From Spreads to Spirals: How Financial Frictions Drive Lumpy Investments*

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Abstract

Using US firm level data we find strong positive correlations between investment and debt issuance spikes, and between the firm investment lumpiness and interest rate spreads. A heterogeneous firm model with both capital adjustment costs and financial frictions predicts that externally financed investment spikes lead to corresponding debt spikes, thereby increasing the overall risk profile of the firm and resulting in higher interest rate spreads. Subsequently, these elevated spreads influence the investment decisions of the firm, creating what we term 'lumpy investment spirals', where investment becomes more irregular and concentrated. This mechanism significantly impacts the mean reversion dynamics of the economy, contributing to deeper and more prolonged recessions.

Keywords : Investment, Financial Frictions, Firm Dynamics
JEL Codes: D53, E44, D21

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1 Introduction

There is substantial evidence of lumpy investment behaviour at the firm level (Caballero & Engel, 1999; Cooper et al., 1999; Cooper & Haltiwanger, 2006). Firms either replace a considerable fraction of capital in one period or are investing very little for multiple periods. At the same time, investment decisions that are not entirely financed by retained earnings are inextricably linked to the firm's financial position and the cost of external finance. In this paper we investigate the interaction between extensive-margin investment decisions and financial frictions and its consequences for the aggregate economy.

Using annual Compustat data on U.S. publicly listed firms, we start by documenting two stylized facts. First, there is a strong co-movement between investment and debt spikes. Investment and debt issuance spikes tend to happen in the same periods, with debt issuance increasing on average 6% following an investment spike Second, there is also a strong positive correlation between financial conditions and investment lumpiness. A firm with a lumpier investment profile is associated with a more precarious financial position and thus external cost of finance. We take this as evidence and motivation that both debt decisions and investment decisions are clearly linked and that large investment events are associated with higher cost of credit.

In order to assess the aggregate consequences of the connection between financial and investment frictions, we set up a heterogeneous firm general equilibrium model that features both financial and investment frictions that can rationalize the observed facts. Firms choose investment to maximize their present discounted value and face fixed adjustment costs of capital, which gives rise to infrequent adjustment of the capital stock. In order to finance this investment, firms may use external funds on which they can choose to default, giving rise to a spread on loans.

A few simplifying assumptions to this setting allow us to theoretically illustrate two key mechanisms whose interaction, to the best of our knowledge, is not explored in the literature so far. First, firms with a lumpier investment profile face higher interest rate spread. A lumpier investment profile means the firm invests less often but larger amounts when active. If investment is externally finance, this leads to larger debt issuance as well. As such, when actively investing, the firms becomes riskier, as the investment profitability is reduced and default probability increased. Thus, financial intermediaries charge a higher spread to lend to these firms. Second, a higher spread will in turn affect the firm's investment decision. If the spread is too high, the firm may decide to postpone the investment, leading to an increase in the investment lumpiness. So, we show that financing conditions directly affect the firm's investment profile and can amplify lumpiness with the presence of fixed capital adjustment costs.

The interaction between these two mechanisms creates a feedback loop that we term lumpy investment spirals. As firms face higher adjustment costs and are lumpier, they are less likely to adjust due to financial frictions. In the following period, given the same target level of capital and some depreciated capital, a non-adjusting firm will need an even higher level of investment. However, this will in turn cause the financial intermediary to charge a higher spread, creating an endogenous lumpy investment spiral.

We then proceed to test a quantitative general equilibrium model fit to the data along three testable theoretical implications. First, the lumpy investment spiral creates a strong link between the firm's investment profile and the interest rate spread it is charged, which, qualitatively, matches the strong empirical correlation between the two.

Second, the presence of the financial friction in a lumpy investment model generates flatter investment hazard rates, for which we find empirical support. A model with fixed capital adjustment costs but with no financial friction predicts strongly increasing hazard rates. The introduction of the financial friction causes some firms to fall into the lumpy investment spirals and explains in part the flattening of the investment hazard rates.

Third, a model with financial frictions but without lumpy investment is not able to generate lumpy financing. As firms mainly use debt to finance investment, the fact that investment is lumpy will cause debt issuance to be equally lumpy. We find empirical support for this prediction, with close to 60% of firms not issuing debt in a given period, and when they issue, it tends to be in large scale relative to their total assets.

