Capital Embodied Structural Change *

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Abstract

Structural change is one of the most robust features of modern economic growth. Capital-embodied technological change (CETC), or the decline in the marginal consumption cost of investment, is a prominent source of modern economic growth. What is the role of CETC for structural change? We build new measures of sectoral CETC in the US from the decline in the price of new capital goods used in each sector between 1948 and 2020. We document faster CETC in services than in manufacturing than in agriculture; and show that CETC, when paired with the observed path of the labor share, can trace the dynamics of the price of output in agriculture relative to manufacturing and, to a less extent, that in services relative to manufacturing. To quantify the role of CETC for structural change we build a parsimonious model that accommodates sector-specific CETC through the usage of distinct bundles of equipment, as well as an endogenous sectoral labor share, which mediates the passthrough between CETC and structural change. Via counterfactuals, we find that CETC is the primary driver of the reallocation of output away from agriculture and accounts for a third of the reallocation towards services. The importance of CETC for labor reallocation into services increases post 1990s, and it is a source of acceleration in labor productivity.

JEL codes: O13, O47, Q10.
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1 Introduction

The reallocation of economic activity across broadly defined sectors, i.e. structural change, is a robust feature of modern economic growth. Consensus is growing in the literature that sectoral disparity in productivity growth is a key force behind such reallocation, along with shifts in consumer demand driven by income effects (**??**). At the same time, capital embodied technological change (CETC) has been identified as a prominent source of aggregate productivity growth in modern economies (**?**). Existing analysis of CETC in economies that accommodate structural change primarily focus on environments where capital services are uniform across sectors, assuming a singular capital good for production. Such approach overlooks disparities in capital bundles across sectors, which are particularly important when rates of technological advancements embedded in capital types are heterogeneous. This paper investigates the extent to which structural change is influenced by heterogeneous sectoral capital embodied technological change, arising from disparities in sectoral investments in capital types with differing embodied technologies.

Solow defines (capital) embodied technological change as a shift in the marginal cost of investment relative to consumption, as opposed to factor neutral technological change, which shifts output for a given level and composition of the capital stock (**?**). Competitive markets imply that shifts in this marginal cost map to the path of the relative price of investment to consumption. Our approach is to use this mapping to measure CETC across capital equipment goods along with a constant returns to scale sectoral investment aggregator to generate a measure of CETC at the sectoral level. In line with Solow's definition, our measure of CETC encompasses the multiple forces that could drive a change in the marginal cost of producing new capital goods in consumption units, including, for example, structural change in their production inputs as recently highlighted by **?** and **?**.

We start by documenting systematic disparities in CETC across sectors, using quality-adjusted prices across 24 equipment categories, supplemented with NIPA equipment deflators when such specific data is not available. Sector specific deflators for new capital are constructed by weighting equipment prices with nominal investment shares per equipment type and sector reported in the industry accounts of the Bureau of Economic Analysis (BEA). We find that the price of new capital relative to consumption has declined at an annualized rate of 1.5% in agriculture, 2.4% in manufacturing, and 4.8% in services, in the US between 1948 and 2020. These are sizable differences in the rate of CETC across sectors and are entirely driven by systematic differences in the composition of equipment across sectors.

CETC shapes structural change by changing sectoral labor productivity which, in turn, is reflected in sectoral output prices. Under the assumption of constant returns sectoral production technologies and competitive markets for labor, we show how to link the path of the relative sectoral output price to that of the sectoral user cost of capital. Under constant interest rates, this usercost is proportional to the relative price of investment to consumption and therefore, CETC. The passthrough between CETC and sectoral output prices is mediated by the capital expenditure share in value added. The reason is that the elasticity of labor productivity to CETC is a function of this capital share. Whether faster CETC in a sector translates into a lower output price depends on how intensively the sector uses capital relative to others. We measure the capital expenditure share across sectors residually from computing labor expenditure shares, following the methodology in **?**.[1] The resulting expenditure paths are consistent with the extensive literature that documents a decline in the labor share in manufacturing starting in the 1980s and accelerating in the 2000s (**?**). The labor share in services is instead flat during our sample period, while the labor share in agriculture decreases in the first half of the sample and is relatively stable in recent years.

Armed with the sectoral paths of CETC and capital expenditure share, we document that CETC tracks the path of the relative price of agriculture to manufacturing and the relative price of services to manufacturing quite well, although the magnitudes in the latter are smaller than in the data. The relative price of agriculture to manufacturing falls in the data, suggesting that labor productivity in agriculture rises faster than in manufacturing. So how does CETC track this movement given that CETC in agriculture is slower than in manufacturing? Agriculture is more capital intensive than manufacturing at the beginning of the sample, when disparities in CETC across these two sectors are relatively small. The relative price of services to manufacturing rises in the data, suggesting that labor productivity growth in services is slower than in manufacturing. How does CETC track this movement and, in particular, the acceleration starting in the 1980s if CETC in services is the fastest? The capital expenditure share in manufacturing raises dramatically since the 1980s, compensating for faster CETC in services. We conclude that both levels and movements in the sectoral capital expenditure shares are quantitatively important for the correlation that we uncover between sectoral output prices and CETC.

Because factor expenditure shares are endogenous to the path of technological change, we rely on a structural general equilibrium model to account for the role of CETC in driving movements in economic activity across sectors. We build a model of structural change where sector-specific CETC has a role in shaping the path of sectoral labor productivity.

[1] **?**'s methodology allocates proprietors' income equally between capital and labor within a sector. This is an important source of returns to labor in agriculture and some services where workers are mostly self-employed.

In the model, technological change is both labor augmenting and embodied in capital (as in ?). While labor augmenting technology is exogenous and heterogeneous across sectors, heterogeneity in CETC stems from disparities in the bundles of capital used by different sectors in production. To accommodate endogenous movements in expenditure shares, we work with production technologies that display arbitrary elasticities of substitution between capital and labor in each sector, as in ??. In discussing the channels that drive shift in expenditure shares from CETC, we micro-found these sectoral production technologies as the outcome of firms choosing whether to produce with capital intensive or labor intensive technologies, as in the seminal work of ?.[2]

We quantify the model economy to measure the role of CETC in driving structural change by asking how much of the reallocation of labor across sectors observed in the US between 1948 and 2020 would have occurred absent labour augmenting technological change.[3] We align the model economy to the paths of sectoral prices and capital shares between 1984 and 2020, as well as to sectoral output and employment in 1984. We allow for heterogeneous elasticities of substitution between capital and labor across sectors, which we estimate from the covariation of the capital share with the capital output ratio. Under such parameterization, the model is consistent with the path of structural change observed in the data: it generates all of the reallocation of employment out of agriculture and 71% of the reallocation toward services. Via counterfactuals, we find that CETC accounts for all of the model-generated outflow of employment from agriculture and for 27% of the inflow toward services. The feedback between CETC and the capital share is important for this quantification. Without considering it, i.e. in a model with exogenous capital shares, CETC would be attributed 60% of the reallocation of employment out of agriculture and 38% of the reallocation into services. That is, the feedback effect between CETC and the capital share doubles the role of CETC for agriculture reallocation and decreases that for services by 11p.p.. This is because CETC pushes up the capital share in agriculture relative to manufacturing (due to more substitutability between capital and labor in the sector) and so pushes the price of agricultural goods relative to manufacturing down; while CETC generates the opposite effects for services. Lastly, in isolation, CETC in the manufacturing sector is an important driver of structural change as, alone, it generates 2/3 of the cross-sectoral reallocation of employment.

By placing CETC at the forefront of the drivers of sectoral productivity growth, a new emphasis is given to the role of capital accumulation in facilitating structural change. Un-

[2]The benchmark specification extends the work of ? to multiple sectors, and separates the sources of input intensification from CETC.

[3]Currently, in our counterfactual, we assume a fixed savings rate, which we measure from consumption data net of housing, and introduce a wedge in capital demand to match the observed paths of both capital and labor share.

derstanding the underlying factors behind productivity growth is important because sectoral disparities in these productivity paths may not be set in stone. Post-second world-war evidence suggests that in developed countries, agriculture is a fast productivity growth sector, followed by manufacturing, and then services (?). However, our findings reveal that CETC is the slowest in agriculture and the fastest in services. As sectors become more capital intensive, the significance of CETC as a driver of productivity growth increases, potentially leading to a reversal of current productivity growth trends.[4] Our documentation of sectoral CETC relates to an extensive literature that duels with the nature of technological change and the extent to which its nature is embodied or disembodied from capital or other factors of production (?). To the extent that embodied technology is reflected in the decline in the relative price of equipment investment to consumption, our findings indicate that in relative terms, CETC is of first order importance for agricultural productivity growth (supporting previous cross-country findings in ?), while either technology embodied in labor or disembodied altogether, is relatively more important in the service sector.

There is a growing literature that documents structural change within investment, i.e. shifts in the composition of goods used to produce investment goods in the economy (? and ?). Our main empirical facts are robust to adjusting the path of the relative price of investment to consumption for the cost of inputs from the consumption sector in the production of equipment, and therefore eliminating the role of possibly factor neutral productivity from input producer into the price of investment. In other words, even if we account for shifts in the input bundle for the production of capital, the remaining variation in the relative price of investment to consumption across sectors follows our benchmark results.[5] We therefore focus the analysis on sectoral heterogeneity in the bundles of capital used in production. Our proposed framework can be readily extended to allow for a rich vintage structure for capital, disparities in investment rates across capital types, as well as to think about policies geared towards boosting investment in different sectors and ultimately productivity.

Importantly, emphasizing the role of the changes in the capital expenditure share for the passthrough between CETC and employment reallocation across sectors, links our work to the extensive literature analyzing shifts in the demand for skills, and the role of capital-skill complementarities in production, ??. Our findings suggest that these complementarities have become more relevant as a driver of sectoral reallocation in employment as the US economy developed. For example, systematic changes in the bundle of capital used in the service sector, imply that CETC only accounts for a third of the reallocation towards services

[4] Some early evidence of this phenomenon is documented by ? in agriculture and, more recently, BEA estimates suggests that productivity growth in services may be surpassing that in manufacturing.

[5] See ? for a study of the investment network in a structural change model where, differently from us, capital expenditure shares are held constant along the equilibrium path.

throughout the period, but that such a role has increased substantially after the 1990s. At the same time, systematic changes in the bundle of capital used in manufacturing, imply that CETC accounts for most of the reallocation out of agriculture in the beginning of the sample, and its role has been declining towards the end of the sample.

2 Sectoral CETC

This section measures sectoral growth rates in relative prices, which are proportional to sectoral labor productivity as well as sectoral measures of capital-embodied technological change, CETC. Our study focus on equipment capital and software (one of the categories of intellectual properties products). We measure CETC in each capital type from the (quality-adjusted) relative price of investment to consumption, a measure of the marginal cost of capital when factor markets are competitive.

We construct a time-series of quality-adjusted prices exploiting the estimates in **?** (which are available from 1948 to 1983) and then extrapolate his measures using the prices reported by the BEA.[6] Because we abstract from structures, we remove the real state and construction sectors for the sectoral aggregation. We also abstract from the government sector. Sectors are aggregated following **?** into three categories: agriculture, manufacturing (including mining and manufacturing) and services.

Let I_s be a sector-specific aggregator of investment in J capital types, $I_s(\{X_{js}(t)\}_j)$. The aggregator displays constant returns to scale so the change in the price of sectoral investment is a weighted average of the price of each investment type, with weights equal to the sectoral investment shares for each type $\kappa_{js}(t) \equiv \frac{P_j^x(t)X_{js}(t)}{\sum_j P_j^x(t)X_{js}(t)}$,

$$\frac{\dot{P}_s^x}{P_s^x} = \sum_j \kappa_{js}(t)\frac{\dot{P}_j^x}{P_j^x}.\text{[7]} \tag{1}$$

This relative price in each sectors provides in turn a measurement of the efficiency units embodied in new capital used by the sector. Some of that technology may be inherited from

[6]Appendix C displays the path of prices without quality adjustment (NIPA). Most of the differences between the series occur at the beginning of the series and are concentrated in agriculture, consistent with the findings in **?**. We could have alternatively followed the methodology in **?** which projects quality-adjusted series on BEA deflators. However, using a linear projection method over more than 30 years of data stretches the validity of the assumptions of the econometric model. Quantitatively, the difference between the quality-adjustment and the BEA series is not too large, in part because for the equipment with arguably fastest shifts in quality, i.e computers and communication equipment, BEA introduces quality adjustments and we use the raw series rather than an extrapolation.

[7]This is the correct aggregator when the composite of investment in each sector displays constant returns to scale, **?**.

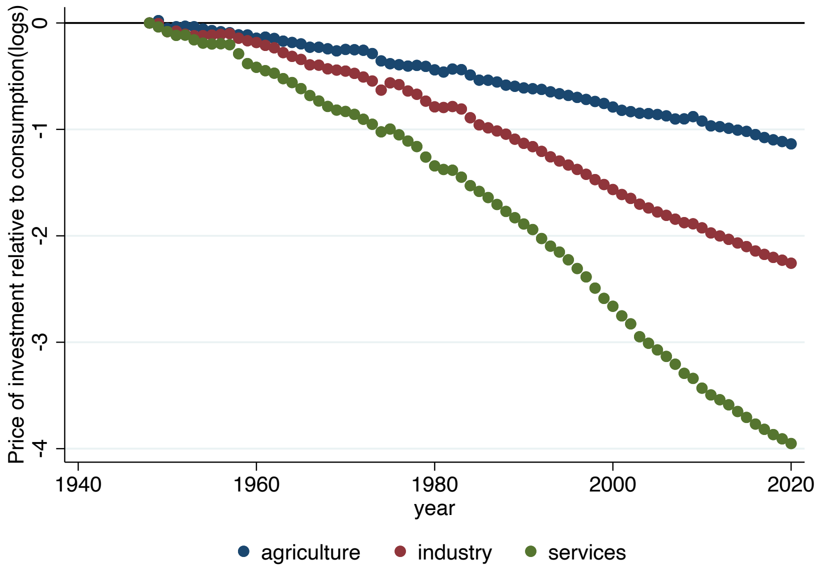


Figure 1: Log-price of investment relative to consumption.

