ are high enough. Higher liquidity premia result in lower loans price Q_t^l which exacerbate the decline in net worth according to a financial accelerator mechanism.

The optimality conditions of the financial intermediary are a key ingredient to determine how the loan price depends on the entrepreneurs' leverage η_t and on aggregate variables. In particular, we can rewrite equations (31) and (32) as two pricing equations for loans to entrepreneurs:

$$E_t\Lambda_{t+1}^{b}R_{t+1}^{l,b}=1 \tag{33}$$

where the banks stochastic discount factor, Λ_{t+1}^{b}, depends on whether or not the constraint binds at time t as follows:

$$\Lambda_{t+1}^{b}=\begin{cases}\frac{\Omega_{t+1}}{E_t\Omega_{t+1}R_{t+1}^{d}} & if\ \mu_t=0\\ \frac{\Omega_{t+1}}{\theta^l+E_t\Omega_{t+1}R_{t+1}^{d}\left(1-\frac{1}{\phi_t^b}\right)} & if\ \mu_t>0\end{cases} \tag{34}$$

Multiplying (33) by Q_t^l we get an expression for entrepreneurs' loans price:

$$Q_t^l=E_t\Lambda_{t+1}^{b}\left\{\left[1-F_t\left(\bar{\xi}_{t+1}\right)\right]\frac{\left(c_l+\left(1-\lambda_l\right)Q_{t+1}^{l}\right)}{\pi_{t+1}}+\gamma_l\frac{Q_t^kR_{t+1}^{k}}{\eta_t}\int_0^{\bar{\xi}_{t+1}}\xi_{t+1}dF_t\left(\xi_{t+1}\right)\right\} \tag{35}$$

When the constraint does not bind the model is equivalent, at a first order, to a framework in which banks are just a veil, and loans are directly priced by households.[8] When bank net worth is low enough to cause the bankers' incentive constraint to bind, loans are priced with the stochastic discount factor $\frac{\Omega_{t+1}}{\theta^l+E_t\Omega_{t+1}R_{t+1}\left(1-\frac{1}{\phi_t^b}\right)}$, which moves inversely with bank leverage. Hence, in the region where banks are constrained, loan prices will incorporate also a liquidity premium which will move inversely with bank net worth.

From equation (35) we can compute the derivative of the loan price with respect to en-

[8] A first order approximation in a neighborhood of the steady state where the constraint is slack satisfies $\psi_t\approx 1$ and $\Lambda_{t+1}^{b}\approx\Omega_{t+1}\approx\Lambda_{t+1}$.

trepreneurial leverage as

$$\frac{\partial Q_t^l}{\partial \eta_t} = -E_t \Lambda_{t+1}^b \left\{ \frac{\partial \bar{\xi}_{t+1}}{\partial \eta_t} f_t\left(\bar{\xi}_{t+1}\right) \left[Q_t^l R_{t+1}^l - \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t} \bar{\xi}_{t+1} \right] + \gamma_l \frac{Q_t^k R_{t+1}^k}{\left(\eta_t\right)^2} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t\left(\xi_{t+1}\right) \right\}$$
$$+ E_t \Lambda_{t+1}^b [1 - F_t\left(\bar{\xi}_{t+1}\right)] \left(1 - \lambda_l\right) \frac{\partial Q_{t+1}^l}{\partial \eta_{t+1}} \eta_{\eta,t+1}\left(\eta_t\right) \quad (36)$$

The top term in (36) includes the impact of entrepreneurs' leverage on next period default threshold and on the expected recovery rate in case of default. Because higher η_t^l results in a higher expected probability of default and in a lower recovery rate, this term is negative. The bottom term in (36) captures the effect of current leverage choice on future leverage, as measured by the derivative of the entrepreneur's leverage policy function $\eta_{\eta,t+1}\left(\eta_t\right) = \frac{\partial \eta_{t+1}}{\partial \eta_t}$.[9] As noted by Gomes et al. (2016), long-term debt introduces an incentive for entrepreneurs to dilute the value of preexisting debt by increasing leverage, and rational lenders take this effect into account. The computation of this derivative complicates the numerical solution of the model, because it cannot be computed with standard perturbation methods. In the appendix, we describe a method which uses global solution techniques to capture the local dynamics of this derivatives around the steady state of the model.

Finally, the evolution of bankers' aggregate net worth N_t^b will be given by

$$N_t^b = \sigma_b \left\{ R_t^{l,b} Q_{t-1}^l L_{t-1} - R_t^d D_t \right\} + T^b N_{t-1}^b \quad (37)$$

where L_t and D_t are aggregate loans ad deposits, and T^b represents a transfer to new bankers proportional to past net worth. Aggregate credit supply is then given by:

$$\phi_t^b = \frac{Q_t^l L_t}{N_t^b}. \quad (38)$$

2.5 Final good producers

The final good Y_t is a CES composite of different intermediate varieties, given by

$$Y_t = \left[\int Y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (39)$$

[9] We are abusing notation here since the policy function is the individual entrepreneur's policy function for leverage as a function of individual entrepreneur's leverage. See Appendix for details.

so that the demand for each variety will be given by

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\varepsilon} Y_t \tag{40}$$

where the aggregate price level for the home good is given by

$$P_t = \left[\int \left(P_t\left(i\right)\right)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{41}$$

2.6 Intermediate goods Producers

The intermediate good is produced with Cobb-Douglas technology by monopolistically competitive intermediate-goods firms

$$Y_t = (K_{t-1})^{\chi} H_t^{(1-\chi)} \tag{42}$$

where H_t and K_{t-1} are aggregate labor and capital

If we define p_t^m as real marginal costs, then real wages and the real rental rate on capital will satisfy

$$W_t = p_t^m (1-\chi)\frac{Y_t}{H_t} \tag{43}$$

$$z_t = p_t^m \chi \frac{Y_t}{K_{t-1}} \tag{44}$$

Retailers with monopoly power sell the variety $Y_t(i)$ at price $P_t(i)$ subject to Calvo-style frictions. In each period a fraction of retailers $1-\theta^p$ can reset their price, while the remaining fraction index their prices to past inflation according to $P_t(i) = \pi_{t-1}P_{t-1}(i)$.