The model results suggest that both sides of the spiral are strong enough to affect aggregate outcomes. First, increases on the fixed capital adjustment cost parameter, which increases investment lumpiness, are enough to generate higher average interest rate spread all else equal, consistent with the micro theoretical and empirical evidence that lumpier firms pay a higher spread. Second, the quantitative model suggests that a higher spread leads to a lumpier economy, evidence that the micro mechanism highlighted with the theoretical model is strong enough to survive aggregation. Lastly, we show that gains in terms of capital allocation from loosening the financial friction are greater in the presence of a real friction than in an economy without one.

Lastly, we assess the importance of the spiral for the propagation of aggregate TFP and financial shocks. As the spiral causes the investment hazard rates to become flatter, this will affect the overall mean reversion of the economy. Overall, this causes recessions to be deeper and more prolonged.

Literature Empirical evidence on lumpy investment, such as that provided by Cooper et al. (1999) and Cooper & Haltiwanger (2006), documents the prevalence of lumpy investment, characterized by irregular, large-scale capital expenditures. These studies use plant-level data to demonstrate the presence of lumpy investment and its significant impact on aggregate investment volatility. Further, Whited (2006) demonstrates the significance of financial frictions for investment lumpiness using hazard rate estimation, although their focus is exclusively on a subsample of firms in Compustat. In contrast, our analysis utilizes all available data from 1974 to 2019. Our dataset also includes richer data on financial variables and lumpiness both in the cross-section and the panel, allowing for a more detailed analysis of how financial conditions influence investment behavior. Additionally, Nilsen & Schiantarelli (2003) provide evidence of investment lumpiness but with downward-sloping hazard rates, whereas Cooper et al. (1999) find upward-sloping rates. We extend this literature by showing that the shape of the hazard rate depends significantly on the financial position of firms, demonstrating that financial health plays a crucial role in investment patterns.

Building on this empirical foundation, there is a large literature on explaining lumpy investment dynamics in quantitative models. Caballero & Engel (1999), Khan & Thomas (2008), Bachmann et al. (2013), and Winberry (2021) study the effect of lumpy investment dynamics at the microeconomic level on aggregate outcomes. The focus in these papers is mostly on the real side, studying the importance of adjustment costs. Our model extends this literature by incorporating the financial side, where firms can default, leading to a risk premium. This addition provides a more comprehensive view of investment dynamics, including the importance of financing conditions in influencing firms' lumpy investment decisions.

Conversely, our work is also related to the literature on firm financing conditions and investment decisions. Studies by Ottonello & Winberry (2020), Khan & Thomas (2013), Begenau & Salomao (2018), and Cloyne et al. (2023) have explored how financial frictions impact investment behavior. These studies illustrate that financial constraints can significantly affect the timing and magnitude of investment, thereby influencing aggregate economic outcomes. For instance, Jiao & Zhang (2022) closely examine this interaction, although they do not address aggregate implications in depth. In a similar vein, Melcangi (2024) focuses on the interplay between financial frictions and lumpy investment, particularly in the context of employment, providing valuable insights into how financing conditions affect labor market dynamics. Additionally, Bazdresch (2013) show that financing costs are crucial for explaining investment dynamics but do not dissect the interaction in a detailed model. A study close to ours is Senga et al. (2017), which also considers default risk. We add to this literature by incorporating financing frictions, highlighting the combined effects of financial constraints and default risk on investment behavior. Likewise, Koby & Wolf (2020) present a model with collateral constraints and financing frictions, studying their effects on investment decisions. However, we further advance this by fully modeling priced default risk and spreads on financing costs, offering a more nuanced understanding of how these factors influence investment dynamics.

Outlook The rest of the paper is structured as follows. Section 2 presents the stylized facts. Section 3 describes the heterogeneous firms model, the theoretical mechanisms and the model fit to the data. Section 5 presents the quantitative model results. Section 6 concludes.

2 Stylized facts

In this section, we use annual data from Compustat on U.S. publicly listed firms between 1972 and 2019 to document two stylized facts: i) a strong correlation between interest rate spreads and investment lumpiness; ii) synchronization between investment and debt spikes.

2.1 Data description

Investment We follow Ottonello & Winberry (2024) in most of the data cleaning and also in terms of the investment rate, which is defined as the ratio of capital expenditures (Compustat code CAPX) over lagged plant, property, and equipment (Compustat code PPEGT). Furthermore, we follow the literature and define a large or lumpy investment event whenever the investment rate is above 20% (Cooper & Haltiwanger, 2006; Bachmann & Ma, 2016). More details on both the data cleaning procedure and descriptive statistics can be found in Appendix A.1

Firm observables We also employ other firm observables such as leverage, liquidity and interest rate expenses and construct them as is standard in the literature. We also extensively use distance to default as a proxy for the external finance spread. We construct the distance to default measure by using in CRSP data on stock prices and follow Gilchrist & Zakrajšek (2012) in the methodology.