The price of investment relative to consumption is normalized to 1 in 1948 and the picture displays log of prices. Source: BEA and own computations.

the efficiency of the goods used for the production of equipment. and some may be specific to the production of capital. When the technology that transforms consumption goods into new capital is linear (i.e. the inputs in the production of capital and goods are the same) the relative price of investment to consumption maps to the inverse of CETC. Hence, equation 1 defines CETC at the sectoral level, A^x_s, as an investment weighted average of CETC for each equipment type. When the technology for production of new capital is not linear in consumption goods, factor neutral technology in the production of inputs is also reflected in the relative price of investment to consumption. Our benchmark results follow the linear specification, and Section 2.1 discusses robustness when we accommodate the empirically relevant composition of inputs in the production of new capital.

We start by reporting the change in the relative price of investment to consumption in each of our three sectors. The decline in the price of capital is slower in agriculture than in manufacturing and both of them are slower than in services. Disparities in these declines are purely a consequence of the compositional differences in the investment bundle across sectors and time, κ_{js}. Hence, an implication of these differences is that the service sectors are relatively more intensive in capital types with strong declines in the relative price of investment to consumption. These average differences are certainly present in the data, but in addition, the service sector has changed the bundle composition of capital towards equipment types with faster CETC.[8] These differences in CETC amount to an annualized growth rate of embodiment of 1.5% in agriculture, 3.2% in manufacturing and 4.3% in services between 1948 and 2020.

agriculture	manufacturing	services
1.5%	3.2%	4.3%

Table 1: Price of investment relative to consumption, annualized declines 1948-2010

Source: BEA and own computations.

Let production technologies be $Y_s(t) \equiv F_s(K_s(t), A^n_s(t)N_s(t))$ where A^n_s is total factor productivity, and F is a production technology that displays constant returns to scale. In a standard model of structural change —i.e. F_s has common factor shares across sectors, and unitary elasticities of substitution between capital and labor, e.g. **?**— differences in total factor productivity map into differences in output per worker and inversely to relative output prices across sectors. Hence, faster productivity growth in agriculture relative to manufacturing translates into a decline in the relative price of agriculture to manufacturing. When factor shares are different across sectors but the elasticity of substitution in capital and labor is unitary, relative prices are also inversely proportional to TFP but now capital-output ratios need not be equalized across sectors. Capital-deepening (either through investment or aggregate CETC) shifts labor productivity and differentially so across sectors that are more or less capital intensive, as in **?**.

We showed however, systematic disparities in the bundles of capital used by different sectors, which in turn leads to systematic differences in the cost of capital across sectors. In other words, there is no such a thing as an aggregate capital stock from which sectors rent services. Disparities in the the cost of capital through the type of equipment used for production have a direct impact on capital-output ratios, beyond disparities in factor shares.

Often the path of productivity across sectors is inferred from relative output prices, assuming factor price equalization in the market for labor across sectors. In particular,

$$\frac{P_s(t)}{P_{s'}(t)} = \frac{1-\alpha_{s'}(t)}{1-\alpha_s(t)} \frac{\overbrace{Y_{s'}(t)/N_{s'}(t)}^{\text{labor prod.}}}{Y_s(t)/N_s(t)} = \frac{1-\alpha_{s'}(t)}{1-\alpha_s(t)} \frac{F_{s'}(K_{s'}(t)/N_{s'}(t), A^n_{s'}(t))}{F_s(K_s(t)/N_s(t), A^n_s(t))} \tag{2}$$

where $\alpha_s \equiv \frac{F_k K}{F}$. Hence, the mapping between total factor productivity and relative output

[8]Figure 9 presents the series of prices when we fix investment weights to their values at the beginning of the sample and at the end of the sample. The decline in prices is slower than in the benchmark if fixing weights at the beginning of the sample.

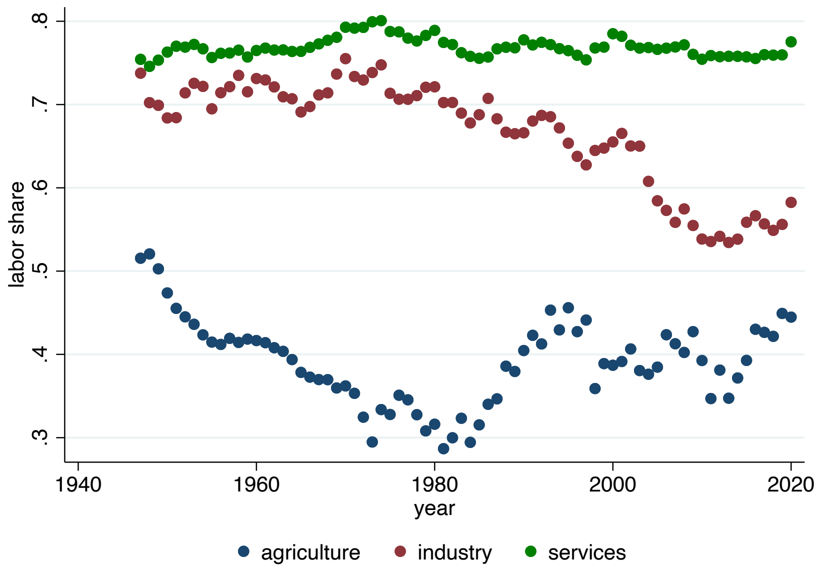


Figure 2: Labor expenditure shares

Labor compensation as a function of total value added in the sector. Estimates include proprietors income, which has been distributed equally between labor and capital. Construction and real state are abstracted away. Source: GDP by industry reported BEA and own computations.

prices depends on disparities in labor expenditure shares, $1 - \alpha_s$, as well as disparities in capital-labor ratios, both of which are potentially affect by sectoral CETC.

We compute labor expenditure shares from the ratio of the compensation of employees in each sector and value added, as reported in the components of GDP by industry compiled by the BEA and following the methodology of **?**, see Figure 2. We find systematic differences in the level of expenditure shares across sectors, consistent with **?**; and importantly, differences in their paths across sectors. As it is well known in the literature, the labor share in manufacturing declines post 1980s and this decline accelerates post 2000s. Perhaps less known is that the labor share in agriculture also falls sustainedly in the first half of our sample period, pre 1980s, and remains stable afterwards. These movements affect the pass-through between capital-labor ratios and relative output prices, which we quantify next.

Relative output prices and CETC. To make the link between relative prices and CETC explicit we add the assumption of competitive input markets. Capital-labor ratios are:

$$\frac{K_s(t)}{N_s(t)} = \frac{W(t)}{R_s(t)} \frac{F_{k,s}(t)}{F_{n,s}(t)} = \frac{W(t)}{R_s(t)} \frac{\alpha_s(t)}{1 - \alpha_s(t)}.[9]$$

[9]The last equality exploits the homogeneity of F, so that its marginals are homogeneous of degree zero in inputs.

In what follows, we drop the time indexes for notational convenience.

Proposition 1 *When firm minimize costs, the growth rate of relative prices across sectors satisfies*

$$\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} = \frac{\dot{\mathcal{A}}_{s'}}{\mathcal{A}_{s'}} - \frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s} + \frac{\alpha_s}{1-\alpha_s}\left(\frac{\dot{R}_s}{R_s} - \frac{\dot{P}_s}{P_s}\right) - \frac{\alpha_{s'}}{1-\alpha_{s'}}\left(\frac{\dot{R}_{s'}}{R_{s'}} - \frac{\dot{P}_{s'}}{P_{s'}}\right) \tag{3}$$

where $\frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s}$ *corresponds to the change in labor augmenting technological change as well the weighted changes in expenditure shares, i.e.* $\frac{\dot{\overline{1-\alpha_s}}}{1-\alpha_s} + \frac{\alpha_s}{1-\alpha_s}\frac{\dot{\alpha}_s}{\alpha_s}$.

Proofs can be found in Appendix B.

Proposition 1 is not a structural decomposition of the path of prices, but it is informative as of channels that are relevant along the equilibrium path, as well as how the results in these papers compare to other channels previously studied in the literature. Proposition 1 shows that the difference in the change in relative output prices depends on the difference in labor augmenting technological change, as in **?**, and in the marginal product of capital, i.e. the ratio between the user cost of capital and the sectoral output price. In economies where the usercost of capital is equalized across sectors and there are no sectoral disparities in factor shares, only technological change drives relative prices. When the user cost of capital is the same across sectors, but factor shares are heterogeneous, the dynamics of sectoral prices relative to the price of consumption shifts affect sectoral disparities in the path of prices, as in **?** when factor shares are constant, or **?** when factor shares are time-varying.

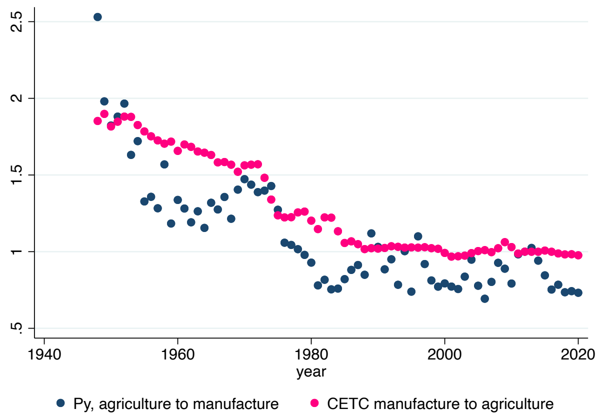
The link between the marginal product of capital and CETC requires further structure on the model, namely an equilibrium path where the interest rate in the economy is constant.[10]

Corollary 1.1 *The growth rate of relative prices across sectors in a growth path where constant interest rates satisfies*

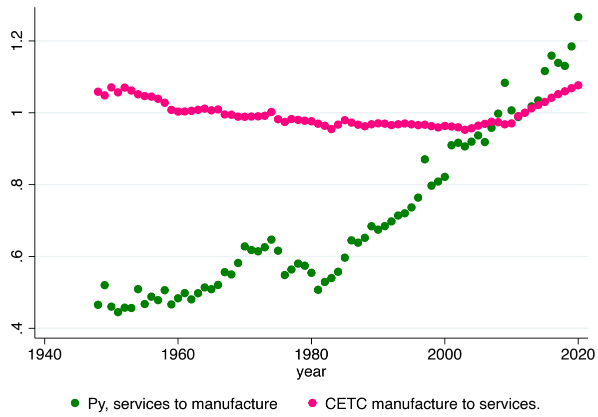
$$\begin{aligned}\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} = {} & \frac{\dot{\mathcal{A}}_{s'}}{\mathcal{A}_{s'}} - \frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s} + \frac{\alpha_{s'}}{1-\alpha_{s'}}\frac{\dot{A}^x_{s'}}{A^x_{s'}} - \frac{\alpha_s}{1-\alpha_s}\frac{\dot{A}^x_s}{A^x_s} + \\ & \frac{\alpha_{s'}}{1-\alpha_{s'}}\left(\frac{\dot{P}_{s'}}{P_{s'}} - \frac{\dot{P}_c}{P_c}\right) - \frac{\alpha_s}{1-\alpha_s}\left(\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_c}{P_c}\right)\end{aligned}$$

An equilibrium path with a constant interest rate implies that the user cost of capital relative to the price of consumption maps into CETC. If the economy displays structural

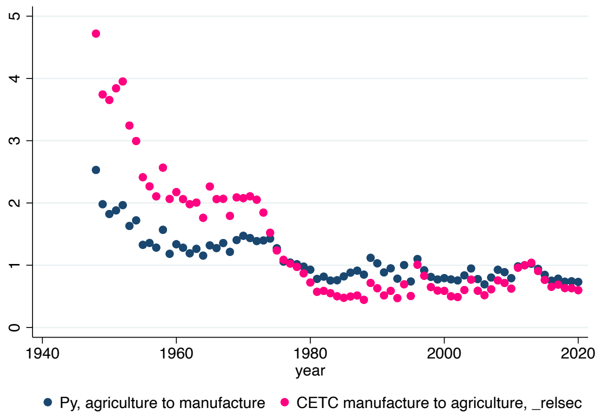
[10] This interest rate is determined by the Euler equation of households, which are the only agents making dynamic decisions.



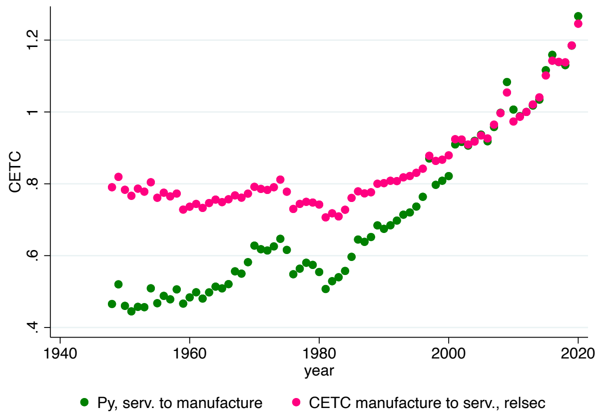
(a) Agriculture



(b) Services



(c) Agriculture



(d) Services

Figure 3: Relative prices and CETC.