Retailers that reoptimize at time t choose the optimal (real) price p_t^0 to solve:

$$\max E_t \sum_{i\geq 0} \Lambda_{t+i}\left(1-\theta_p\right)^i \left[\frac{p_t^o \pi_{t,t+i}^{nt}}{\pi_{t,t+i}}\left(\frac{p_t^o \pi_{t,t+i}^{nt}}{\pi_{t,t+i}}\right)^{-\varepsilon} Y_{t+i} - p_{t+i}^m \left(\frac{p_t^o \pi_{t,t+i}^{nt}}{\pi_{t,t+i}}\right)^{-\varepsilon} Y_{t+i}\right],$$

which yields a standard optimality condition:

$$p_t^o = E_t \sum_{i\geq 0} \frac{\Lambda_{t+i}\left(1-\theta_p\right)^t \left(\frac{\pi_{t,t+i}^{nt}}{\pi_{t,t+i}}\right)^{-\varepsilon} Y_{t+i}}{E_t \sum_{j\geq 0} \Lambda_{t+j}\left(1-\theta_p\right)^t \left(\frac{\pi_{t,t+j}^{nt}}{\pi_{t,t+j}}\right)^{1-\varepsilon} Y_{t+j}} \frac{\varepsilon}{(\varepsilon-1)} p_{t+i}^m, \tag{45}$$

where $\pi^{nt}_{t,t+i}$ is the inflation of a retailer that does not reoptimize before $t+i$.

2.7 Capital goods production

Capital producers sell capital at price Q^k_t and face convex adjustment costs. They solve:

$$\max E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left[Q_{t+i} I_{t+i} - I_{t+i} - \frac{\gamma_k}{2} (\frac{I_{t+i}}{I_{t+i-1}} - 1)^2 I_{t+i} \right]$$

Optimality implies the following relation between the price of capital and investment:

$$Q^k_t = 1 + \frac{\gamma_k}{2} (\frac{I_t}{I_{t-1}} - 1)^2 + \gamma_k \frac{I_t}{I_{t-1}} (\frac{I_t}{I_{t-1}} - 1) - \beta \Lambda_{t+1} \gamma_k \left(\frac{I_{t+1}}{I_t} \right)^2 (\frac{I_{t+1}}{I_t} - 1) \tag{46}$$

2.8 Monetary policy, shocks, and market clearing

Monetary policy follows a simple inertial Taylor rule that responds to inflation according to

$$\log \left(R^g_{i,t} \right) = (1 - \rho_r) R_{SS} + \rho_r \log \left(R^g_{i,t-1} \right) + \kappa_\pi \log (\pi_t) + \varepsilon^m_t. \tag{47}$$

We assume that $\varepsilon^m_{i,t}$ is a random shock $\varepsilon^m_{i,t} \sim N(0, \sigma^m)$. We also assume that the cross-sectional dispersion of entrepreneurial risk σ^l_t follows an AR(1) process

$$\sigma^l_t = \rho_\sigma \sigma^l_{t-1} + \varepsilon^\sigma_t \tag{48}$$

where ε^σ_t captures a risk shock as defined in Christiano et al. (2014).

Market clearing in the goods and investment market requires

$$Y_t - \nu(1 - \gamma_l) \int_0^{\bar{\xi}^l_t} \xi^l_t dF_t(\xi^l_t) = C_t + I_t + G \tag{49}$$

$$K_t = (1 - \delta) K_{t-1} + I_{it} \tag{50}$$

where the term $(1 - \gamma_l) \int_0^{\bar{\xi}^l_t} \xi^l_t dF_t(\xi^l_t)$ represents default costs, G is government spending, and δ represents capital's depreciation rate. The parameter ν governs what share of default costs translate into a loss of real resources.

3 Numerical Exercises

3.1 Calibration

Table 1 reports the calibration for our baseline model. We calibrate the model so that in steady state the leverage constraint on financial intermediaries is not binding, and, consequently, the bank liquidity premium is zero. In order to take into account the nonlinearities arising from the occasionally binding incentive constraint for banks, we solve the model using the piecewise linear solution algorithm of the OccBin toolkit (Guerrieri and Iacoviello, 2015). In section 3.3 and 3.4 we consider shocks small enough not to trigger the leverage constraint, whereas in section 3.5 we study the implications of larger shocks, or sequences of small shocks, which could push the model in the constrained region.

Most macroeconomic parameters are calibrated to standard values from the literature. For household preferences, we assume log utility and a discount factor equal to .995, implying a steady state real interest rate of 2 percent. We set the habit parameter to 0.75, in line with, for instance, Christiano et al. (2014), the inverse Frisch elasticity is set to unity.

The capital share in production and the capital depreciation rate are set to 0.33 and 0.025 respectively. We set the parameter governing the elasticity of the price of capital to investment, γ_I^k, equal to 2, a value in the range of existing estimates (for example, Gertler and Karadi (2011) use 1.7 and Justiniano, Primiceri, and Tambalotti (2010) estimate a value around 2.5).

The elasticity of substitution across final goods varieties is calibrated to obtain a 10 percent markup in steady state, and the Calvo stickiness parameters θ^p and θ^w are set to standard values from the literature, implying a slope for the Phillips curve for prices and wages of around 0.05. Monetary policy follows a standard inertial Taylor rule responding to inflation, and we assume a constant government spending G equal to 20 percent of output in steady state.

The parameters pertaining to the entrepreneurs and the financial intermediary are specific to our model. We set λ_l to obtain a loan duration of 7 years, a value in line with the mean bond duration reported by Gilchrist and Zakrajšek (2012). The parameters T^e, γ_l and σ_l are jointly selected to hit the following targets: i) a corporate leverage of 0.5 as in Christiano et al. (2014); ii) a spread on corporate loans of 110 basis points, in line with the average BAA-AAA spread over 1980-2019; iii) an annual default rate in steady state of 1.5 percent, between the values of

1 percent of Gomes et al. (2016) and 2 percent of Christiano et al. (2014). We assume that 100 percent of default costs entail a resource loss, as in Christiano et al. (2014) and Gomes et al. (2016). We set $\bar{\omega} = .01$ implying a steady state dividend/output ratio of about 4.5 percent, in line with the data. The parameter governing the equity adjustment costs, α, is a novel parameter which plays an important role in our model, because it determines whether long-term debt dampens or amplifies macroeconomic shocks. We describe the empirical procedure we employ to discipline this parameter in the next section.

As regards the bank's parameters, we assume a bank's survival rate σ_b equal to .98, in line with Gertler and Karadi (2011). The parameters θ_l and T^b are jointly calibrated to obtain a steady state bank leverage of 9.5 and a level of bank net worth about 5 percent above what required by the leverage constraint.