Cross-sectional lumpiness measures To measure lumpiness firm investment and debt issuance we restrict the sample to firms that show up in the data at least 10 years so that we capture the behaviour of the firm's over a sufficiently long period of time. We then construct a number of cross-sectional definitions of firm debt and investment lumpiness. Our main measure of lumpiness is a Herfindahl-Hirschman measure of concentration:

$$\text{HHI}_i^x = \left(\sum_{t=1}^{T} \left(\frac{x_{it}}{\sum_{l=1}^{T} x_{il}} \right)^2 | x_{it} > 0 \right) \tag{1}$$

where x_{it} is firm i's investment I or debt issuance b at time t, and T the total number of periods of firm observations in the sample. We also restrict to observations when investment is positive, so that this measure can be interpreted as a Herfindahl-Hirschman index for positive investment events. Similar to the literature on firm concentration, this measure captures the concentration of firm investment or debt issuance in specific periods. A higher value thus indicates a higher degree of lumpiness at the firm level.

For robustness, we consider two additional measures. First, we consider an investment Gini index, with the objective of capturing how concentrated firm's investment is over time. Second, we consider a coefficient of variation, which is standard deviation of investment normalized by its mean. For additional information on the definition of these variables please refer to Appendix A.5.

2.2 Stylized facts

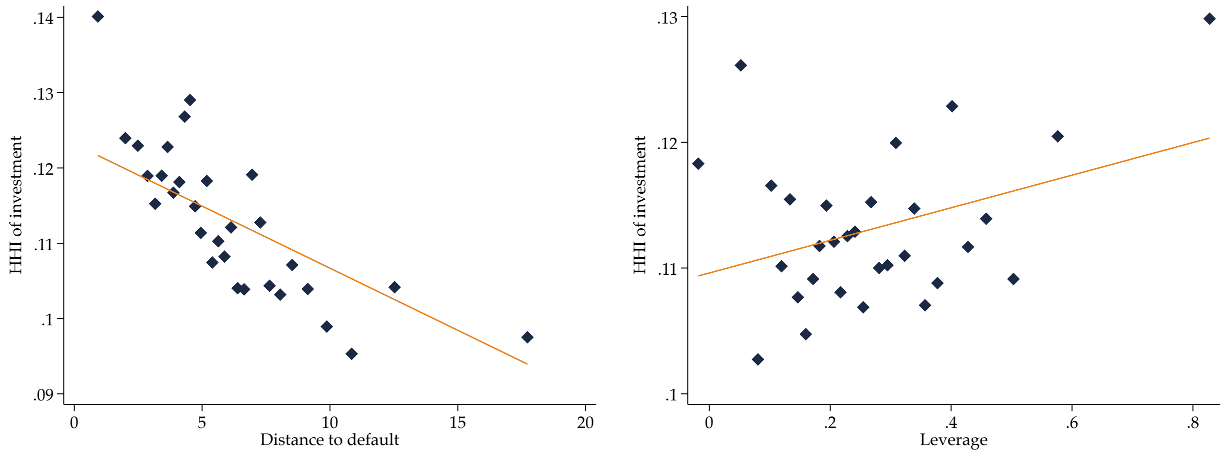
Stylized fact 1: Higher lumpiness correlates with the financial position We first document the strong positive correlation between investment lumpiness and the financial position of firms. We test this correlation by running cross-sectional regressions in the following fashion.

$$\text{Lumpiness}_i = \alpha + \gamma_j + \beta \text{ Financial Position}_i + \mathbf{\Gamma} \mathbf{Z}_i + \epsilon_i, \tag{2}$$

where γ_j is the sector fixed effect for firm in sector j, β is the correlation coefficient of interest and $\mathbf{Z}_i$ is a set of firm controls such as average size, and sales growth.

Figure 1 displays the graphical evidence of this correlation with distance to default and leverage. We show the binned scatterplot between the residualized average distance to default and leverage of a firm and the Herfindahl-Hirschman index of investment of that firm. The solid line depict the regression line (i.e. β) whilst the scatter points are binned on a firm level with 30 total bins. It is clear that there is a lumpiness is negatively

Figure 1: Investment concentration and financial position



Note. This figure depicts the binned histograms of between distance to default and HHI in investment on the LHS. On the RHS the x-axis measures leverage instead. In both cases there are 30 bins (dark blue squares) and the regression line (solid orange line). We control for various observables as illustrated in Equation (2)

correlated with distance to default and positively with leverage. We interpret this as reduced form evidence that firms with more lumpy investment schedules, i.e. firms that have a high HHI index, also tend to be closer to default and consequently face a higher spread. Of course, there are many additional co-founders here that we cannot control for.