The price of value added in a sector relative to manufacturing and CETC in manufacturing relative to a given sector in Panels (a) and (b). In Panels (c) and (d) we include also the impact from disparities in factor shares across sectors. Source: BEA and own computations.

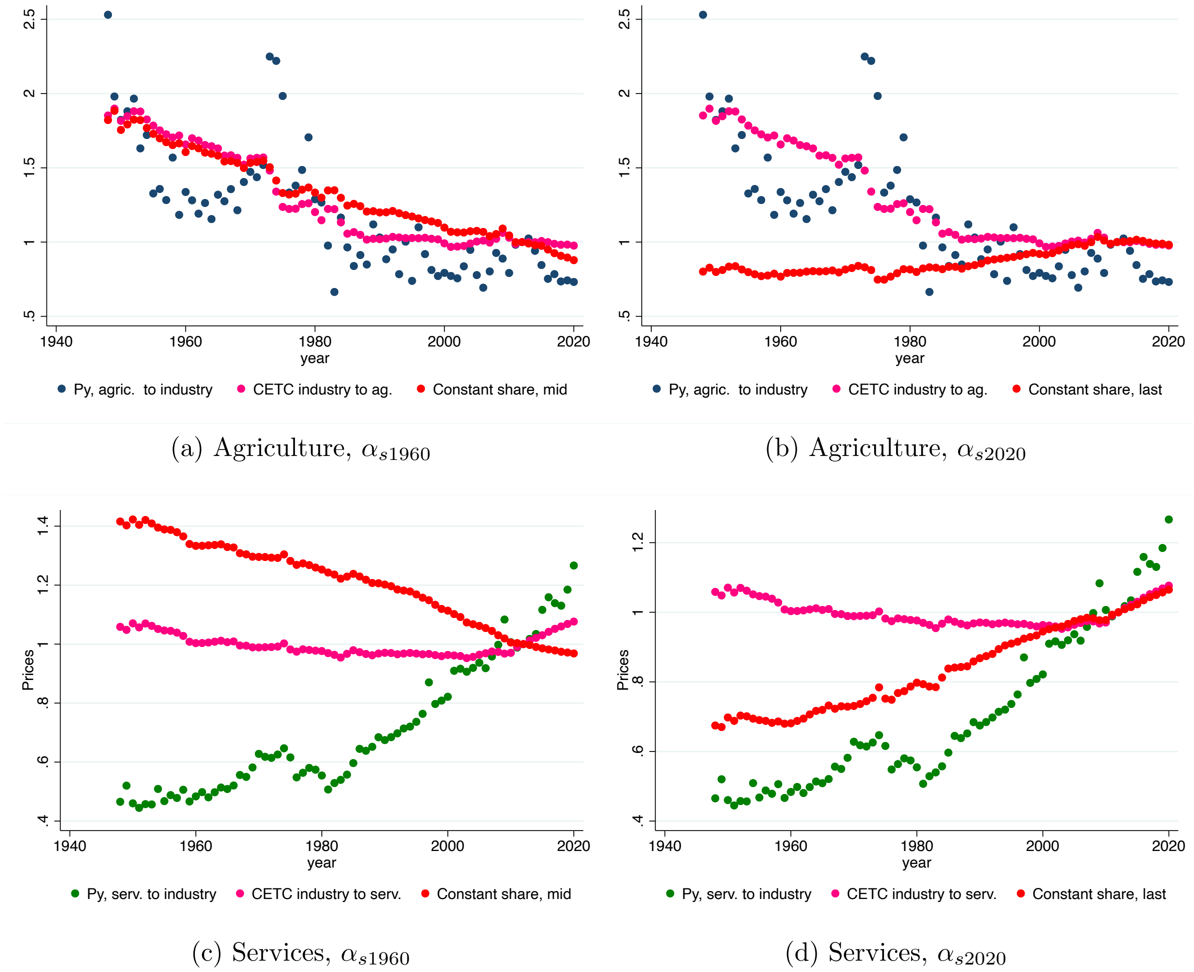
change, the price of sectoral output and the price of consumption will in general differ. While the measurement of CETC is unaffected, the components of the residual variation in relative output prices that are not explained by CETC include the price of sectoral output relative to consumption along the transition. In an economy with different sectoral investment bundles but where the capital intensity is the same in all sectors, this additional channel disappears. To the extent that factor shares are in turn endogenous to CETC, this is yet another channel affecting the incidence of CETC on relative prices.

For measurement purposes, it is useful to disaggregate the measure of sectoral CETC as an investment weighted measure of CETC in each equipment type that comprises the sectoral aggregator, so

$$\begin{aligned}\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} &= \frac{\dot{\mathcal{A}}^n_{s'}}{\mathcal{A}^n_{s'}} - \frac{\dot{\mathcal{A}}^n_{s}}{\mathcal{A}^n_{s}} + \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}\sum_j \kappa_{js}(t)\frac{\dot{A}^x_j}{A^x_j} - \frac{\alpha_{s}(t)}{1-\alpha_{s}(t)}\sum_j \kappa_{js}(t)\frac{\dot{A}^x_j}{A^x_j} + \\ &\quad \frac{\alpha_{s'}}{1-\alpha_{s'}}(\frac{\dot{P}_{s'}}{P_{s'}} - \frac{\dot{P}_c}{P_c}) - \frac{\alpha_{s}}{1-\alpha_{s}}(\frac{\dot{P}_{s}}{P_{s}} - \frac{\dot{P}_c}{P_c}) \qquad (4)\end{aligned}$$

Equation 4 is our main accounting measure. We observe relative prices, as well as construct measures of the last two terms from the relative price of investment to consumption in each equipment and sectoral investment shares. These two terms are the contribution of CETC for the shift in relative prices.In addition, relative prices are proportional to labor productivity given a constant returns technology for production, so the above condition also measures the contribution of CETC to labor productivity. Because we are interested in changes through time, both measures are normalized to 1 in the base year 2012.

Figure 3 Panel (a) displays the path for the relative of agriculture to manufacturing as well as the contribution of CETC. We find that movements in CETC track closely the path of relative surprises, suggesting that in an accounting sense, CETC was likely the main driver of differences in labor productivity across sectors. In other words, the residual variation in relative output prices that is **not** accounted by CETC (adjusted by the capital expenditure share) is relatively small. Figure 3 Panel (b) displays the paths for services relative to manufacturing. Qualitatively, the measure of relative CETC tracks the dynamics of relative prices in the second half of the sample, particularly post-2000. This is in part driven by the increased importance of high-skill services within the sector and the fact that CETC in those sectors better tracks their relative prices. Quantitatively, the changes in measured CETC (corrected by the expenditure shares) are substantially lower in magnitude than the change in relative output prices, which implies higher residual variation accounted for disparities in total factor productivity or the standard intensification effect from disparities in factor



(a) Agriculture, α_{s1960}

(b) Agriculture, α_{s2020}

(c) Services, α_{s1960}

(d) Services, α_{s2020}

Figure 4: Contribution of CETC to labor productivity, constant α_s.

Panel (a) displays the relative price of output in agriculture and Panel (b) the relative price of output in services relative to manufacturing. In pink, we show the path of CETC in manufacturing relative to each of these sectors. In red, we report the contribution of CETC when fixing the capital shares to their value in their initial and final sample years. Source: BEA and own computations.

shares.

Figure 3 Panels (c) and (d) present the role of CETC when we also account for disparities in capital shares. In Services, disparities in factors shares and the movement in relative prices increase the role of CETC for the dynamics of relative prices.

2.1 Discussion

Capital expenditure shares. These findings are somehow surprising given that the ranking of CETC across sectors is opposite to the path of total factor productivity across sectors that has been widely documented in the literature, where productivity growth in agriculture is fastest in agriculture than in manufacturing and in services (**?**). Hence, the direction of the component of relative prices accounted for by CETC is driven by the expenditure share adjustment.

To assess the role of time-varying factor shares for the contribution of CETC to relative labor productivity, we compute a counterfactual path of CETC fixing the labor share to their initial and final levels in the sample, see top panel of Figure 4 for agriculture and bottom for services. On average, agriculture is the most capital-intensive sector followed by manufacturing and services, with capital expenditure shares of 61%, 33% and 23% respectively. These qualitative disparities remain through the sample, but its magnitude do not. Manufacturing becomes more capital intensive (and closer to agriculture), which explain the upward pressure on the relative price of agriculture to manufacturing when computed with end-of-sample expenditure shares. The same capital intensification, makes services and manufacturing more dissimilar in terms of expenditure shares, again generating upward pressure on relative output prices as observed in the data.

CETC affects relative output prices both directly, through the rate of sectoral embodiment, $\frac{\dot{A}^x_s}{A^x_s}$, and indirectly, through movements in the capital expenditure shares induced by shifts in the usercost of capital. Hence, to fully assess the role of CETC it is necessary to account for this feedback which we tackle in the next section.

Investment aggregator. There is a growing literature that argues that the way in which economies combine inputs to produce investment and consumption goods is systematically different, that these combinations shift along the process of structural change, (**?** and **?**. A natural question is to what extent the protracted decline in the price of capital relative to consumption reflect changes in the inputs used for the production of investment. Our main accounting, equation 4, does not require assumptions on the nature of the final output aggregators. The introduction of distinct investment and consumption aggregators changes the mapping between the relative price of investment in each equipment type and the productivity of the investment sector, modifying the interpretation in Corollary 1.1. Given our definition of embodiment, i.e. a shift in the marginal cost of producing capital; changes in labor-augmenting technological change in the production of inputs that are disproportionally used in investment relative to consumption, would lower their cost of production and in turn lower the price of investment relative to consumption. These shifts in technology are embodied in capital because they are associated to the change in the marginal cost of producing capital and the economy takes advantage of it through investment in new capital. Empirically, we can apportion technological change that is purely associated to the production of capital and the one that arises in the production of other goods in the economy.

Cost minimization implies that the price of investment in each equipment type relative to consumption is a combination between productivity residuals in the production of each equipment and the sectoral output productivities of the sectors providing inputs, weighted by their role in the investment aggregator. These sectoral output productivities are reflected in sectoral prices. So, empirically, we can construct alternative price indexes for equipment that remove the effect of changes in the price of inputs. We exploit historical capital-flows tables from **?** to discipline the input bundles used in the production of equipment, and construct our counterfactual index eliminating any sector other than those producing equipment.

Table 2 presents our implied measures of CETC and shows that the path of the counterfactual sectoral price of investment is very similar to our benchmark estimate. The reason for this is that most inputs in the production of equipment are sourced from other equipment sectors, not elsewhere in the economy.

Table 2: Annual CETC by sector

	benchmark	capital-flow adj
agriculture	-1.52%	-1.62%
manufacture	-3.07%	-2.87%
services	-5.25%	-4.97%

Benchmark estimates and estimates adjusting from the changes in the price of inputs not produced in the equipment sector. Own estimates based on BEA's and **?**.

2.2 Cross-country evidence

Before discussing the model economy, we present evidence from the path of relative prices and the price of equipment relative to consumption across countries. In this exercise we use value added and output deflators from the 2017 edition of EUKLEMS (29 countries) and capital prices and investment from the corresponding capital input files.[11] The price of capital by sector is available for 24 of these countries.[12]

We compute the relative price of investment in each equipment category j relative to consumption, λ_j^k, as the ratio of the price index for investment (variable Ip_j for each j category) to the deflator of aggregate Value Added (variable VA_P). The **categories of capital** available include "CT" "Cult" "IT" "OCon" "OIPP" "OMach" "RD" "RStruc" "Soft DB" "TraEq" and and overall stock "GFCF". Use as **base year 2010** (consistently with the dataset) and compute the growth rate in λ_j^k for all j and for every period available.

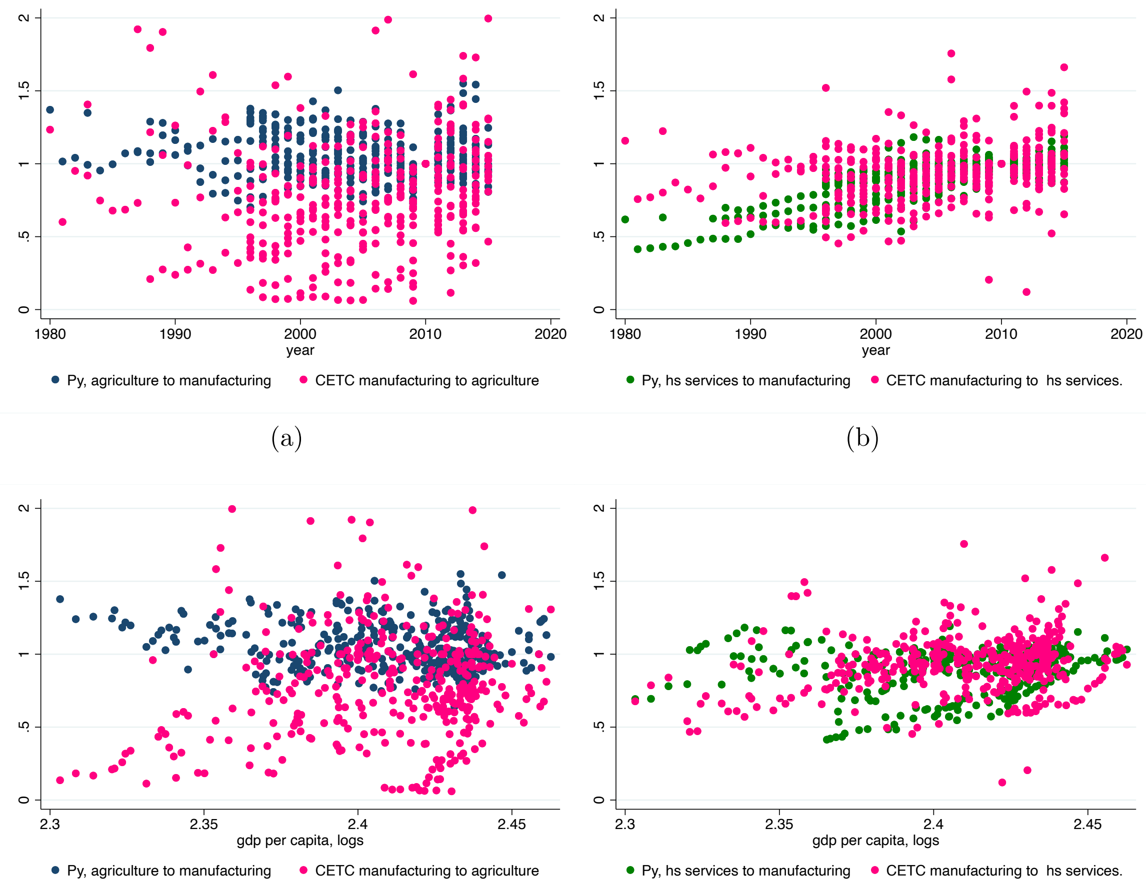
[11] We are working on extending the sample to the 2007 edition, as well as including LAKLEMS.
[12] Croatia, Malta, Rumania, Cyprus and Czech Republic have no data on sectorial capital.