3.2 Estimation of the equity adjustment cost parameter

The parameter α controls the quadratic adjustment costs of equity issuance. It is critical in determining the effect of long-term debt in transmitting shocks to the economy. We use our model-implied restriction on this parameter to directly estimate its value. To a first order, the value of α will only affect the optimality condition for entrepreneurs' equity injections. In the model, returns on equity injections are defined as

$$R^x_{t+1} = \frac{\int_{\bar{\xi}_{t+1}}^{\infty} v_{t+1}\left(\eta_t, \xi_{t+1}\right) dF_t\left(\xi_{t+1}\right)}{Q^k_t - Q^l_t \eta_t}.$$

The associated optimality condition, given by (17) and reprinted here for convenience, shows how α relates to the expected return to equity injections:

$$E_t \Lambda_{t,t+1} R^x_{t+1} = \frac{1}{f'\left(\omega_t\right)} = \frac{1}{1 - \bar{\alpha} - \alpha(\omega_t - \bar{\omega})} \tag{51}$$

We then linearize (51) to obtain:

$$E_t r^x_{t+1} - r_{t+1} = \frac{\alpha}{1 - \bar{\alpha}} d\omega_t \tag{52}$$

where r^x_{t+1} and r_{t+1} are percent deviation of R^x_{t+1} and the real risk free rate from steady state.[10]

We use firm-level balance sheet data from Compustat to estimate this relationship in the

[10]The parameter $\bar{\alpha}$ is a very small number and does not affect the regression in a meaningful way.

data. Our sample consists of approximately 21,000 US non-financial firms between 1985q1 and 2023q1.[11] We define the return on equity, $R^x_{i,t+1}$, as the time $t+1$ cum-dividend market value of equity of firm i over the same firm's time t ex-dividend book value of equity, $R^x_{i,t+1} = \frac{V_{i,t+1}}{X_{i,t}}$. The market value of equity, $V_{i,t+1}$, is measured as the end-of-quarter market value of equity (using Compustat items `cshoq` and `prccq`). The book value of equity, $X_{i,t}$, is given by `ceqq`. Net dividend payouts are given by dividends (`dvy`), minus net repurchases (`sstkyq - prstkcyq`). This lets us define ω_{it} as one minus the fraction of net payouts to shareholders over the cum-dividend book value of equity.

We run the following regression using Compustat data:

$$\frac{R^x_{i,t+1}}{1+r_{t+1}} = \alpha\omega_{i,t} + \delta_i + \delta_t + \nu_{it}, \tag{53}$$

where r_{t+1} is the quarterly realized real rate and δ_i and δ_t are firm- and time fixed effects.

This approach produces a value of $\alpha = 8.54$ with a standard error of 0.26. The result is robust to the inclusion of the fixed effects.

3.3 The role of equity adjustment costs

We start by illustrating the role of equity adjustment costs in transmitting a pure inflation shock in a version of our economy without nominal rigidities and without real costs of default, i.e. $\nu = 0$. We consider the case of no equity adjustment costs and infinite equity adjustment costs, and compare the responses with long-term debt and one period short-term debt.[12]

When it is costless to adjust equity injections at the margin, i.e. $\alpha = 0$, the optimality conditions for the entrepreneur's problem can be summarized by the following three equations:

$$K_t = \frac{\omega_t N_t}{Q^k_t - Q^l_t \eta_t}, \tag{54}$$

$$1 = \frac{Q^k_t}{\left(Q^k_t - Q^l_t \eta_t\right)} E_t \Lambda_{t,t+1} \int_{\bar{\xi}_{t+1}}^{\infty} \left(\xi_{t+1} - \bar{\xi}_{t+1}\right) R^k_{t+1} dF_t\left(\xi_{t+1}\right), \tag{55}$$

[11] Details about the data can be found in the Appendix.

[12] In the appendix, we show that when α is exactly equal to zero, the first order conditions of the entrepreneur's problem do not select an optimum, but a saddle point. Therefore, we can think of the case with no equity adjustment cost that we study below as a calibration in which adjustment costs are arbitrarily small. In addition, it can be shown that as α goes to zero, the derivative of the leverage policy function, η_η, goes to zero as well.

$$\left(1 - F_t\left(\bar{\xi}^l_{t+1}\right)\right) E_t \Lambda_{t,t+1} \frac{R^l_{t+1}}{\pi_{t+1}} = (1 + \epsilon_{\eta t}) - \epsilon_{\eta t} \frac{(1-\lambda_l)\,\eta_{t-1}}{\eta_t} \frac{1}{\pi_t} \frac{K_{t-1}}{K_t}, \tag{56}$$

where $\epsilon_{\eta t} = \frac{dQ^l_t}{d\eta_t} \frac{\eta_t}{Q^l_t} < 0$ is the elasticity of the price of debt.

Equation (54) is the entrepreneur's budget constraint. Equation (55) is the optimality condition for leverage, equation (17), where we used the fact that $f'(\omega_t) = 1 = \varphi_t$ when $\alpha = 0$. Equation (56) is the optimality condition for leverage, equation (21), after dividing through by Q^l_t and using (55).

With no equity adjustment costs, the value of entrepreneurial net worth, N_t, does not matter for capital demand as entrepreneurs can always adjust equity injections ω_t, to satisfy any given level of capital demand. The level of capital demand is determined by equation (55), which requires that the rate of return on capital leaves the entrepreneurs indifferent between paying out dividends and injecting equity. This rate of return will depend on the leverage choice of entrepreneurs, which affects both the total amount of capital per unit of equity, $\frac{Q^k_t}{Q^k_t - Q^l_t \eta_t}$, and the expected default threshold $\bar{\xi}_{t+1}$.

Figure 2 shows the response of the economy to an exogenous decrease in inflation with no persistence.[13] The blue solid line is the economy with long-term debt, $\lambda_l = \frac{1}{28}$, while the dotted red line is an economy with short-term debt, $\lambda_l = 1$. The decline in inflation causes the real value of entrepreneurs debt to increase and hence their net worth to decline. In the economy with short-term debt, entrepreneurs react to this decline in net worth by reducing dividends payouts, hence increasing equity injections, and keeping the allocation unaffected. This is because with short-term debt, the optimal choice of leverage is not affected by the decline in inflation. In contrast, with long-term debt, the increase in the real value of debt raises the entrepreneurs benefits from leverage, as shown by the last term in equation (56). This is because the larger value of debt makes it more profitable for the entrepreneur to dilute the preexisting creditors position by increasing leverage and hence decreasing the price of debt. The increase in leverage causes expected defaults to increase and the required expected return on capital implied by optimal equity injections in equation 56 to rise. As a result, in the economy with long-term debt, investment declines in response to a temporary decline in inflation. This mechanism captures the *debt overhang channel* described, for example, in Gomes et al. (2016).

[13] This is the same experiment as in Proposition 1 of Gomes et al. (2016). In our flexible price economy we implement this shock by removing the monetary policy rule and replacing it with $\pi_t = exp(\epsilon^\pi_t)$ where ϵ^π_t is an i.i.d. random variable.