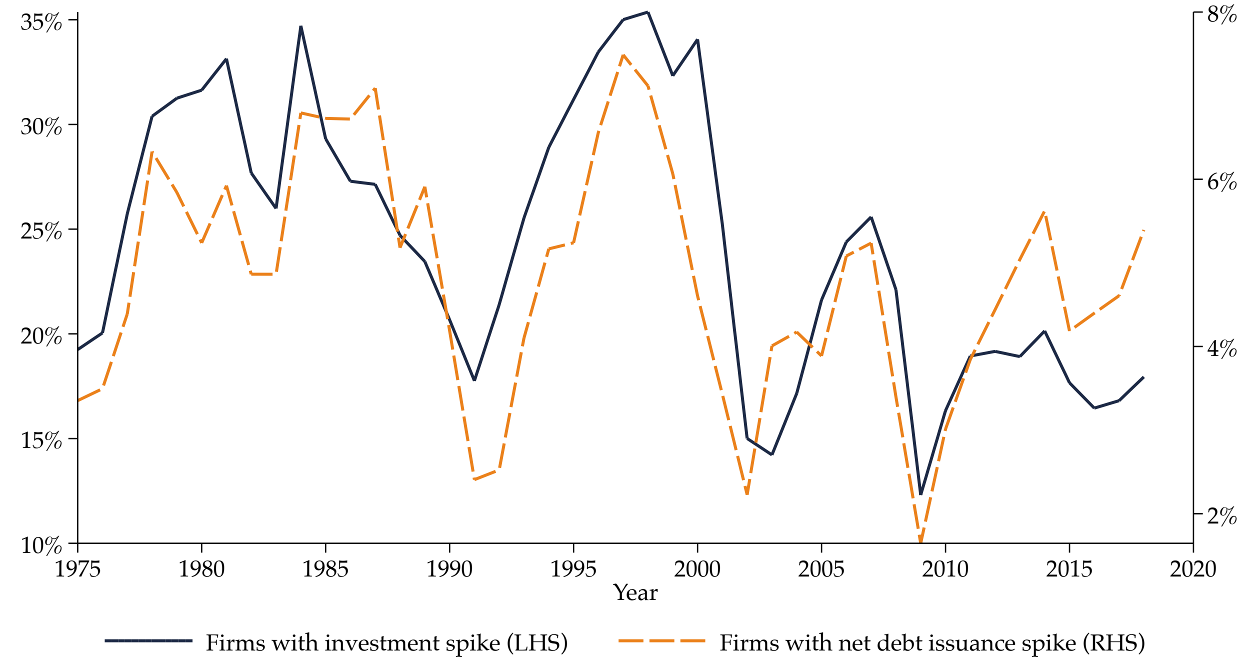
We additionally test the correlation coefficients with different proxies for external finance dependence, such as the liquidity ratio, interest rate expenses and age. Results are robust to a number of alternative proxies for financial position. In addition, we test the correlation when using either annual or quarterly data, as well as with the different measures of investment lumpiness at the firm level. Results can be found in Tables Tables A2 to A7 in Appendix A.3.

Stylized fact 2: Investment and debt spikes co-move. Second, we document a strong co-movement between investment and debt spikes.[1] Figure 2 plots the percentage of firms with an investment rate (blue line) and net debt issuance rate (orange line) above 20% in a given year. It is noticeable the strong co-movement between the two series. Periods with a higher fraction of large investment are also characterized with by a higher fraction of firms with large net debt issuance rates.

Additionally, to support this finding, we run a local projection on debt issuance following an investment spike. We find an investment spike to be correlated with an average increase of debt issuance of 6% on impact. Results can be found in Figure A4 in

[1] A debt spike is defined as net debt issuance higher or equal than 20% of total liabilities.

Figure 2: Percent of firms with investment and net debt issuance spikes over time



Note. This figure plots both the percent of firms with a investment spike and with a net debt issuance spike between 1975 and 2018. The left axis measures the percent of firms with an investment spike whilst the right axis measures the percent of firms with a net debt issuance spike.