The sectorial aggregation in each country follows the analysis for the US. The decline in the price of investment relative to consumption is a Torqvinst index with weights equal to the nominal share of investment in a sector s for capital j at each point in time, ω_{js} (nominal investment in national currency, variables I_j for j the capital category). We initialize the price of capital relative to consumption in sector s to 1 in the base year.

Our main findings are summarized in Figure 5. On average, the path of CETC (weighted by the proper elasticity), tracks the path of the relative price of sectorial output through time. On average across countries the path for CETC in industry relative to services tracks the path of relative output prices, as predicted by the theory. In the cross-country evidence the correlation between CETC in industry relative to agriculture and the path of relative output prices is weaker than what we documented in the US. However, data quality might be important particularly in agriculture.[13] Figure 5 also displays the path of relative prices and CETC against income per capita.[14]

[13]Although we lack time series information on quality adjusted prices of equipment in agriculture, we do have differences across countries in this price. If we use the estimate of CETC in agriculture from **?** we observe a stronger correlation than the one uncovered with the price of capital reported in KLEMS.

[14]The next iteration of this data analysis will include poorer countries from LatinAmerica.



(a) (b)

(c) Agriculture (d) Services

Figure 5: Contribution of CETC to labor productivity, average across countries.

Panel (a) displays the relative price of output in agriculture and Panel (b) the relative price of output in services relative to industry. In pink, we show the path of CETC in industry relative to each of these sectors. We report cubic polynomials of the cross-country data. Panel (c) and Panel (d) report the same statistics against income per capita, in constant 2017 PPP dollars. Source: EUKLEMS, PWT and own computations.

3 A model of capital-embodied structural change

To study the role of CETC for structural change, we build a parsimonious multi-sector model where each sector uses heterogeneous bundles of capital, and capital expenditure shares are endogenous to the price of each of these bundles relative to the cost of labor. Labor is homogeneous and frictionlessly allocated across sectors. Final goods are a composite of goods produced by these sectors and can be allocated to consumption or investment. A unit of final output invested towards capital of type j generates $A_j^x(t)$ capital services.

3.1 Demand structure

Consider a standard continuous time problem of a representative household with constant intertemporal elasticity preferences over a consumption aggregate $C(t)$. The household inelastically supplies $N(t)$ units of labor which earns a wage $W(t)$, and invest in capital of different types $j \in \{1, , .., J\}$, $X_j(t)$. This investment is distributed into three different sectors for the accumulation of sectoral capital, which can be rented to firms in the economy at a rental rate, $R_s(t)$. Capital in each sector depreciates at a rate $\delta_s \in (0, 1)$. A bond $B(t)$, which is priced and pays off in units of consumption, is in zero net supply. This bond prices the (consumption-based) interest rate, $R(t)$, given the rate of discount υ.

The household's problem is therefore:

$$\max_{C(t),X_j(t),X_{js}(t),K_s(t),B(t)} \int_{t=\tau}^{\infty} e^{\upsilon(t-\tau)} \frac{C(t)^{1-\theta}}{1-\theta},$$

subject to

$$P_c(t)C(t) + \sum_j P_j(t)X_j(t) + P_c(t)\dot{B}(t) = W(t)N(t) + \sum_s R_s(t)K_s(t) + r(t)P_c(t)B(t),$$

$$\dot{K}_s(t) = I_s(\mathbf{X_s}(t)) - \delta_s K_s(t), \tag{5}$$

$$X_j(t) = \sum_s X_{js}(t),$$

where $\mathbf{X_s}$ is a vector of investment of each equipment type in sector s, $\mathbf{X_s} \equiv [X_{1s}, X_{2s}, ..., X_{Js}]$ and I_s is a homogeneous of degree one aggregator.

The amount of household labor allocated to different sectors should be consistent with the labor supply,

$$\sum_s N_s(t) = N(t). \tag{6}$$

To this consumption-investment problem, we add structural transformation. Define an output aggregator

$$Y(t) = (\sum_{s=a,m,s} \omega_{ys}^{\frac{1}{\sigma_y}} Y_s(t)^{\frac{\sigma_y - 1}{\sigma_y}})^{\frac{\sigma_y}{\sigma_y - 1}}$$

for an elasticity of substitution, $\sigma_y \in [0, +\infty)$. Final output can be used to produce final consumption goods or inputs for investment goods, $\chi_j(t)$,

$$C(t) + \sum_j \chi_j(t) = Y(t).^{15} \tag{7}$$

3.2 Production structure

Producers in the investment sector maximize profits $P_j(t)X_j(t) - P_c(t)\chi_j(t)$ subject to a linear technology for production

$$X_j(t) = A_j(t)\chi_j(t), \tag{8}$$

with productivity trend, $\dot{A}_j(t) = \gamma_j(t)A_j(t)$.[16]

Producers of sectoral goods need to perform a measure 1 of activities to generate output. They choose which activities to perform with each factor of production and how much of each factor to allocate to a given activity:

$$\max_{k_i(t), n_i(t), m(t)} P_s(t)F(k_i(t), n_i(t)) - R_s(t)\int_0^1 k_i(t) - W(t)\int_0^1 n_i(t)di.$$

subject to their production technology

$$Y_s(t) \equiv F(k_i(t), n_i(t)) = \left[\int_o^1 y_i(t)^\rho di\right]^{\frac{1}{\rho}},$$

$$y_i(t) = \zeta_i(t)^{\frac{\rho-1}{\rho}} Z_s(t)^{\frac{\rho-1}{\rho}} n_i(t) \qquad i \in [0, 1],$$

$$y_i(t) = k_i(t) \qquad i \in [0, m(t)],$$

$$\dot{Z}_s(t) = \gamma_{Z_s} Z_s(t).$$

[15] The model can be readily extended to a non-homothetic final output aggregator, as well as distinct output aggregators for consumption and investment goods.

[16] In the extensive tradition that studies investment-specific technological change, e.g. **?** and followers, the rate of technological change is assumed constant, $\gamma_j(t) = \bar{\gamma}_j$. At this level of generality, we allow for a time-varying growth rate, and later narrow its path as we characterize equilibrium allocations.

Capital is equally productive across different activities, but labor is not: there exist a profile ζ_i, increasing in the activity index i, which characterizes the productivity of labor in each activity. In addition, there is a common component of labor productivity across activities in the sector, Z_s. Differences across economies in the supply of skills that are relevant to a given sector would map into this common trend, while the relative productivity of workers across production processes would better map to the profile ζ_i.

3.3 Optimal allocations

Key features of the allocation in this economy include the amount of labor allocated to each sector, the capital-labor ratios, the capital-expenditure shares, and the shares of output in total value added.

Expenditure shares across activities. We start characterizing the input allocation in each sector. Optimality requires

$$W(t) = p_i(t)(Z_s(t)\zeta_i(t))^{\frac{\rho-1}{\rho}},$$

$$R_s(t) = p_i(t),$$

for a price of each activity that follows $p_i(t) = P_s(t)\left(\frac{Y_s(t)}{y_i(t)}\right)^{1-\rho}$. An implication of this optimal allocation is that expenditure shares in all activities within a sector are the same.

Measure of mechanized activities. When the cost of performing an activity is lower with capital than with labor, the activity is mechanized.

$$W(t)Z_s(t)^{\frac{1-\rho}{\rho}}\zeta_i(t)^{\frac{1-\rho}{\rho}} \geq R_s(t). \tag{9}$$

Any activity below $m(t)$ that satisfies such a condition will be allocated no labor and the optimal amount of capital, i.e. the activity is "mechanized".

This condition presents the key tradeoff between the rate of decline in the usercost of capital, which is linked to CETC; and the improvements in labor productivity.

Capital and labor allocation per activity. A consequence of the equalization of the expenditure shares across activities is that all mechanized activities get the same amount of capital

$$k_i(t) = \frac{K_s(t)}{m_s(t)}. \tag{10}$$

Output across non-mechanized activities depends on labor productivity and so does the labor allocated to them,

$$n_i(t) = \frac{\frac{1}{\zeta_i(t)}}{\mathcal{Z}_s(t)} \frac{N_s(t)}{1 - m_s(t)}, \tag{11}$$

where $\mathcal{Z}_s \equiv \frac{1}{1-m_s(t)} \int_{m_s(t)}^{1} \frac{1}{\zeta_i(t)} d_i$, a geometric average of the labor productivity across activities. The amount of labor allocated to activities with higher labor productivity is relatively lower.

It is indeed low enough that output in non-mechanized activities with higher labor productivity is lower than in activities with lower labor productivity. This is again a consequence of the equalization of expenditure across activities.

$$y_i(t) = \frac{P_s(t)^{\frac{1}{1-\rho}} Y_s(t)}{W(t)^{\frac{1}{1-\rho}} (Z_s(t)\zeta_i(t))^{\frac{1}{\rho}}}, \tag{12}$$

$$y_i(t) = \frac{P_s(t)^{\frac{1}{1-\rho}} Y_s(t)}{R_s(t)^{\frac{1}{1-\rho}}}. \tag{13}$$

Capital expenditure share. The expenditure share of capital, which is the sum of the expenses across mechanized activities, can be described using equation 10

$$\alpha_s(t) \equiv \int_o^{m_s(t)} \frac{p_i k_i}{P_s(t) Y_s(t)} = \frac{R_s(t) K(t)}{P_s(t) Y_s(t)}.$$

We can further describe the capital expenditure share in sector s as proportional to the mechanization rate, $m(t)$ and a function of the marginal product of capital using equation 13:

$$\alpha_s(t) = m_s(t) \left(\frac{P_s(t)}{R_s(t)} \right)^{\frac{\rho}{1-\rho}}. \tag{14}$$

When there are complementarity in production across activities $\rho < 0$, the capital expenditure share increases in the marginal product of capital, $\frac{R_s}{P_s}$, as well as the mechanization rate. In a one sector economy, the marginal product of capital is simply a function of CETC because the sectoral price equals the price of consumption. In an economy with a non-trivial and heterogeneous path for sectoral prices, the marginal product is not only a function of CETC but also of the relative price of output to consumption.[17]

sectoral output and factor augmenting technology. Given the optimal capital and labor demand, output in each sector can be written as

[17]Note that **?** sets up an economy with structural change with no differences in the prices of sectoral output. The authors assume this when imposing a linear aggregation of real output into GDP.

$$Y_s(t) = \left[(\underbrace{(1-m_s(t))^{\frac{1-\rho}{\rho}} A^n_s(t)}_{b^n(t)=\text{labor augmenting}} N_s(t))^\rho + (\underbrace{m_s(t)^{\frac{1-\rho}{\rho}} A^k_s(t)}_{b^k(t)=\text{capital augmenting}} \quad \tilde{K}_s(t))^\rho \right]^{\frac{1}{\rho}}, \tag{15}$$

where the elasticity of substitution between capital and labor services is $\frac{1}{1-\rho} < 1$, and where capital has been described as product between the stock in units of the top technology available, and the efficiency of the top technology $A^k_s(t)$, i.e. $K_s = A^k_s(t)\tilde{K}_s$. Then, the capital-augmenting term is a function of the share of activities that are mechanized, and the efficiency of the top technology. The labor-augmenting term is in turn a function of the average labor productivity in activities that are not mechanized, as well as its common component within the sector, $A^n_s(t) \equiv \frac{Z_s(t)}{\mathcal{Z}_s(t)}^{\frac{\rho-1}{\rho}}$.

Proposition 2 *When $\rho \to 0$, the technology converges to a Cobb-Douglas form with output elasticity to capital equal to m_s and output elasticity to labor equal to $1 - m_s$, for arbitrary $m_s \in (0, 1)$.*

Hence, in the limit when $\rho \to 0$ in all sectors, the economy converges to **?**.

Perhaps most important to the properties of allocations along the development path, one can use the expression for output in each sector and the optimality condition for labor (in terms of the labor expenditure share) to describe:

$$Y_s(t) = \frac{b^n_s(t)}{(1-\alpha_s(t))^{\frac{1}{\rho}}} N_s(t). \tag{16}$$

This representation of output highlights the key departures from an economy where the output elasticity of capital is constant. Output per worker is a function of the labor augmenting term as in the plain vanilla model of structural change. But output per worker is also a function of the endogenous cost-share of capital, which responds to the cost of capital relative to labor. When capital and labor are complementary, output per worker is lower in sectors with higher labor share.