Turning to the case of infinite equity adjustment costs, i.e. $\alpha = \infty$, the optimality conditions are given by:

$$K_t = \omega \frac{K_{t-1}\left(\xi_t R_t^k Q_{t-1}^k - \frac{\left(c_l + (1-\lambda_l) Q_t^l\right)}{\pi_t}\eta_{t-1}\right)}{\left(Q_t^k - Q_t^l \eta_t\right)} \tag{57}$$

$$\omega_t = \bar{\omega} \tag{58}$$

$$\left(1 - F_t\left(\bar{\xi}_{t+1}^l\right)\right) E_t \Lambda_{t,t+1} \varphi_{t+1} \frac{R_{t+1}^l}{\pi_{t+1}} = \gamma_t\left(1 + \epsilon_{\eta_t}\right) - \epsilon_{\eta_t} \frac{\left(1-\lambda_l\right)\eta_{t-1}}{\eta_t}\frac{\varphi_t}{\pi_t}\frac{k_{t-1}}{k_t} \tag{59}$$

In this case, as entrepreneurs cannot adjust equity injections, the response of net worth is key in determining the overall response of the economy. When net worth declines entrepreneurs want to increase their leverage, but their ability to do so is limited by the effect of higher leverage on expected default. Two features will determine the overall response of investment to a given shock: the magnitude of the decline in net worth and the degree to which entrepreneurs increase leverage in response to this decline.

Figure 3 shows the response of the economy to the same decrease in inflation as in Figure 2 but in the case of infinite adjustment costs of equity. The decline in net worth caused by the increase in the real value of debt leads to a decline in investment in both the economy with long-term debt, blue solid line, and in the economy with short-term debt, red dotted line.[14] The investment drop however is dampened in the economy with long-term debt. This is due to two reasons. First as entrepreneurs increase leverage the price of debt goes down, dampening the drop in net worth. Second, the elasticity of leverage to a given drop in net worth is higher with long-term debt because of the extra benefit of leverage that comes from diluting pre-existing creditors. Due to this debt overhang channel, leverage is higher in the economy with long-term debt, even with a smaller drop in equity, and this dampens the contraction in investment. Interestingly, the same channel that is responsible for the drop in investment when there are no costs of adjusting equity, causes a smaller drop when it is impossible to adjust equity.

3.4 Debt maturity and financial shocks

We now turn to study the response of our baseline model to financial shocks. In our baseline, parameters are as reported in Table 1. In particular, with respect to the model version we

[14] The economy with infinite equity adjustment costs and short-term debt is very similar to the framework considered in Bernanke et al. (1999) and Christiano et al. (2014).

used in the previous section, here we turn on nominal rigidities and the real costs of default, and we set equity adjustment costs in line with the empirical evidence in section 3.2. The estimated value of $\alpha = 8.54$ implies that it is quite costly for firms to adjust their equity in response to shocks. The magnitude of the shocks considered in this section is small enough that intermediaries' financial constraint is never binding in the simulations, so that we can ignore the banks' balance sheet for these experiments.

Figure 4 reports the impulse responses to a monetary policy shock which causes the policy rate to increase by 100 basis points on impact. The red dashed line reports the response of the economy with short-term debt. In this case, higher interest rates cause a decline in the price of capital Q_t^k and lower inflation, which result in a drop in firms' net worth and in higher default rates. Lenders charge a higher default premium and restrict the supply of credit, amplifying the contraction in investment and in net worth through a standard financial accelerator channel, as in Bernanke et al. (1999). Despite a reduction in dividend payouts by more than 20 percent, firms' equity declines by about 3 percent and aggregate investment declines by about 2.5 percent. The blue line in figure 4 shows that the response to the same shock is dampened if we consider long-term debt. This result is due to the two forces described in the previous section. First, the price of long-term debt declines by about 0.7 percent in response to a higher path of the policy rate, dampening the decline in entrepreneurial net worth. Second, the debt dilution incentive results in a higher leverage compared to the short-term debt model, supporting the demand for capital. These effects are absent with short-term debt, and they become stronger the longer the maturity of corporate loans. Compared to the model with short-term debt, with long-term debt equity declines 50 percent less, investment declines 40 percent less and output by about 15 percent less. [15]

Figure 5 reports the impulse responses of our model to an unanticipated risk shock, ϵ_t^σ, with a persistence of 0.97, as in Christiano et al. (2014). A higher value of σ_t^l increases the future dispersion of the idiosyncratic entrepreneurial shocks, implying persistently higher future defaults. As a result, lenders contract the supply of credit sharply, as witnessed by the decline in firms' leverage. A higher external finance premium implies a lower investment demand

[15] The smaller difference in the output response between the long-term debt model and the short-term debt model, compared to the investment response, is due to the lower investment to output ratio in the steady state of the short-term debt model.

and, through a decline in Q_t^k, a lower net worth which ignites the standard financial accelerator mechanisms. However, also with this type of shock, the response of the economy with long-term debt is quite smaller compared to the one with short-term obligations. In fact, higher future defaults imply a large and persistent drop in the price of long-term debt, Q_t^l, which prevents net worth from declining. In addition, due to the debt overhang effect, the decline in leverage is quite smaller with long-term debt. As a result, the drop in investment and output are about 70 percent and 50 percent smaller with long-term debt. By estimating a model nested within our framework, with infinite equity adjustment costs and short-term debt, Christiano et al. (2014) find that this financial shock is the main driver of business cycle fluctuations. However, the impulse responses in figure 5 suggest that the effects of the risk shock on real and financial variables are very sensitive to the assumptions on corporate debt maturity.

3.5 Debt maturity and financial intermediaries

Part of the reason why long-term debt delivers a dampening of the effects of contractionary financial shocks in our model is that lower loan prices mean lower real value of firms' debt and higher net worth, which is a relevant state variable for aggregate investment when equity adjustment costs are large enough. However, long-term debt represents an asset on the lenders' balance sheet, so that a decline in Q_t^l causes a redistribution of resources from financial intermediaries to firms, and a deterioration in banks aggregate value of equity, as shown in 37.[16] In the quantitative experiments performed so far, the decline in N_t^b was not large enough to cause the banks leverage constraint to bind for a prolonged period.