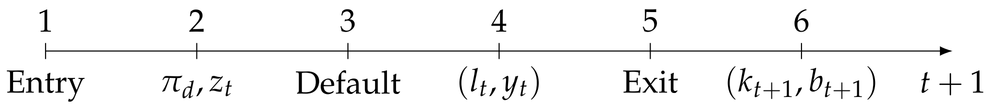
Appendix A.4.

The two stylized facts presented in this section highlight that lumpy investment and debt decisions are correlated and that the decision of large investments is inextricably linked to (the change of) the external finance position. More specifically, the evidence suggests that firms with a lumpier investment profile are more risky and pay higher spreads. To understand the mechanisms that connect the two decisions, as well as its implications for macro outcomes, we build a structural model that is introduced in the next section.

3 Structural model

We now describe a structural general equilibrium model with heterogeneous firms to analyze the interaction of real and financial frictions. Time is discrete and infinite. There are three types of agents. First, there is a fixed mass of heterogeneous firms $j \in [0, 1]$ that face variations in productivity and are subject to fixed capital adjustment costs and financial frictions. Second, the representative financial intermediary provides non-contingent one-period loans to the firms, with loan rates based on their individual characteristics and choices. Finally, the household is modelled as a simple representative agent who works,

Figure 3: Within period timing of individual firm



saves and consumes.

3.1 Firms

At the beginning of each period, a firm's state is given by its idiosyncratic productivity z_t, predetermined capital stock k_t and its level of debt b_t. The firm is also aware of the distribution Φ_t of the individual states (z_t, k_t, b_t) and aggregate productivity A_t, rationally expecting the dynamics of these aggregate allocations. Define $S_t \equiv \{\Phi_t, A_t\}$ as the collection of aggregate states at time t in order to ease on notation. The within-period timing of any period t is given below (see also Figure 3).

1. A mass $\overline{\mu}_t$ of new firms enters the economy. This mass is always equal to the firms that exit the economy so that the mass of firms in production of total firms stays constant. Entrants enter with initial capital k_0, debt b_0 and productivity drawn from the stationary distribution. They then proceed as incumbents.

2. Idiosyncratic TFP shocks and the i.i.d. exogenous exit shocks are realized. If the firm receives the exit shock, it will continue to the production stage and then exit.

3. Conditional on the exogenous realizations and the level of capital and debt, firms decide whether to default or not. If the firm defaults, the firm permanently and immediately exits the economy whilst the lender can recover a fraction of capital.

4. Conditional on not defaulting, firms enter the production stage. They choose labour input l_t, produce output y_t and pay the fixed operational cost F_c.

5. If a firm was hit by the stochastic, exogenous death shock with probability π_d and decided not to default, it pays out cash on hand and exits.

6. Conditional on survival and non-default, a firm chooses the investment size $I_t \equiv k_{t+1} - (1-\delta)k_t$ and future stock of debt b_{t+1}. The dividend will be paid out at this point.

Default decision We assume surviving firms default if there is no choice that guarantees non-negative dividends. If the firm decides to default, it will permanently exit the economy prior to production and avoids debt repayment. However, it forgoes current profits and the non-depreciated capital stock, achieving a value of 0. Define the default indicator for a continuing firm as:

$$\chi_t^{Cont.}(z,k,b) = \mathbf{1}\left(\left\{(k',b')|x_t(z,k,b) - k' + q_t(z,k',b')b' - w_t\xi\mathbf{1}(k' \neq k^c) \geq 0\right\} = \phi\right). \quad (3)$$

$$\text{where:} \quad (4)$$

$$x_t(z,k,b) \equiv \pi_t(z,k) + (1-\delta)k - b - F_c.$$

Firms who are subject to the exit shock still have the option to default before production. Firms default before exit if continuation would imply negative cash-on-hand x:

$$\chi_t^{Exit}(z,k,b) = \mathbf{1}\left(x_t(z,k,b) < 0\right). \quad (5)$$

So we can define the composite default indicator as:

$$\chi_t \equiv \pi_d\chi_t^{Exit}(z,k,b) + (1-\pi_d)\chi_t^{Cont.}(z,\xi,k,b). \quad (6)$$

Production Each firm operates using the following Cobb-Douglas production function, where the inputs are labor n_t and the capital k_t:

$$y_t = A_t z_t k_t^{\alpha} n_t^{\gamma} \quad (7)$$

with $\alpha > 0$, $\gamma > 0$ and $\alpha + \gamma < 1$. Both the idiosyncratic and the aggregate productivity shock follow a log-AR(1) process:

$$\log(z_{t+1}) = \rho_z \log(z_t) + \sigma_z \epsilon_{t+1}^z, \quad \text{where} \quad \epsilon_{t+1}^z \sim N(0,1). \quad (8)$$

$$\log(A_{t+1}) = \rho_A \log(A_t) + \sigma_A \epsilon_{t+1}^A, \quad \text{where} \quad \epsilon_{t+1}^A \sim N(0,1). \quad (9)$$

where $0 \leq \rho_z < 1$, $0 \leq \rho_A < 1$, $\sigma_z > 0$ and $\sigma_A > 0$.