Households' savings decisions. The dynamic decisions of the household are characterized by the Euler equation associated to the accumulation of capital of type s,

$$\theta \frac{\dot{C(t)}}{C(t)} = R(t) - \upsilon = \frac{R_s(t)\iota_{js}(t)}{P_j(t)} - \delta_s - \upsilon + (\frac{\dot{P_j}(t)}{P_j(t)} - \frac{\dot{P_c}(t)}{P_c(t)} - \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)})$$

where $\iota_{js}(t) \equiv \frac{\partial I_s(\mathbf{X_s})}{\partial X_{js}}$, i.e. the partial of the investment aggregator in sector s to capital of type j in sectoral capital s. The return to a unit invested in equipment j at cost P_j which yields ι_{js} additional units of sectoral capital is $\frac{R_s(t)\iota_{js}(t)}{P_j(t)}$ net of the effective discount rate, $\delta_s + \upsilon$, and the change in the value of the sectoral capital.

The relative price of investment to consumption can be computed from the zero profit condition in the production of investment goods, so that $\frac{\dot{P}_j(t)}{P_j(t)} - \frac{\dot{P}_c(t)}{P_c(t)} = -\frac{\dot{A_j(t)}}{A_j(t)}$.

$$\theta\frac{\dot{C(t)}}{C(t)} = \frac{R_s(t)\iota_{js}(t)}{P_j(t)} - \delta_s - \upsilon - \frac{\dot{A_j(t)}}{A_j(t)} + \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)}$$

Cost minimization in the investment aggregator implies that $\frac{P^x_s(t)}{P_j(t)}\iota_{js}(t) = 1$. The change in the value of capital is then $(\frac{\dot{P}_j(t)}{P_j(t)} - \frac{\dot{P}_c(t)}{P_c(t)} - \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)}) = \frac{\dot{P}^x_s(t)}{P^x_s(t)} - \frac{\dot{P}_c(t)}{P_c(t)}$.

$$\theta\frac{\dot{C(t)}}{C(t)} = \frac{R_s(t)}{P^x_s(t)} - \delta_s - \upsilon - \frac{\dot{A^x_s(t)}}{A^x_s(t)} \tag{17}$$

In other words, we can describe the dynamics for the accumulation of capital in the economy as a function of the sectoral capital composite, and its rate of embodiment. Likewise, these Euler equations impose a series of no-arbitrage conditions on the value of the marginal product of capital across sectors.

We can describe consumption in units of investment of an arbitrary sector as $\tilde{C}_s(t) \equiv \frac{P_c(t)}{P^x_s(t)}C(t)$. Then, the Euler equation(s) read

$$\theta\frac{\dot{\tilde{C}}_s(t)}{\tilde{C}_s(t)} = \frac{R_s(t)}{P^x_s(t)} - \delta_s - \upsilon - (1-\theta)\frac{\dot{A^x_s(t)}}{A^x_s(t)}. \tag{18}$$

Then, with logarithmic preferences, $\theta = 1$ the Euler equation is independent of the relative price of investment to consumption, i.e. CETC, similarly to **?**.

3.4 Equilibrium

Definition: *The* ***competitive equilibrium*** *is fully characterized by the Euler Equation, 17, the law of motion for capital in each sector, 5, the optimal capital and labor allocation 10 and 11, the optimal measure of mechanized activities in each sector, 9 and the feasibility constraints of the economy, 6, 8 and*

$$C(t) + \sum_j \chi_j = Y(t);$$

as well as the transversality condition for each capital type, $\lim_{t\to\infty} e^{\upsilon t}C(t)^{-\theta}K_j(t) = 0$.

With the definition of equilibrium at hand, we can explore what features of the economy would admit a generalized balanced growth path as described below. In what follows we assume logarithmic preferences for final consumption.

Definition: *A* ***generalized balanced growth path (GBGP)*** *is an allocation where the interest rate in the economy is constant,* $\bar{R} \equiv R(t) = \frac{R_s(t)}{P_s^x(t)} - \delta_s$.

The existence (or lack of) a GBGP depends critically on the properties of the rental rate of capital in units of investment. Due to complementarities in the output aggregator, the sector with slowest output growth overtakes the economy, call this sector s. The optimality condition for capital implies that the rental cost of capital (in units of investment) is

$$\frac{R(t))}{P^x(t)} = \frac{P_s(t)}{P_c(t)} \left(\frac{Y_s(t)}{\tilde{K}_s(t)} \right)^{1-\rho} (b_s^k)^\rho \tag{19}$$

where $\frac{P_s(t)}{P_c(t)} = \left(\frac{Y(t)\omega_s}{Y_s(t)} \right)^{\frac{1}{\sigma_y}}$. Hence, the sector with slowest output growth displays the fastest growth of capital in efficiency units $\tilde{K}_s(t)$ whenever the elasticity of substitution between capital and labor is higher than the elasticity of output across sectors, $\sigma_y < \sigma_{kn} \equiv \frac{1}{1-\rho}$, which is the empirically relevant case. Conversely, the sector with the fastest growth in output should display the slowest growth in capital in efficiency units. But a permanently shrinking capital stock in efficiency units (sectoral output) would eventually hit the non-negativity constraint for stocks (output), so it is not feasible along a GBGP. Hence, the BGP is only achieved in the limit.

Before describing the long-run allocations, it is important to highlight the main differences between our economy and others studied in the literature. Consider a version with no disparities in CETC across sectors, $\gamma_s^x = \gamma^x$, but where sectoral output follows the CES structure posed before. Balanced growth for this class of technologies requires pure labor-augmenting technological change **?**, or $b_s^k(t) = b_s^k$.

Proposition 3 *Along an allocation with constant interest rate and capital-augmenting technological change,* ***relative output*** *follows*

$$\frac{Y_s(t)}{Y_{s'}(t)} = \frac{\omega_{s'}}{\omega_s} \left(\frac{\phi_s}{\phi_{s'}} \frac{\alpha_s(t)}{\alpha_{s'}(t)} \right)^{-\sigma_y \frac{1-\rho}{\rho}},$$

where $\phi_s \equiv \left(\frac{R}{P^x} \frac{1}{b_s^{k\,\frac{1}{\rho}}} \right)^{\frac{\rho}{1-\rho}}$ *is a constant.*

Proposition 3 implies that the limiting sector has also the slowest increase in the capital-expenditure share, which occurs whenever capital and labor are complementary, $\rho < 0$). The representation of output in this Proposition also highlights the main difference to **?**, where constant capital-augmenting technological change maps one-to-one to a constant capital share. This is a consequence of the 1-sector nature of their economy and the fact that they study a static allocation, i.e. capital fully depreciates.[18] Indeed, the representation of the capital expenditure share in equation 14, makes it clear that the expenditure share is a function of the price of sectoral output, which differs from the price of consumption. In a one-sector economy, these two prices are identical.

3.5 Long run properties

We now characterize what happens along the limiting BGP.

Sectoral Output. In the steady-state the growth rate of output in the sector follows from equation 16,

$$g_{Y_s} = \frac{1-\rho}{\rho}(g_{1-m_s} + g_{A_s}) = \frac{\rho-1}{\rho}(g_{Z_s} - g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i})). \tag{20}$$

Given that $\rho \leq 0$ in the limiting sector, growth occurs as A_s declines. It does so for three reasons: (a) there are less activities performed by labor, (b) the productivity of labor in newly mechanized activities is higher (lower $\frac{1}{\zeta_{m_s}(t)}$ on the margin), (c) the productivity of labor in all activities grows exogenously at rate g_{Z_s}. the first two reasons imply that average productivity across activities is falling, $g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i} < 0$.

Aggregate Output and Wages. To compute the equilibrium wage, aggregate over the optimal demand for labor in sector. Assuming a common elasticity of substitution between capital and labor across sectors, wages in the long-run converge to

$$W(t) \to \frac{1}{N(t)^{1-\rho}} (\omega_{ys})^{\frac{1}{\sigma_y}} \left(\frac{Y_s(t)}{Z_s(t)} \int_{m_s}^{1} \frac{1}{\zeta_i(t)} d_i\right)^{1-\rho}$$

for the sector s that overtakes the economy. Using the growth rate for sectoral output 20 we can compute the equilibrium growth rate of wages, which as expected, follows the growth rate of output:

$$g_w = \frac{\rho-1}{\rho}(\gamma_{Z_s} - g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i}) \tag{21}$$

[18] In this case, the rental rate of capital equals the price of investment and 14 can be rewritten as $\alpha_s(t) = m_t(\frac{P_s(t)}{P_c(t)}\frac{P_c(t)}{P_s^x}\frac{P_s^x}{R_s(t)})^{\frac{\rho}{1-\rho}} = m_t(\frac{P_s(t)}{P_c(t)}A_s^k(t)\frac{P_s^x}{R_s(t)})^{\frac{\rho}{1-\rho}}$. In the setup of **?** the latter equation equals, $\alpha_s = b_s^{k\,\frac{1}{1-\rho}}$.

which is an increasing function of the labor augmenting term for $\rho < 0$. In other words,

$$g_w = g_Y = g_{Y/N}.$$

Usercost of Capital and Mechanization. Using the ratio of the expenditure shares in capital and labor, we can compute the growth rate in the usercost of capital as a function of the mechanization rate and the growth rate in labor augmenting technology

$$\frac{\alpha_s}{1-\alpha_s} = \frac{R_s(t)K_s(t)}{W(t)N_s(t)} = \frac{m_s(t)}{\int_{m_s}^1 \frac{1}{\zeta_i} d_i} \left(\frac{W(t)}{R_s(t)}\right)^{\frac{\rho}{1-\rho}}.$$

so that

$$g_{R_s} = \frac{1-\rho}{\rho}(g_m - (g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i} - \gamma_{Z_s})) + g_w$$

$$g_{R_s} = \frac{1-\rho}{\rho} g_m. \tag{22}$$

Hence, the reason for which in this model the capital share is constant is that when the cost of capital falls, there is a proportional increase in the share of newly automated tasks, compensating for the upward pressure on the capital expenditure share from more mechanized activities. Notice that the impact of mechanization on the efficiency of labor generates even more upward pressure on the expenditure ratio, but this effect is exactly compensated with an increase in wages.

The growth rate of the usercost of capital is linked to CETC through the Euler equation, equation 17, which in turn pins down g_m as a function of factor neutral productivity and CETC, i.e. $g_{R_s} = -\gamma_{A_s^k}$; so that

$$g_{m_s} = \frac{\rho}{\rho-1}\gamma_{A_s^k} > 0.\text{[19]} \tag{23}$$

By definition, as activities become mechanized, the measure of unmechanized activities shrinks. Incidentally, the rate of shrinkage is also the rate at which $m_s(t)$ reaches its limit

$$g_{1-m_s} = \frac{\dot{m}_s}{1-m_s(t)} = -g_{m_s}\frac{m_s(t)}{1-m_s(t)}$$

[19]The usercost of capital is only exactly proportional to CETC when the rate of embodiment is constant, the interest rate is constant and the expenditure share in each equipment type in the investment aggregator is constant. The latter occurs along the transition path if the investment aggregator displays unitary elasticity across capital types. Still, the economy's interest rate might not be constant away from the limiting BGP.

or what is the same

$$g_{m_s} = -g_{1-m_s}\frac{1-m_s(t)}{m_s(t)}. \tag{24}$$

There are two takeaways from this expression. The second term in the LHS is monotonically decreasing so two scenarios can realize: (a) if the growth rate of mechanized activities is constant, then the growth rate of unmechanized activities has to be monotonically increasing; or (b) the growth rate of unmechanized activities is constant, and the growth rate of mechanized activities is monotonically decreasing. The first case cannot arise because m_t is bounded between 0 and 1. Hence, if $g_{m_s} > 0$ is declining then so is the rate of decline in the usercost of capital, see equation 22, $g_{m_s}, g_{R_s} \to 0$. A slow-down in the rate of decline in the usercost of capital, also imposes restrictions on the rate of CETC through the Euler equation.

The optimality condition for the set of automated tasks implies,

$$g_w + \frac{1-\rho}{\rho}(\gamma_{Z_s} + g_{\zeta_{m_s}}) = g_{R_s}.$$

Replacing the expressions for the growth rates of wages and the usercost, equations 21 and 22,

$$g_{\zeta_{m_s}} = g_{m_s} - g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i} \tag{25}$$

Let the trend in labor productivity be $-q = (g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i} - \gamma_{Z_s})$. Along the BGP, q should be constant (?). Then, the rate of shrinking of the productivity across activities is proportional to the growth in productivity within the sector.[20]

Totally differentiating the expression for labor productivity and using the Leibniz rule,

$$-\frac{\dot{m_s}(t)}{\zeta_{m_s}(t)} = -q + \gamma_{Z_s}$$

Then,

$$\zeta_{m_s}(t) = \frac{g_{1-m_s}(1-m_s(t))}{q+\gamma_{Z_s}},$$

or what is the same, $\gamma_{\zeta_{m_s}} = g_{1-m_s}$. Using equation 25, we conclude that $q = g_{1-m_s} + \gamma_{Z_s}$ and constant.

Finally, using the expression of the usercost of capital is easy to see that in the limit, capital and output grow at the same rate.

[20]Before convergence in m_s, the threshold is also a function of CETC, $g_{\zeta_{m_s}} = \frac{\rho}{\rho-1}g_{A_s^x} - g_{\int_{m_s}^{1}\frac{1}{\zeta_i}d_i}$.