Figure 6 presents the impulse responses to a monetary policy shock twice as large as the one used in Figure 4. The dashed line represents the model response if banks did not face any constraint. In this case the response of all variables is simply twice the responses shown in figure 4. In particular, higher interest rates cause the price of long-term loans to decline by about 1.5 percent on impact, resulting in a decline in bank's net worth of about 10 percent. When banks are unconstrained, they can make up for the lower equity by raising more deposits, making their balance sheet irrelevant for the aggregate dynamics of the model. The solid line, instead, takes

[16] At the same time, lower unexpected inflation causes a transfer from borrowers, who are short nominal debt, to lenders, who are long nominal assets, on impact. However, in our experiments, the effect of lower loan prices dominates.

into account the possibility that the financial constraint on banks becomes binding. Lower bank net worth causes the leverage constraint in 32 to bind. As a result, banks offload loans from their balance sheet, causing a liquidity premium on corporate lending, $E_t(R^{l,b}_{t+1} - R^d_{t+1})$ to materialize, as shown in the bottom right panel of figure 6. This additional bank spread causes a larger decline in Q^l_t compared to the model with unconstrained banks, amplifying the decline in bank net worth according to the financial accelerator mechanism described in Gertler and Kiyotaki (2010), and leading the economy into a constrained regime lasting eight quarters. Banks' liquidity premium is linked to an endogenous contraction in credit supply which forces firms to leverage up less compared to the model without constrained intermediaries (middle panel of figure 6). This effect is akin to the reduction in credit caused by an exogenous risk shock, and it dominates the positive effect on investment coming from the lower debt value. As a result, investment and output decline almost 50 percent more than in the model with unconstrained banks.

In order for the model to enter the region in which the banks' leverage constraint binds, we do not necessarily need a very large monetary policy shock. Figure 7 reports the response of our baseline model to a sequence of small monetary shocks for six periods followed by a larger shock of the same magnitude as the one used in the experiment in figure 4. These shocks cause an initial slow decline in the price of debt and in bank net worth. After 5 quarters the constraint starts binding slightly, but the amplification with respect tho the unconstrained model, as measured by the difference between the solid and dashed line, is small. However, given the low level of bank net worth after six quarters, the larger shock in the seventh quarter pushes the system well into the constrained region, as suggested by the spike in the bank spread. As a result, after seven quarters the amplification coming from the banking sector is quite substantial. Figure 7 also reports the response to the same sequence of shocks of the model with short-term debt. Importantly, without a time varying price of loans bank net worth moves much less, and it actually increases over time due to the positive effects of unexpected deflation on the real value of one-period nominal loans. As a result, the financial sector never approaches the constrained region. Interestingly, the baseline model with long-term debt initially delivers some dampening in the response of investment compared to the short-term debt model, in line with the evidence from the previous sections, but once bank net worth declines enough the effect on investment and output becomes much larger with long-term debt.

Our model also allows us to study the interaction between banks leverage constraints and firms constraints on issuing equity. Figure 8, replicates the experiment of figure 6 but assuming that firms can freely adjust equity, that is $\alpha = 0$. Interestingly, in this case, a binding leverage constraint for banks results in a smaller economic contraction. The lower value of loans brings the financial intermediaries to the constraint in the first period, as witnessed by the spike in the bank spread. However, with $\alpha = 0$ firms can substitute away from more expensive debt towards equity. As a result, leverage declines persistently, implying lower default rates and a smaller debt overhang effect, which results in a higher path of investment. Hence this experiment suggests that firms' equity adjustment costs are also important for evaluating the macroeconomic effects of shocks originating in the financial sector.

The exercises in this section suggest that the balance sheet of the lenders is another important element to assess the implications of longer debt maturity for the transmission of monetary policy. For example, the events of March 2023, when Silicon Valley Bank, and other regional lenders, suffered very sudden bank runs caused by the deterioration of the value of their long-term assets, due to the steep monetary tightening performed by the Federal Reserve, highlight a mechanism similar to the one described in figure 6 and 7.

4 Conclusion

In the U.S. most of corporate debt is long-term. In addition, debt typically represents the main financing channel for firms. This paper studies the macroeconomic implications of these stylized facts in a medium scale New-Keynesian model.

Our main contribution is to show that the interaction between debt maturity and equity adjustment costs is key to gauge the dynamic response of aggregate investment to macroeconomic shocks. When firms cannot adjust equity, longer debt maturity generates some insurance against negative shocks. This result is due to two forces: First, the price of outstanding long-term debt typically declines with contractionary financial shocks, dampening the deterioration of firms' net worth; second, a debt dilution incentive causes firms' leverage to increase more, supporting aggregate investment. After estimating equity adjustment costs consistent with empirical data on firms' net dividend payouts, we show that these effects can result in a significantly smaller effect of monetary shocks and risk shocks, compared with a model with short-term debt.

Finally, we extend the model by introducing financial intermediaries facing an occasionally binding leverage constraint. We show that the effect of financial shocks can be amplified if the decline in the value of long-term corporate debt is large enough to cause a credit crunch.

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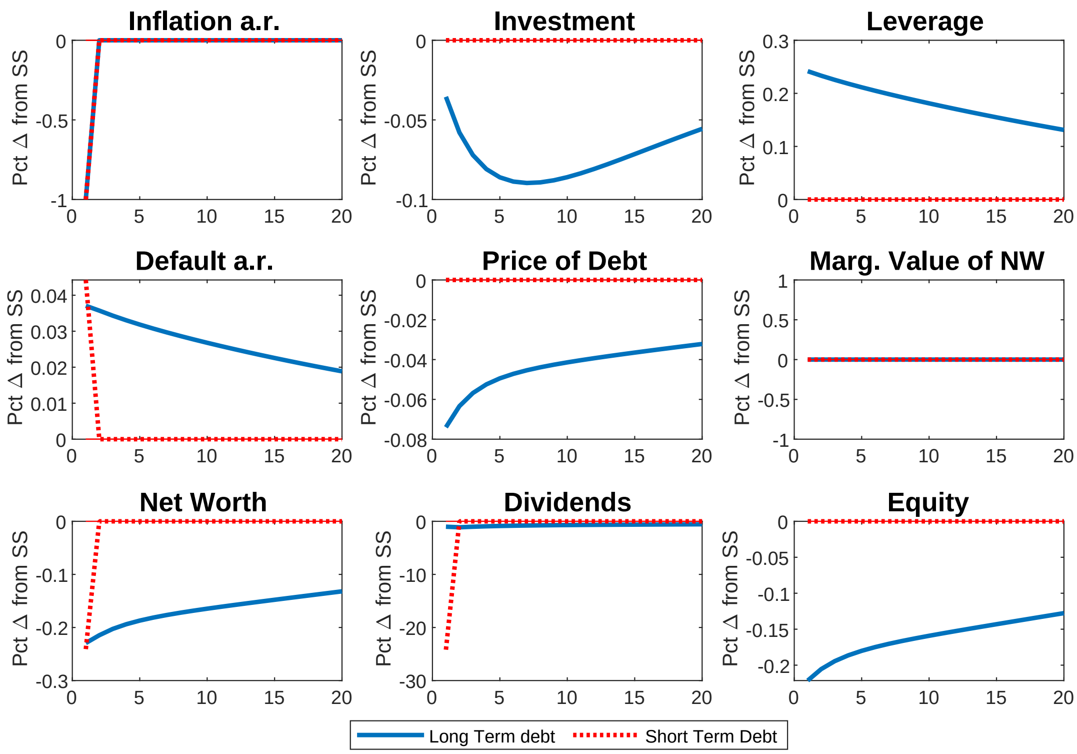
Table 1: Calibration