Based on this production technology, the optimal labor demand $n_t(k_t, z_t)$ and production volume $y_t(k_t, z_t)$ is determined from the following static maximization problem:

$$\max_{n_t} A_t z_t k_t^{\alpha} n_t^{\gamma} - w_t n_t \quad (10)$$

where w_t denotes the aggregate wage. This optimization results in the following expressions:

$$n(k_t, z_t) = \left(\frac{\gamma A_t z_t k_t^{\alpha}}{w_t}\right)^{\frac{1}{1-\gamma}} \tag{11}$$

$$y(k_t, z_t) = (A_t z_t)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t^{\frac{\alpha}{1-\gamma}} \tag{12}$$

and, finally, the static profit of a firm conditional on optimal labour demand can be expressed as:

$$\pi_t(k_t, z_t) = (1-\gamma) y_t(k_t, z_t). \tag{13}$$

Real constraints Each firm owns the capital stock and makes an investment decision. Firm investment is subject to a fixed capital adjustment cost friction. In particular, the firm incurs the fixed adjustment cost ξ whenever next period's capital is different than non-depreciated capital:

$$k_{t+1} \neq (1-\delta) k_t. \tag{14}$$

3.2 Financial intermediaries

To finance investment the firm can use either internal resources or external resources. The only source of external finance is non-contingent one-period debt, b, on which the firm may have to pay a spread. We assume financial markets are perfectly competitive and so the spread is pinned down by the zero expected profit condition and will depend on the default probability of the firm. The pricing schedule is given by:

$$Q_t(z, k', b') = \mathbb{E}_t \left[\Lambda \left(1 - \chi_{t+1} \left(1 - \min\left\{ \frac{\theta(1-\delta)k'}{b'}, 1 \right\} \right) \right) \right], \tag{15}$$

where the term $\min\left\{\frac{\theta(1-\delta)k'}{b'}, 1\right\}$ is what the lender recovers in case the firm defaults, with θ being the recovery rate, this is, the share of the value of capital the lender is able to recover.

3.3 Firm's problem

Incumbents. As previously outlined, a firm starts the period with a given idiosyncratic productivity z shock, capital stock k and debt b. The firm has two different decision stages, conditional on surviving the exogenous exit shock. First, it decides whether to default or not. Then it decides either to invest or not and how much to borrow b, to invest i and dividends to distribute d.

Consider the problem of a continuing firm - one which does not default nor suffers the exogenous death shock. The value of a continuing firm is given by

$$V_t(z,k,b) = \max\left\{V_t^a, V_t^{na}\right\}, \tag{16}$$

where V_t^a is the value if the firm pays the adjustment cost and decides to invest, while V_t^{na} is the value under no adjustment. V_t^a is defined as follows:

$$\begin{aligned} V_t^a(z,k,b) = \max_{k',b'} \{ & x_t(z,k,b) - \xi w_t - k' + Q_t(z,k',b')b' \\ & + \Lambda_t \mathbb{E}_t[\pi_d(1-\chi_{t+1}^{Exit}(z',k',b'))x_t(z',k',b') \\ & + (1-\pi_d)(1-\chi_{t+1}^{Cont.}(z',k',b'))V_{t+1}(z',k',b')]\}, \end{aligned} \tag{17}$$

subject to:

$$D_t(z,k,b,k',b') = x_t(z,k,b) - \xi w_t - k' + Q_t(z,k',b')b' \geq 0,$$

When the firm decides to adjust the capital stock, it can freely choose k', but it has to pay the adjustment cost ξ, denoted in labour units. The firm also decides the debt amount b', on which it will receive $Q_t b'$ units today and will have to repay b' tomorrow. The continuation value takes into account the possibility under which the firm suffers the exit shock and does not default, in which case the firm exits with a value equal to its cash-on-hand. In case the firm neither suffers the exit shock nor defaults, it will have a continuation value $V_t(z',k',b')$. The value under default is zero, which happens when the firm violates the no-equity issuance constraint.