3.6 Detrending and steady state

Sectoral output in each sector should be detrended by labor productivity, $A_s^n(t)$. Define the trend in aggregate output as

$$a(t) \equiv \left(\sum_s \omega_s^{\frac{1}{\sigma_y}} (b_s^n(t))^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{\sigma_y}{1-\sigma_y}}$$

, for $b_s^n(t) \equiv (1 - m_s(t))^{\frac{1-\rho}{\rho}} A_s^n(t)$.

The trend in sectoral capital is

$$a_s^k(t) \equiv a(t) A_s^k(t).^{21}$$

With these trends, we can define detrended consumption, output, sectoral output and sectoral capital as follows

$$c(t) \equiv \frac{C(t)}{a(t)}, \quad y(t) \equiv \frac{Y(t)}{a(t)}, \quad y_s(t) \equiv \frac{Y_s(t)}{a(t)}, \quad k_s(t) \equiv \frac{K_s(t)}{a_s^k(t)}.$$

3.7 Transition dynamics

With the detrended economy, the transition dynamic of the problem is summarized by the following conditions

$$\theta \frac{\dot{\tilde{c}}_s(t)}{\tilde{c}_s(t)} = r_s(t) - \delta_s - \upsilon$$

where $\tilde{c}$ corresponds to the detrended consumption expenditure in units of investment and $r_s(t)$ is the detrended value of the marginal product of capital in each sector, which satisfies

$$p_s(t) \frac{y_s(t)}{k_s(t)}^{1-\rho} A_s^x(t)^\rho m_s(t)^{1-\rho} (\frac{a(t)}{a_s(t)})^\rho = r_s(t).$$

The detrended sectoral output satisfies

$$y_s(t) = \left[N_s^\rho + m_s(t)^{1-\rho} \left(\frac{a(t)}{a_s(t)} \right)^\rho A_s^k(t)^\rho k_s(t)^\rho \right]^{\frac{1}{\rho}}.$$

The dynamics of capital in the detrended economy are

[21] Here we take a shortcut and describe the trend in sectoral CETC, but this trend is an investment weighted sum across CETC of different equipment types.

$$\frac{\dot{k}_s(t)}{k_s(t)} = \frac{y(t)}{k_s(t)} - \frac{c(t)}{k_s(t)} - \sum_{s' \neq s} \frac{x_{s'}(t)}{k_s(t)} - \delta_s - \gamma_{A_s^k} - \gamma_a$$

for $x_{s'} \to 0$ for any sector that is not the one overtaking the economy.

Prices of sectoral output satisfy:

$$p_s(t) = \omega_s^{\frac{1}{\sigma_y}} (\frac{a_s(t)}{a(t)})^{\frac{\sigma_y - 1}{\sigma_y}} \left(\frac{y(t)}{y_s(t)}\right)^{\frac{1}{\sigma_y}}$$

Combining the optimality condition for capital with that of labor, we obtain a characterization of the static allocation, and equilibrium capital-labor ratios.

Finally, the system is closed with a transversality condition for each sectoral capital, $\lim_{t \to \infty} \exp(-(\upsilon + (1-\theta)\gamma_a - \gamma_{A^x})t)c(t)^{-\theta} k_s(t) = 0$. For log utility, the transversality condition requires $\upsilon > \gamma_{A^x}$, which is trivially satisfied absent CETC in the limit, or if agents have a relatively high discount for time.

4 The role CETC for structural change

To quantify the role of CETC for structural change, we run an accounting exercise on a calibrated version of the model described in the previous section. We start our exercise by feeding the path of observed sectoral CETC to the model along and calibrate the path of sectoral labor augmenting technological change from the path of relative prices and average output growth in the data. We then run counterfactual exercises where we measure the contribution of CETC to the reallocation of labor across sectors – that is, our measure of structural change.

In our quantification we depart from the model described in the previous section on four dimensions. First, we work with an exogenous savings decision – that is, we assume that the household consumes a fixed share of output in each period and saves the rest: $C(t) = \eta(t)Y(t)$. Second, we allow sectors the elasticity of substitution between capital and labor to be sector specific, i.e. we allow ρ_s to vary by sector. Third, we introduce wedges in the sectoral capital demand, $\tau_s(t)$, (i.e. equation 19)

$$\tau_s(t) R_s(t) = P_s(t) m_s(t)^{1-\rho} \left(\frac{Y_s(t)}{K_s(t)}\right)^{1-\rho}, \tag{26}$$

where the price of consumption is normalized to 1. This wedge allows us to target capital in the sector along with the capital share as measured in data residually from the labor share

Table 3: Calibration.

Parameter		Value			Target
output elast.	σ_y		0.01		?
discount factor	β		0.98		?
savings rate	$1-\eta$				data
CETC	A^x_s				data
final output TFP	D		0.33		aggregate output per worker in 1948
		Agr.	Manuf.	Serv.	
elast. of substitution, k-l	$\frac{1}{1-\rho}$	1.23	0.84	0.74	own estimates
output shares	ω_{ys}	0.11	0.38	0.52	employment shares, 1948
mechanization rate	m_s				capital share
labor aug. tech., 1948	$b^n_s(1948)$				sectoral output, 1948
labor aug. tech. change: relative	$g_{b^n_s/b^n_m}$				relative prices
labor aug. tech. change: level	$g_{b^n_m}$				output per worker

Note: The table shows the value of the calibrated parameters and their targets. Sectoral values are reported in the following order: agriculture, manufacturing, services.

and the CRS assumption. Last, we augment final good aggregator to add a TFP term, D, that we assume constant over time:

$$Y(t) = D\sum_s [\omega_s^{\frac{1}{\sigma_y}} Y_s(t)^{\frac{\sigma_y-1}{\sigma_y}}]^{\frac{\sigma_y}{\sigma_y-1}}.$$

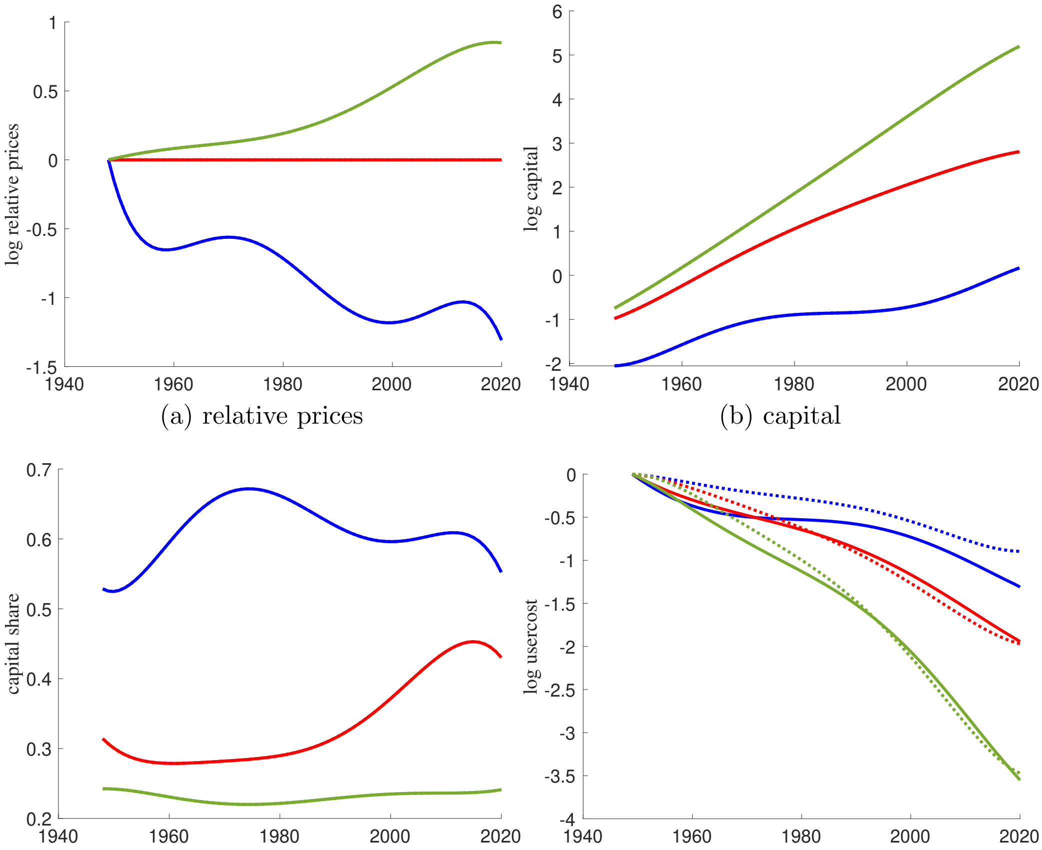
This terms allows us to match aggregate output, along with sectoral output, in one year.

Currently, we are running the quantification under exogenous mechanization rate m_{st}. As our results below reveal, a feedback effect from CETC to the mechanization rate might be quantitatively important and so we plan to introduce it in future updates of the paper.

4.1 Calibration

We calibrate our model to the US economy between 1948 and 2020. Our calibration strategy is summarized in Table 3 and described below.

Pre-set parameters. We set the discount factor, β, and the elasticity of substitution across occupational outputs, σ_y, in line with **?**. The latter parameter implies preferences that approach Leontieff. **?** show that absent non-homotheticicities, Leontieff substitution between sectors provides a best fit to long-run data. We estimate the elasticity of substitution between capital and labor across sectors from data on expenditure shares and capital labor ratios.



(a) relative prices

(b) capital

(c) capital share

(d) usercost

Figure 6: Model fit on targets.

The figure shows the model fit on the targeted sectoral trends in relative prices, capital, capital share and usercost. Model moments are in solid lines and data moments are in striped lines. Agriculture is in blue, manufacturing is in red, services is in green.

We estimate the following regression: Estimate ρ_s from:

$$\ln(\alpha_s(t)) = \beta_{2s} \ln(\frac{K_s(t)}{Y_s(t)}) + t\beta_{1s} + \beta_{0s} + \epsilon_{st},$$

where β_{2s} gives us an estimate for ρ_s and ϵ_{st} is an i.i.d. error term that augments the structural equation.

Lastly, we measure the savings rate and CETC directly form the data. We compute the savings rate in each year from consumption data net of housing. Instead, we infer CETC from the path of the relative price of investment to consumption as described in Section 2.

Targeted moments. We calibrate the remaining parameters to match relevant features of the structural transformation process in the US between 1948 and 2020. In the first year of our sample, 1948, we target the sectoral employment allocation using the sectoral shares in final good production, ω_{ys}, and sectoral labor productivity, using labor augmenting technology $b_s^n(t)$, by combining sectoral output demand,

$$\left(\frac{N_s(t)}{N_{s'}(t)}\right)^{\frac{1}{\sigma_y}} = \left(\frac{\omega_{ys}}{\omega_{ys'}}\right)^{\frac{1}{\sigma_y}} \left(\frac{b_s^n(t)}{b_{s'}^n(t)}\right)^{\rho} \left(\frac{Y_s(t)/N_s(t)}{Y_{s'}(t)/N_{s'}(t)}\right)^{1-\rho-\frac{1}{\sigma_y}},$$

and the sectoral production function. In addition, we also target the level of aggregate output using the TFP in final good production, D.

In all other years in our sample, we target (a) the capital share, to parameterize the mechanization rate $m_s(t)$:

$$\alpha_s(t) = m_s(t)^{1-\rho} \left(\frac{Y_s(t)}{K_s(t)}\right)^{-\rho};$$

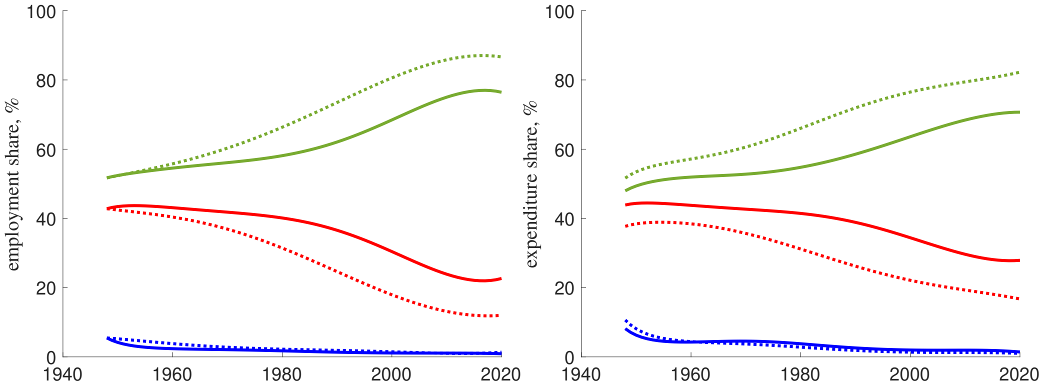
(b) the path of relative output prices to parameterize the path of sectoral labor-augmenting technology relative to the manufacturing sector, using the optimality conditions for sectoral output demand and the manufacturing sector as baseline:

$$g_{\frac{P_s}{P_m}} = -\frac{1}{\sigma_y} g_{\frac{Y_s}{Y_m}};$$

(c) the path of labor productivity in the manufacturing sector to parameterize the path of labor-augmenting technology in the sector, using the sectoral production function. Finally, the model-implied wedge on sectoral capital demand can be recovered from equation 26.