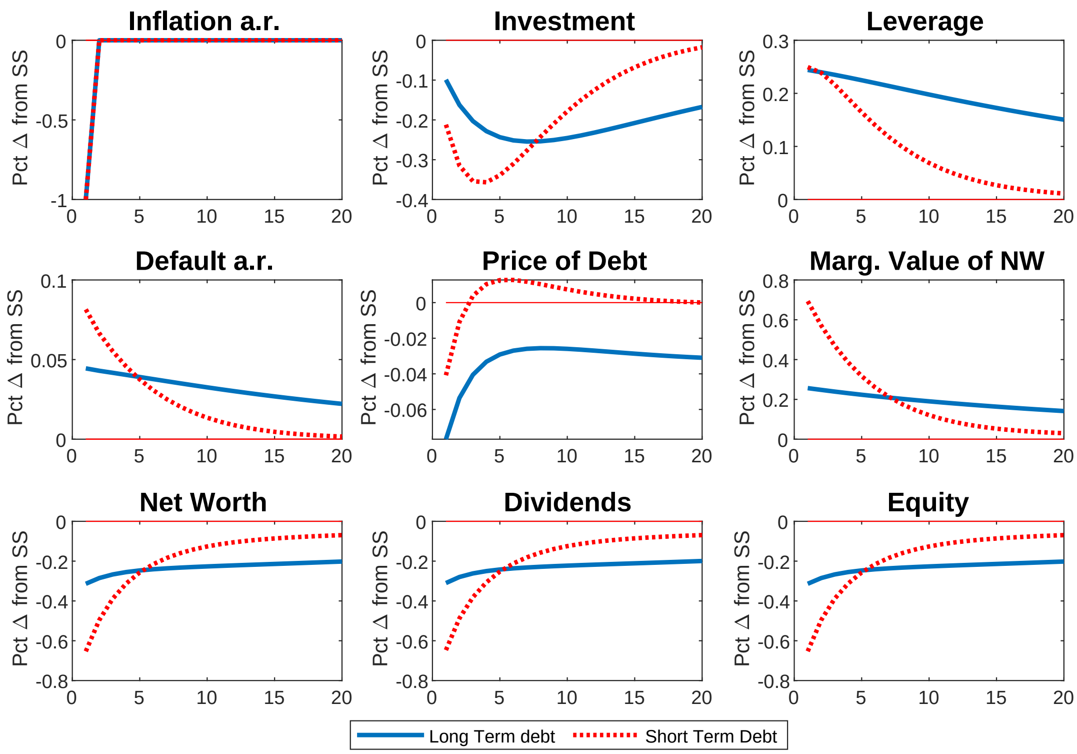
Parameter	Value	Description	Target/Source
Households			
β	.995	Discount factor	$R_{SS} = 2\%$
σ	1	IES	standard
φ	1	Inverse Frisch El.	Standard
h	.75	Habit parameter	Christiano et al. (2014)
Entrepreneurs			
λ_l	.0357	Debt duration	7yrs loan maturity
T^e	.06	Transfer to new ent.	$\eta^l = .5$
γ_l	.54	Loans recovery rate	$4(R^l - R) = 1\%$
σ_l	.2566	Variance idiosync. risk	1.5% Ann. default rate
ν	1	Share of real default costs	Gomes et al. (2016)
$1 - \bar{\omega}$	.01	Dividend Payout	$Div/Y = .045$
α	8.54	Adjustment cost on equity	see text
Financial Intermediaries			
θ^l	.1	Divertable Loans	see text
T^b	.01	Transfer to new ent.	$\phi_b = 9.5$
σ^b	.98	Bankers survival rate	Gertler and Karadi (2011)
Intermediate production			
χ	.33	Capital share	Standard
ε	11	Elasticity of substitution	Standard
θ^p	0.8	Calvo price stickiness	Standard
θ^w	0.8	Calvo wage stickiness	Standard
Investment production			
δ_k	.025	Capital depreciation rate	Standard
γ_I^k	2	Capital Inv. adj. cost	Gertler and Karadi (2011)
Government			
κ_π	1.5	Taylor coeff. on inflation	Standard
ρ_r	0.8	Taylor rule inertia	Standard
G/Y	0.2	Gov. expenditure over GDP	Standard

Figure 2: Inflation shock in model with flexible prices: no equity adjustment costs



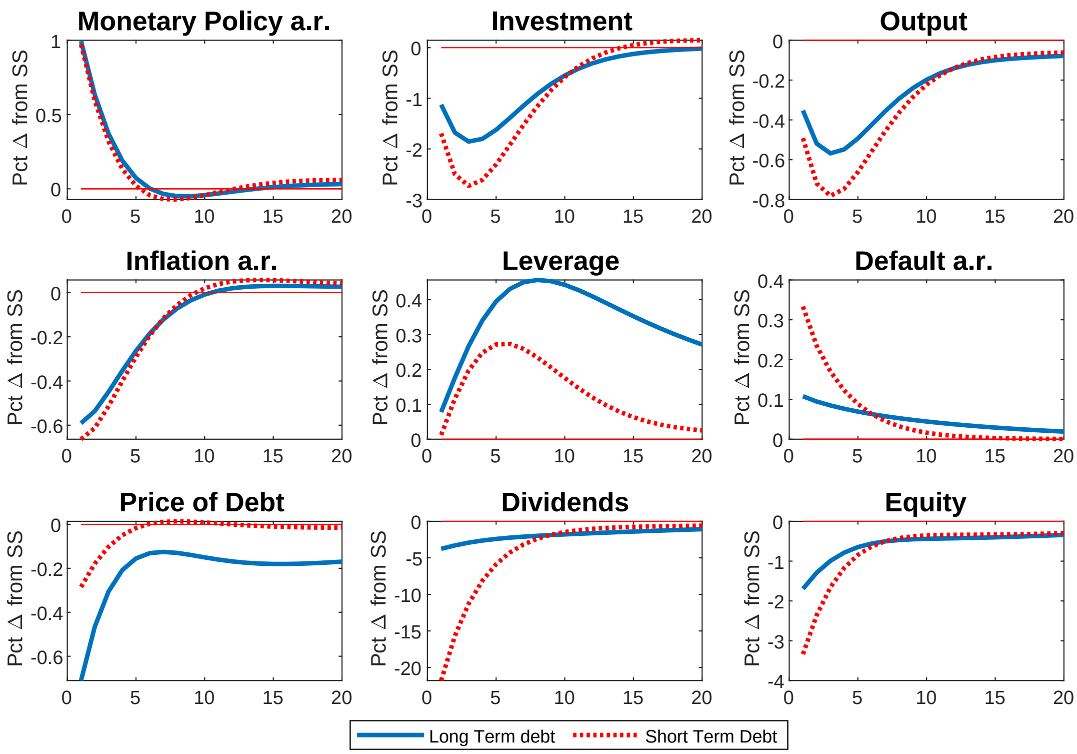
Notes: This figure shows the impulse response to a one time shock to inflation in a simple version of the main model with flexible prices and no real default costs.