The value under no adjustment V_t^{na} is given by:

$$\begin{aligned} V_t^{na}(z,k,b) = \max_{b'} \{ & x_t(z,k,b) - (1-\delta)k + Q_t(z,(1-\delta)k,b')b' \\ & + \Lambda_t \mathbb{E}_t[\pi_d(1-\chi_{t+1}^{Exit}(z',(1-\delta)k,b'))x_{t+1}(z',(1-\delta)k,b') \\ & + (1-\pi_d)(1-\chi_{t+1}^{Cont.}(z',(1-\delta)k,b'))V_{t+1}(z',(1-\delta)k,b')]\} \end{aligned} \tag{18}$$

subject to:

$$D_t(z,k,b,(1-\delta)k,b') = x_t(z,k,b) - (1-\delta)k + Q_t(z,(1-\delta)k,b')b' \geq 0, \tag{19}$$

The main differences relative to the adjustment case are that $k' = (1-\delta)k$ and the firm does not incur the capital adjustment cost.

Entrants. Entry in this model is exogenous. We assume there is a mass, $\overline{\mu}_t$, of entrants equal to the mass of firms exiting after defaulting or receiving a death shock. The entrants are assumed to enter with initial debt b_0 and initial capital k_0. The initial productivity of each entrant, z_0, follows the same processes as the incumbents' productivity. Note that firm entry takes place at the beginning of a period, and entrants start as incumbents given their initial state, (z_0, k_0, b_0).

3.4 Household

We close the model by introducing a representative household, who consumes, saves, and supplies labor. The household specification closely follows Khan & Thomas (2008) and Bachmann et al. (2013). Specifically, we assume a log utility and disutility for indivisible labor supply in the following form:

$$U(C, L) = \log(C) - \eta L, \tag{20}$$

where C is consumption; L is the labor supply; η is the disutility parameter for labor supply.

The recursive formulation of the household's problem is as follows:

$$V_t(a, B) = \max_{C, a', B', L} \quad \log(C) - \eta L + \beta \mathbb{E} V_{t+1}(a', B') \tag{21}$$

$$\text{s.t. } C + \Lambda_t a' + \frac{1}{R_t^f} B' = w_t L + a + B \tag{22}$$

where a is the state-contingent equity portfolio value; B is the risk-free bond; Λ_t is the discount factor for the equity portfolio; R_t^f is the risk-free interest rate.

From the first order condition with respect to state contingent saving a', we characterize the discount factor as follows:

$$\Lambda_t \equiv \beta \frac{C_t}{C_{t+1}}. \tag{23}$$

As in Khan & Thomas (2008), we define $P_t := 1/C_t$, which we use for normalizing the firm's value function for easier computation.

3.5 Distribution

In order to define an equilibrium we need to derive the distribution of firms. Denote the distribution of firms in production by $\hat{\Phi}_t$. This distribution is composed of all incumbents who do not default and all entrants who do not default. Thus we can it as:

$$\begin{aligned}\hat{\Phi}_t(z,k,b) &= \int \left(\pi_d \left(1-\chi_t^{\text{exit}}(z,k,b)\right) + (1-\pi_d)\left(1-\chi_t^{\text{cont.}}(z,k,b)\right)\right) \mathrm{d}\Phi_t(z,k,b) \\ &+ \bar{\mu}_t \int \left(\pi_d \left(1-\chi_t^{\text{exit}}(z,k_0,b_0)\right) + (1-\pi_d)\left(1-\chi_t^{\text{cont.}}(z,k_0,b_0)\right)\right) \mathrm{d}\Phi^{\text{ent}}(z,k,b) \qquad (24)\end{aligned}$$

and the evolution of the distribution of firms $\Phi_t(z,k,b)$ is given by:

$$\begin{aligned}\Phi_{t+1}(z',k',b') = \int &(1-\pi_d)\left(1-\chi_t^{\text{cont.}}(z,k,b)\right) \mathbb{1}\left\{k_t'(z,k,b)=k'\right\} \\ &\times \mathbb{1}\left\{b_t'(z,k,b)=b'\right\} p\left(\epsilon^z \mid e^{\rho \log z + \sigma_z \epsilon^z} = z'\right) \mathrm{d}\epsilon^z \mathrm{d}\Phi_t(z,k,b) \\ &+ \bar{\mu}_t \int (1-\pi_d)\left(1-\chi_t^{\text{cont.}}(z,k_0,b_0)\right) \mathbb{1}\left\{k_t'(z,k_0,b_0)=k'\right\} \\ &\times \mathbb{1}\left\{b_t'(z,k_0,b_0)=b'\right\} p\left(\epsilon^z \mid e^{\rho \log z + \sigma_z \epsilon^z} = z'\right) \mathrm{d}\epsilon^z \mathrm{d}\Phi^{\text{ent}}(z,k,b) \qquad (25)\end{aligned}$$