Outcomes. Figure 6 shows the fit of the model on the targeted sectoral paths in relative prices, capital, capital share and usercost. The model generates the same trend in the first three moments and a trend close to the one observed in the data for the moment. Figure 7 show the model fit on structural change between 1948 and 2020, which is not target of our calibration exercise. We find that the model generates a decline of 4.5p.p. in the share of agricultural employment, compared to the 4.1p.p. decline observed in the data. At the same time, the model generates an increase in the share of employment in services of 24.7p.p. compared to the observed increase of 34.9p.p.. Hence, the model generates all of the reallocation of employment out of agriculture and 71% of the reallocation toward services.

Our next step is to isolate the drivers of the trend in the targeted moments and, ultimately, of structural change, via counterfactual exercises. Before that, we describe the path of the additional source of technological change that, along with CETC, shapes allocations in the model, that is labor-augmenting technology. Figure 8, left panel, displays the paths of labor-augmenting technology in each sector. he growth rate of labor-augmenting technology



(a) employment (b) nominal expenditure shares

Figure 7: Model fit on structural change.

The figure shows the model fit on the non-targeted path of structural change as measured by the allocation of employment across sectors and the nominal sectoral expenditure shares. Model moments are in solid lines and data moments are in striped lines. Agriculture is in blue, manufacturing is in red, services is in green.

has been stronger in manufacturing and agriculture (average annual growth rate of 4.0% and 3.9%, respectively) than in services (average annual growth rate of 2.0%), broadly consistently with the gold-standard calibration outcomes of models of structural change. Lastly, the right panel of Figure 8 displays the path of the share of mechanized activities across the three sectors. The mechanization rate influences the path of the capital share and so the elasticity of structural change to CETC. In 1948, the rate of mechanization was highest in agriculture (55% vs 32% in manufacturing and 27% in services), but by 2020 services have the highest rate 66%. Over this period mechanization has been faster in services (an increase of 40p.p.), followed by manufacturing (an increase of 20p.p.) and, lastly, agriculture (an increase of only 6p.p.).

4.2 Counterfactuals

Two sources of technological change drive the paths of relative prices, and so structural change, in our model: sectoral labor-augmenting technological change, $b_s^n(t)$ and sectoral CETC, $A_s^k(t)$. To isolate the role of sectoral CETC for structural change, we run our baseline counterfactual exercise in which we shut down heterogeneity across sectors in labor-augmenting technological change ("no b_s^n"). In the counterfactual economy we set $b_s^n = b^n$ and we parameterize b^n to target growth in aggregate output per worker, as in our calibration. Along with this baseline counterfactual, we run additional exercise in which, along with shutting down heterogeneity across sectors in labor-augmenting technological change,

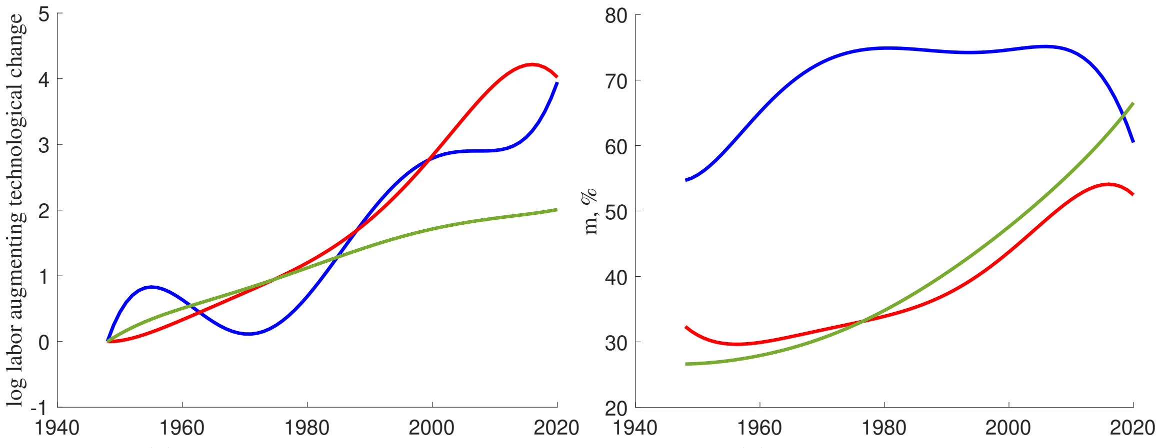


Figure 8: Paths of labor-augmenting technology and mechanization.

Panel (a) displays the mechanization rates $m_s(t)$ while Panel (b) displays labor-augmenting technology, $b_s^n(t)$. Source: Own computations.

Table 4: Counterfactuals: the role of CETC for structural change.

	data	model	no b_s^n	no b_s^n & only CETC in agri	manu	services
agriculture	-4.1	-4.6	-4.9	-5.0	-4.1	-4.1
manufacturing	-30.8	-20.2	-4.6	-6.7	-18.1	8.4
services	34.9	24.7	9.6	11.7	22.2	-4.3

Note: The table shows the change between 1948 and 2020 in sectoral relative prices and employment shares in the data, as predicted by the model, and as predicted in each counterfactual exercise described in the text. Values are in percent.

we also only allow CETC in one sector at a time ("only CETC in.."). For example, if we only allowed CETC in agriculture, we set $A_s^x(t) = 1\ \forall t$ in the manufacturing and services sectors. This set of exercises allows us to point to the importance of CETC in particular sectors.

Table 4 shows the change in sectoral employment shares between 1948 and 2020, as predicted by our baseline counterfactual and each of the three additional exercises. We measure that CETC accounts for all of the movement of employment out of agriculture and for 27% of the movement toward services. In the counterfactual economy that does not feature sectoral labour-augmenting technological change, the share of employment in agriculture decreases of 4.1% as in the data, while it similarly decreases of 4.6% in the model. At the same time, the share of employment in services increases of 22% in the counterfactual economy, in comparison to 35% in the data and 25% in the model. Turning to sector-specific

CETC, the counterfactuals reveal that CETC in the manufacturing sector is an important driver of structural change as, on its own, it generates 2/3 of the cross-sectoral reallocation of employment. In addition, it is worth noting that CETC in the agricultural sector generates the most sizeable reallocation of employment of of agriculture.

The effect of CETC on structural change operates via two channels. One channel is the one studied in the empirical section. For a given capital share, CETC influences sectoral prices and so the allocation of employment. A second channel operates via the capital share, as CETC shapes the capital shares itself. How is important is the feedback between CETC and the capital share when studying the role of CETC for structural change? To answer this question, we run our baseline counterfactual in an alternative economy where the capital share is set exogenously to its level in the data, that is in an economy with $\rho_s \to 0$ in all sectors. The result are shown in Table 5, panel Employment. When the capital share is held fixed to its value in the calibration exercise, CETC generates an outflow of employment of 2.4p.p. compared to the 4.9 generated when the capital share is allowed to move. Instead, in services, the reallocation of employment is higher under the fixed capital share exercise (13.2%) compared to the baseline exercise where the capital share is allowed to move (9.6%). We conclude that the feedback between CETC and the capital share increases the role of CETC for the reallocation of employment out of agriculture by 60p.p. (that is, it more than doubles the effect) and decreases that for the reallocation toward services by 11p.p.. The mechanics of this result are shown in the same table, panels *Relative prices* and *Capital share*. CETC increases the capital share of agriculture relative to manufacturing due to capital and labor being substitutes in agriculture and complements in manufacturing. The higher capital share in agriculture implies a lower sectoral price and so stronger outflow of employment from the sector. Turning to services, despite the stronger CETC in this sector compared to that in manufacturing and capital and labor being complementary in both sectors, the capital share in services relative to that of manufacturing decreases less when the feedback effect is considered. The closer capital share in services to that in manufacturing implies a less high sectoral price and so weaker inflow of employment toward the sector.

Given the key role of the capital share in shaping structural change and, in particular, the elasticity of structural change to CETC, we decompose growth rate of the capital share between 1948 and 2020, g_{α_s}, in its main components:

$$g_{\alpha_s} = g_{m_s} + \frac{\rho_s}{1-\rho_s}\left(g_{P_s} - g_{\tau_s} - g_{R_s}\right),$$

the growth rate of the mechanization rate g_{m_s}, and the growth rates of the sectoral price g_{P_s} (in real terms), the wedge g_{τ_s} and the usercost g_{R_s}, weighted by ρ_s. The contribution of

Table 5: Counterfactuals: feedback between CETC and the capital share.

	data	model	no b_s^n	fixed α_s each year	1948	2020
Employment						
agriculture	-4.1	-4.6	-4.9	-2.4	-3.5	-2.1
manufacturing	-30.8	-20.2	-4.6	.-10.8	6.9	-11.4
services	34.9	24.7	9.6	13.2	-3.4	13.5
Relative prices						
agriculture	-1.3	-1.3	-1.3	-0.4	-1.4	-0.2
manufacturing	0.0	0.0	0.0	0.0	0.0	0.0
services	0.8	0.8	0.2	0.3	-0.2	0.5
Capital share						
agriculture	2.4	2.4	28.1	2.4	0	0
manufacturing	11.6	11.6	1.9	11.6	0	0
services	-0.1	-0.1	-5.4	-0.1	0	0

Note: The table shows the change between 1948 and 2020 in sectoral employment shares, relative prices and capital shares in the data, as predicted by the model, and as predicted in each counterfactual exercise described in the text. Values are in percent.

the usercost can be further split to highlight the role of CETC. Table 6 shows that CETC shapes the different path of the capital share in services relative to manufacturing. The growth rate of the capital share in services is 37p.p. lower than that in manufacturing. CETC alone generates a growth rate that is 60p.p. lower in services than in manufacturing. Other components of the capital share either generate much smaller differences across the two sectors (the sectoral price and residual component of the usercost) or they generate a higher growth rate in the capital share for services than for manufacturing (the mechanization rate). It is worth noting that the path of the mechanization rate is instead one of the key drivers, along with path of sectoral prices, of the differential growth rate of the capital share in agriculture and manufacturing. Farther, the mechanization rate is the second most important driver of the growth of the capital share in manufacturing and the most important driver of the growth rate of the capital share in agriculture. For these reasons, we deem worth quantifying the feedback between CETC and the machanization rate in future versions of the paper.

Table 6: Decomposition of the capital share.

	α_s	m_s	P_s	τ_s	R_s CETC	R_s other
agriculture	7.7	13.7	-35.4	0.1	21.8	7.5
manufacturing	35.8	52.1	7.5	6.7	-27.3	-3.2
services	-1.0	89.7	-9.7	9.5	-85.7	-4.7
difference to manu						
agriculture	-28.1	-38.4	-42.9	-6.6	49.1	10.7
services	-36.8	37.6	-17.2	2.8	-58.4	-1.6

Note: In the first panel, the table shows the growth rate of the capital share between 1948 and 2020, along with each of its components. The second panel shows the same statistics as a difference to manufacturing. Values are in percent.

5 Final Remarks

We document systematic disparities in the bundle of capital used for production across sectors. These disparities induce sector-specific rates of capital-embodied technological change which we can directly measure in the data from the relative price of sectoral investment to consumption.

Sectoral CETC differentials map into labor productivity differences via capital-expenditure shares, which we also show have been changing across sectors through time. We build a structural model that can accommodate these movements endogenously and therefore propose a theory for the endogenous link between CETC and labor productivity.

Our preliminary results show that labor productivity differentials across sectors (fastest in agriculture, slower in manufacturing and slowest in services) may reverse as CETC becomes ever more important for output production. Hence, our findings may have implications for Baumol's cost disease.

Overall, our findings draw attention to the importance of the composition of sectoral investment for the path of economic growth.

A Data construction.

VA price deflators by sector are Torqvinst price indexes constructed weighting the price deflator of output in each sector (variable VA_P) by nominal value-added weights, (variable VA).

Real measures of value added (in national currency) are the ratio between total value added by sector (aggregated linearly) and the price deflator for output in the sector.

Measures of **employment** correspond to counts of people and total hours of those employed (variables $EMPE$ and H_EMPE). I have also included measures of labor compensation LAB.

Measures of capital include capital compensation rk, (variable CAP) and KLEMS also produces a measure of stock which is a quantity index with base year 2010.

B Proofs

Proof. Proposition 1.