Figure 3: Inflation shock in model with flexible prices: infinite equity adjustment costs



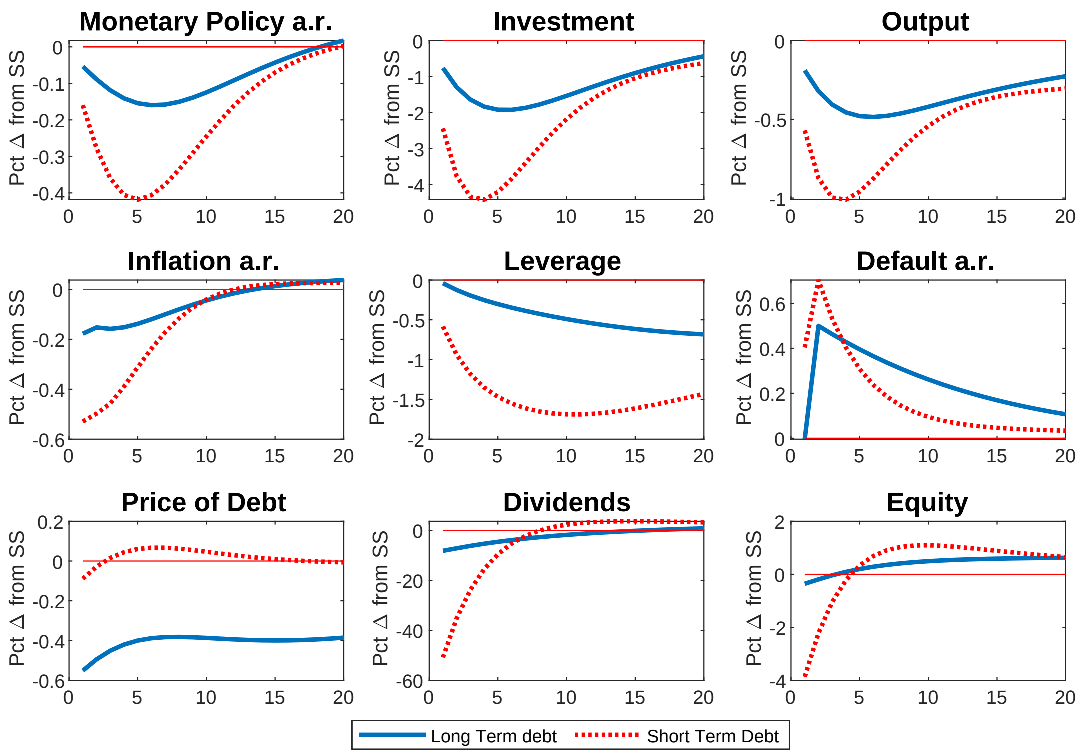
Notes: This figure shows the impulse response to a one time shock to inflation in a simple version of the main model with flexible prices and no real default costs.

Figure 4: Monetary policy shock in the baseline model



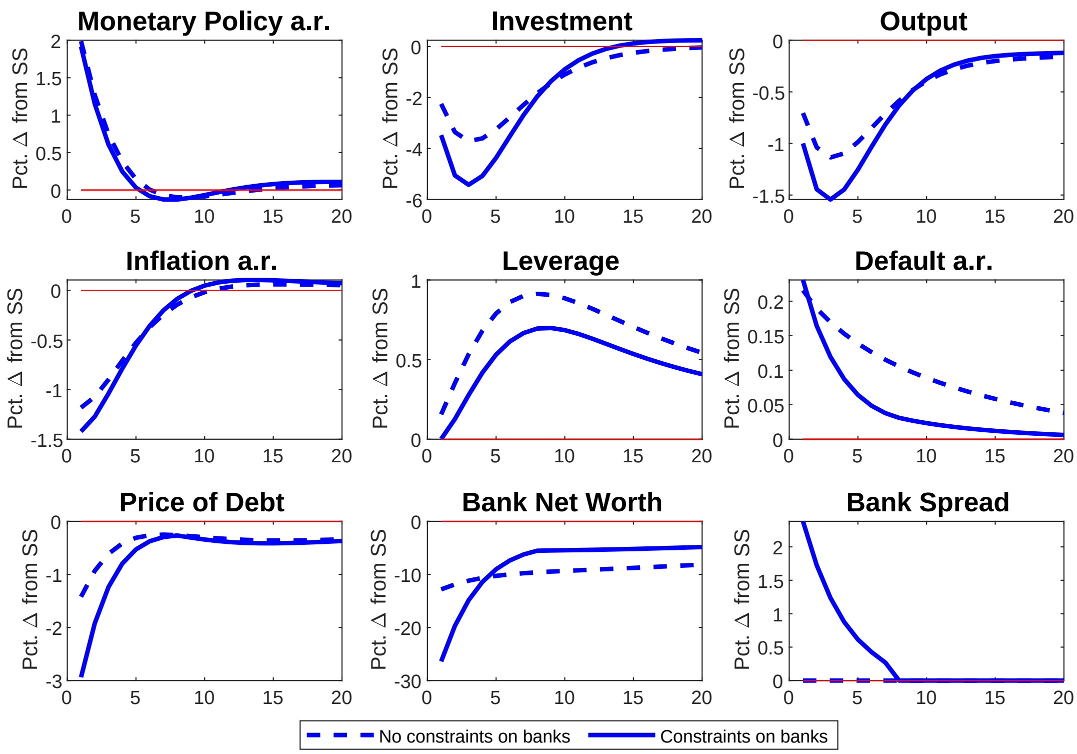
Notes: This figure shows the impulse response to a monetary policy shock in our baseline quantitative model with long-term debt (blue line) compared with an alternative model with short-term debt (red line).

Figure 5: Risk shock in the baseline model



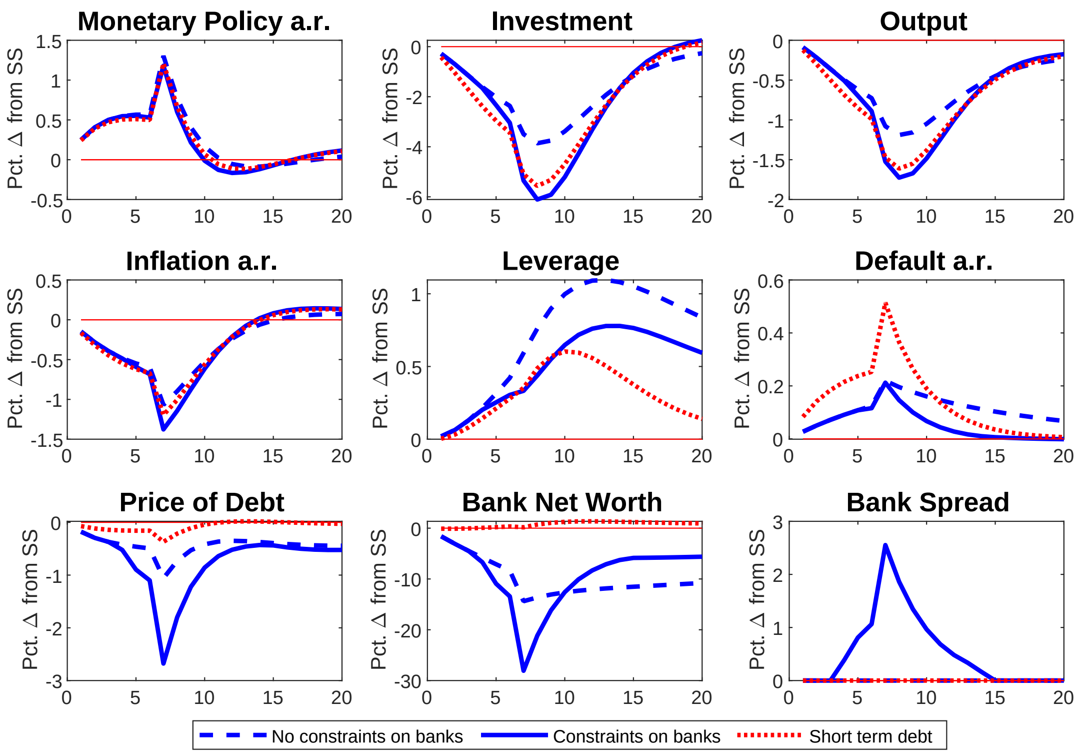
Notes: This figure shows the impulse response to a risk shock in our baseline quantitative model with long-term debt (blue line) compared with an alternative model with short-term debt (red line).

Figure 6: Large monetary policy shock with constrained intermediaries



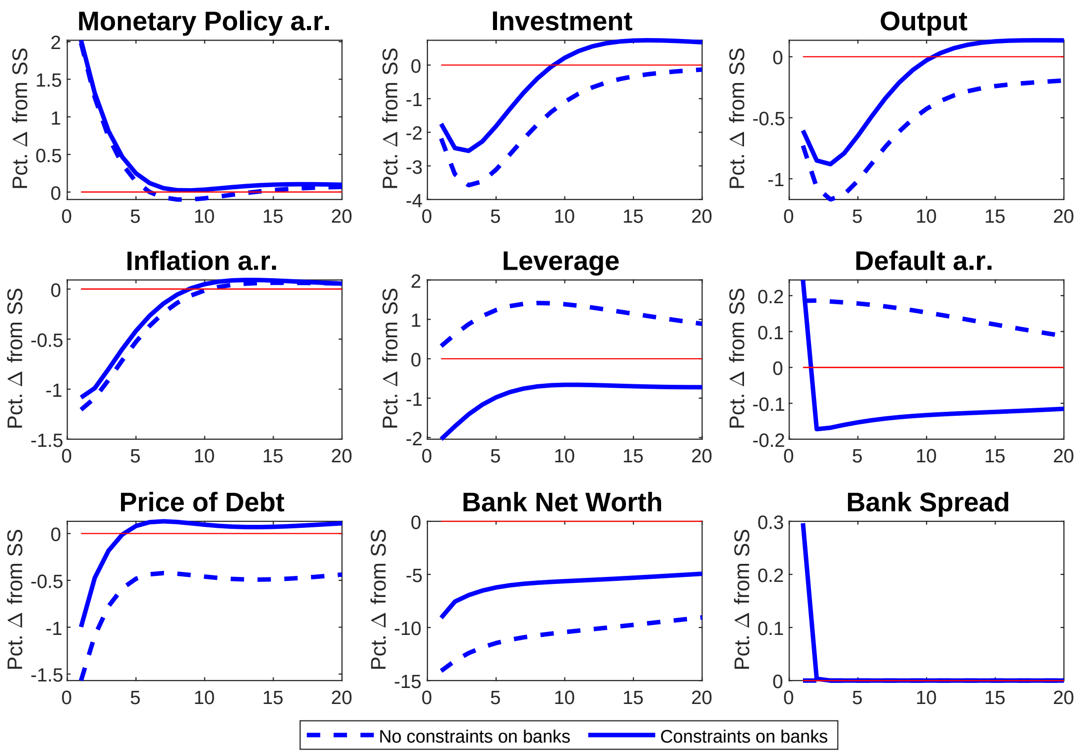
Notes: This figure shows the impulse response to a monetary policy shock in our baseline quantitative model with long-term debt. The dashed blue line reports the model responses when financial intermediaries do not face a leverage constraint, whereas the solid blue line reports the response to the same shock in a version of the model in which banks face occasionally an occasionally binding leverage constraint.

Figure 7: Sequence of monetary policy shocks with constrained intermediaries



Notes: This figure shows the impulse response to a monetary policy shock in our baseline quantitative model with long-term debt. The dashed blue line reports the model responses when financial intermediaries do not face a leverage constraint, whereas the solid blue line reports the response to the same shock in a version of the model in which banks face occasionally an occasionally binding leverage constraint. The red dashed line reports the impulse responses from a version of the model in which debt is short-term and banks face a leverage constraint.

Figure 8: Large monetary policy shock with constrained intermediaries and $\alpha = 0$



Notes: This figure shows the impulse response to a monetary policy shock in our baseline quantitative model with long-term debt. The dashed blue line reports the model responses when financial intermediaries do not face a leverage constraint, whereas the solid blue line reports the response to the same shock in a version of the model in which banks face occasionally an occasionally binding leverage constraint.