Let $k_s(t) \equiv \frac{K_s(t)}{N_s(t)}$, and define output in terms of capital per worker as $f(k_s(t)) \equiv F(\frac{K_s(t)}{N_s(t)}, A^n(t))$. The, wages can be written as,

$$P_s(t)(1-\alpha_s(t))f(\frac{W(t)}{R_s(t)}\frac{\alpha_s(t)}{1-\alpha_s(t)}, A^n(t)) = W(t).$$

The above expression can be linearized around the steady state of the economy as

$$\begin{aligned}\ln(P_s^\star) + \frac{P_s - P_s^\star}{P_s^\star} + \ln(1-\alpha_s^\star) + \frac{(1-\alpha_s)-(1-\alpha_s^\star)}{(1-\alpha_s^\star)} + \ln(A_s^{n\star}) + \frac{f_n A_s^{n\star}}{f}\frac{A_s^n - A_s^{n\star}}{A_s^{n\star}} \\ + \ln(f(k^\star)) + \frac{f_k k^\star}{f}\frac{(k-k^\star)}{k^\star} = \ln(W^\star) + \frac{W-W^\star}{W^\star}\end{aligned}$$

where $x^\star$ corresponds to the s.s. level of variable x.

$$\frac{d\ln(P_s)}{dt} + \frac{d\ln(1-\alpha_s)}{dt} + (1-\frac{f_k k^\star}{f})\frac{d\ln(A_s)}{dt} + \frac{f_k k^\star}{f}\frac{d\ln(k)}{dt} = \frac{d\ln(W)}{dt}$$

using the optimal capital labor ratios

$$\frac{d\ln(P_s)}{dt} + \frac{d\ln(1-\alpha_s)}{dt} + (1-\frac{f_k k^\star}{f})\frac{d\ln(A_s)}{dt} + \frac{f_k k^\star}{f}(\frac{d\ln(W)}{dt} - \frac{d\ln(R_s)}{dt} + \frac{d\ln(\alpha_s)}{dt} - \frac{d\ln(1-\alpha_s)}{dt}) = \frac{d\ln(W)}{dt}.$$

The term $\frac{f_k k^\star}{f} = \alpha_s(t)$ by definition, and therefore

$$\frac{d\ln(W)}{dt}(1-\alpha_s(t)) = (1-\alpha_s(t))(\frac{d\ln(P_s)}{dt} + \frac{d\ln(A_s)}{dt}) + \frac{d\ln(1-\alpha_s)}{dt}(1-\alpha_s(t)) + \alpha_s(t)\frac{d\ln(\alpha_s)}{dt} + \alpha_s(t)(\frac{d\ln(P_s)}{dt} - \frac{d\ln(R_s)}{dt})$$

$$\frac{d\ln(W)}{dt} = \frac{d\ln(P_s)}{dt} + \frac{d\ln(A_s)}{dt} + \frac{d\ln(1-\alpha_s)}{dt} + \frac{\alpha_s(t)}{1-\alpha_s(t)}\frac{d\ln(\alpha_s)}{dt} + \frac{\alpha_s(t)}{1-\alpha_s(t)}(\frac{d\ln(P_s)}{dt} - \frac{d\ln(R_s)}{dt})$$

Substracting the price of consumption in both side and computing the difference between sectors yields the result. ■

Proof. Corollary 1.1 We rewrite this condition as a function of CETC using the relationship between the user cost of capital and CETC through the Euler equation in the long-run:

$$\theta\frac{\dot{C}(t)}{C(t)} = -\delta_s - \nu - \frac{\dot{A}^x_j(t)}{A^x_j(t)} - \frac{\dot{\zeta}^x_{js}(t)}{\zeta^x_{js}(t)} + \frac{R_s(t)}{P_c(t)}\frac{P_c(t)}{P^x_s(t)}.$$

where $\zeta_{js}(t) \equiv \frac{\partial I(\mathbf{X_s})}{\partial X_{js}}$ is the marginal product of investment of equipment j in sector s and $\frac{P_c(t)}{P^x_s(t)} = \zeta_{js}(t)A^x_j(t)$. Define CETC in the sector as the inverse of the relative price of investment in the sector to consumption, $A^x_s(t) = \frac{P_c(t)}{P^x_s(t)}$.

$$\theta\frac{\dot{C}(t)}{C(t)} = -\nu - \delta_s - \frac{\dot{A}^x_s(t)}{A^x_s(t)} + A^x_s(t)\frac{R_s(t)}{P_c(t)}.$$

From the optimality condition for bonds,

$$\theta\frac{\dot{C}(t)}{C(t)} = -\nu + r(t).$$

which implies that if the interest rate in the economy is constant, then, $-\delta_s - \frac{\dot{A}^x_s(t)}{A^x_s(t)} + A^x_s(t)\frac{R_s(t)}{P_c(t)}$ should be constant. In other words, an equilibrium path with a constant interest rate requires $\frac{R_s(t)}{P_c(t)}A^x_s(t)$ constant, or that the user cost of capital in the sector moves inversely proportional to CETC. By definition, $A^x_s(t) = \sum_j \omega_{js}A^x_j(t)$.[22]

■

[22] Along a transition where the interest rate is not constant, the mapping between the usercost of capital and CETC becomes non-linear, but equation 3 still provides a valid link between the growth rate in sectoral prices (and labor productivity), and the usercost of capital.

Proof. Proposition 2.

Apply logs to equation 15 and take the limit when $\rho \to 0$.

$$\lim_{\rho \to 0} \ln(Y_s(t)) = \lim_{\rho \to 0} \frac{1}{\rho} \ln \left((1 - m_s(t)) \left(\frac{A_s^n(t) N_s(t)}{1 - m_s(t)} \right)^{\rho} + m_s(t) \left(\frac{A_s^k(t) \tilde{K}_s(t)}{m_s(t)} \right)^{\rho} \right)$$

Because $m_s(t) + 1 - m_s(t) = 1$ we can apply L'hopital rule and compute the limit.

$$\lim_{\rho \to 0} \ln(Y_s(t)) = (1 - m_s(t)) \ln \left(\frac{A_s^n(t) N_s(t)}{1 - m_s(t)} \right) + m_s(t) \ln \left(\frac{A_s^k(t) \tilde{K}_s(t)}{m_s(t)} \right)$$

which is just a Cobb-Douglas form.

$$Y_s(t) = \frac{A_s^n(t)^{1-m_s(t)}}{(1 - m_s(t))^{1-m_s(t)} m_s(t)^{m_s(t)}} N_s(t)^{1-m_s(t)} (A_s^k(t) \tilde{K}_s(t))^{m_s(t)} \tag{27}$$

■

Proof. Proposition 3.

We can compute **relative output** as the ratio of output in different sectors, equation 16.

$$\frac{Y_s(t)}{Y_{s'}(t)} = \left(\frac{b_{s'}^n(t)}{b_s^n(t)} \right)^{-\frac{1}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)} \right)^{-\frac{1}{\rho}} \frac{N_s(t)}{N_{s'}(t)} \tag{28}$$

The labor allocation follows

$$\frac{N_{s'}(t)}{N_s(t)} = \frac{Y_{s'}(t)}{Y_s(t)}^{1-\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{\omega_s}{\omega_{s'}} \right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{b_{s'}^n(t)}{b_s^n(t)} \right)^{\frac{1}{1-\rho}}$$

Replacing back in the equation for relative output, we obtain

$$\left(\frac{Y_s(t)}{Y_{s'}(t)} \right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} = \left(\frac{b_s^n}{b_{s'}^n} \right)^{\frac{1}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)} \right)^{-\frac{1}{\rho}} \left(\frac{\omega_{s'}}{\omega_s} \right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{b_s^n(t)}{b_{s'}^n(t)} \right)^{\frac{1}{1-\rho}}$$

$$\left(\frac{Y_s(t)}{Y_{s'}(t)} \right) = \left(\frac{b_{s'}^n}{b_s^n} \right)^{-\frac{\sigma_y}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)} \right)^{-\sigma_y \frac{1-\rho}{\rho}} \left(\frac{\omega_{s'}}{\omega_s} \right) \tag{29}$$

Output in the limiting sector will grow slower than elsewhere if labor augmenting productivity growth is slower in this sector (slower decline in b_s^n), or if the labor share falls faster in that sector. In the data however, we have seen a faster decline in the labor share in manufacturing relative to services, which would push in the opposite direction.

To sign the dynamics of the **labor share** in both sectors, we need the dynamics of the capital-labor ratios since

$$\frac{1-\alpha_s(t)}{\alpha_s(t)} = \frac{b_s^n(t)}{b_s^k(t)} \left(\frac{N_s(t)}{\tilde{K}_s(t)}\right)^{\rho}. \tag{30}$$

Using the optimality conditions for labor describe the ratio of **capital-labor ratios** as

$$\frac{P_s}{P_{s'}} \left[\frac{Y_s(t)/\tilde{K}_s(t)}{Y_{s'}(t)/\tilde{K}_{s'}(t)}\right]^{1-\rho} = \left[\frac{\tilde{K}_{s'}(t)/N_{s'}(t)}{\tilde{K}_s(t)/N_s(t)}\right]^{1-\rho} \frac{b_{s'}^n(t)}{b_s^n(t)}$$

The optimality condition for capital 19 implies that the left hand side is a constant along a GBGP if b^k is constant, equal to

$$\frac{R_s(t)/P_s^x(t)}{R_{s'}(t)/P_{s'}^x(t)} \frac{b_{s'}^k}{b_s^k} = \left[\frac{\tilde{K}_{s'}(t)/N_{s'}(t)}{\tilde{K}_s(t)/N_s(t)}\right]^{1-\rho} \frac{b_{s'}^n(t)}{b_s^n(t)} \tag{31}$$

and therefore the capital labor ratios move inversely to the labor augmenting terms b^n. Faster labor productivity growth in a sector implies a shrinking labor augmenting term and therefore higher capital-labor ratios.

Hence, combining equation 31 and 30

$$\left(\frac{R_s(t)/P_s^x(t)}{R_{s'}(t)/P_{s'}^x(t)} \frac{b_{s'}^k}{b_s^k} \frac{b_s^n}{b_{s'}^n}\right)^{\frac{1}{1-\rho}} = \left(\frac{1-\alpha_s(t)}{\alpha_s(t)} \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)} \frac{b_s^k(t)}{b_s^n(t)} \frac{b_{s'}^n(t)}{b_{s'}^k(t)}\right)^{\frac{1}{\rho}}$$

Hence,

$$\frac{1-\alpha_s(t)}{\alpha_s(t)} \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)} = \left(\frac{R_s(t)/P_s^x(t)}{R_{s'}(t)/P_{s'}^x(t)}\right)^{\frac{\rho}{1-\rho}} \left(\frac{b_{s'}^k}{b_s^k} \frac{b_s^n}{b_{s'}^n}\right)^{\frac{1}{1-\rho}}$$

which is only a function of the labor productivity trends along a BGP where the value of the marginal product of capital is constant. If labor augmenting productivity is slower in the limiting sector, then the capital share grows slower in the limiting sector than elsewhere in the economy (assuming capital and labor are complementary, $\rho < 0$).

Define the constant $\phi_s \equiv (R_s(t)/P_s^x(t))^{\frac{\rho}{1-\rho}} \frac{1}{b_s^{k\,\frac{1}{1-\rho}}}$ so that we can write the ratio of the labor shares as

$$\frac{1-\alpha_s}{1-\alpha_{s'}} = \frac{\phi_s}{\phi_{s'}} \left(\frac{b_s^n}{b_{s'}^n}\right)^{\frac{1}{1-\rho}} \frac{\alpha_s}{\alpha_{s'}}$$

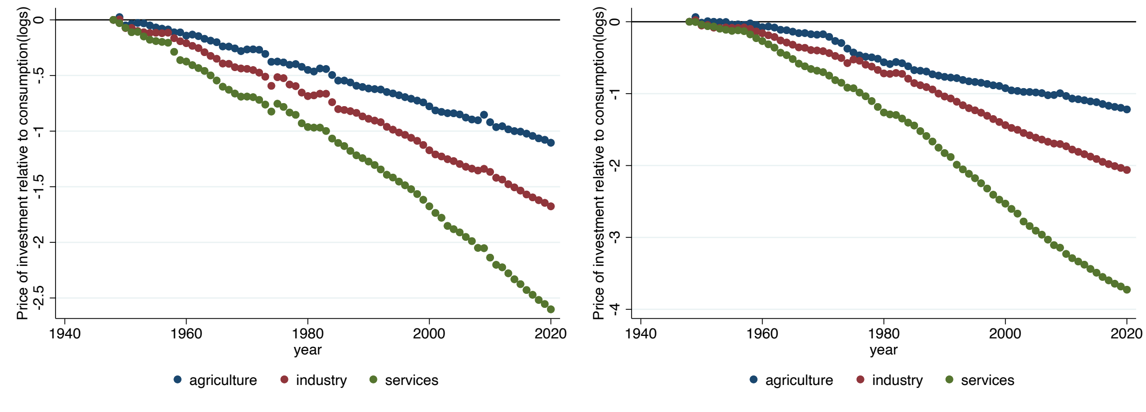
which can be replaced into 29 as

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right) = \left(\frac{b_{s'}^n}{b_s^n}\right)^{-\frac{\sigma_y}{\rho}} \left(\frac{\phi_s}{\phi_{s'}} \left(\frac{b_s^n}{b_{s'}^n}\right)^{\frac{1}{1-\rho}} \frac{\alpha_s}{\alpha_{s'}}\right)^{-\sigma_y \frac{1-\rho}{\rho}} \left(\frac{\omega_{s'}}{\omega_s}\right)$$

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right) = \left(\frac{\phi_s}{\phi_{s'}}\frac{\alpha_s(t)}{\alpha_{s'}(t)}\right)^{-\sigma_y\frac{1-\rho}{\rho}}\left(\frac{\omega_{s'}}{\omega_s}\right).$$

■

C Additional Tables and Figures.



(a) Initial weights, κ_{js1948}

(b) Last weights, κ_{js2020}

Figure 9: Sectoral CETC for different investment weights.

The price of investment relative to consumption is normalized to 1 in 1948 and the picture displays log of prices. Source: BEA and own computations.

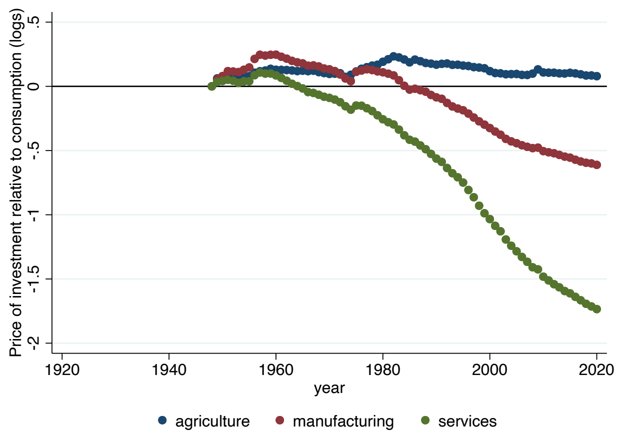


Figure 10: Relative price of investment to consumption (logs). Raw series (BEA), without quality adjustment.