where $p\left(\epsilon^z \mid e^{\rho \log z + \sigma_z \epsilon^z} = z'\right)$ denotes the density of draws ϵ^z such that $e^{\rho \log z + \sigma_z \epsilon^z} = z'$.

3.6 Equilibrium

We are now able to define a recursive competitive equilibrium in this economy.

Definition 1 (Recursive Equilibrium) *An equilibrium of this model is a set of functions $V_t(z,k,b)$, $k_t'(z,k,b)$, $b_t'(z,k,b)$, $Q_t(z,k',b')$, $\Phi_t(z,k,b)$, $\widehat{\Phi}_t(z,k,b)$, Λ_t, p_t, w_t, C_t, and B_t that solve the firm, financial intermediary and household problem and also clear the market for labor, output and assets, as described by the following conditions.*

1. *Production firms optimization: $V_t(z,k,b)$ solves Equation (16) with associated decision rules $k_t'(z,k,b)$ and $b_t'(z,k,b)$.*
2. *Financial intermediaries price default risk according to Equation (15).*
3. *The stochastic discount factor of the household is given by Equation (23). The wage satisfies the intratemporal optimality condition $w_t = \eta C_t$.*
4. *The distribution of firms in production $\hat{\Phi}_t(z,k,b)$ is given by Equation (24) and the evolution of the distribution of firms is given by Equation (25)*

5. *Aggregate investment is defined as* $I_t = K_{t+1} - (1-\delta)K_t$, *where* $K_t = \int k \mathrm{d}\Phi_t(z,k,b) + \bar{\mu}_t k_0$. *Aggregate consumption is defined by* $C_t = Y_t - I_t - F_c$.

6. *Bond market clears* $B_t = \int b d\Phi_t(k,z,b)$.

4 Theory and illustration

4.1 Theoretical predictions

We next outline some theoretical predictions of the model. We make three simplifying assumptions to the main model to be able to derive closed form solutions: 1) no savings: firms cannot save to finance future investment. The only internal resources which the firm has available are current revenues, which are either invested in capital or distributed as dividends at the end of the period; 2) liquidation value is fixed: in case of default, the firm's remaining value is not dependent on the state of the firm; 3) investment is fixed: the size of investment I is exogenous and fixed. [2] Please see Appendix B.3 for more details on the theoretical model.

We start by showing how the size of the capital adjustment cost affects the bond price. Proposition 1 establishes that the bond price is weakly decreasing in the debt. The firm issues debt to finance both investment and the capital adjustment cost. Recall that for simplicity we assume the size of investment to be fixed. So, if investment is fixed, necessarily a higher amount of debt is coming from a higher capital adjustment cost.

Proposition 1 (The monotonicity of the bond price in the real friction)
The bond price decreases in the frictional cost of investment:

$$\frac{\partial}{\partial b} Q \leq 0 \tag{26}$$

Proof. See Appendix B.4. ■

Proposition 2 establishes that firms with a higher liquidation value pay a lower risk spread. There are various sources of the liquidation value in the firm-level operation. One of the most important components is a firm's capital stock. When a firm holds a large capital stock, the liquidation value is high and the spread is small.

Proposition 2 (The monotonicity of the bond price in the liquidation value shifter)
Suppose the liquidation value x *is expressed as* $x = \widetilde{x} + \kappa$, *where* κ *is non-stochastic and* $\widetilde{x}$ *is the*

[2] We later show, with the quantitative model, that even when removing these three assumptions the theoretical mechanisms here identified are quantitatively relevant.

stochastic part of the liqudiation value. The bond price increases in the liquidation value shifter κ:

$$\frac{\partial}{\partial \kappa} Q \geq 0 \tag{27}$$

Proof. See Appendix B.4.ToDo ■

Lastly, Proposition 3 establishes how the bond price affects the firm's investment decision on the extensive margin. A lower bond price, translates into a higher investment threshold and consequently a lower probability of investing.

Proposition 3 (External finance costs elevate investment inactivity)
A firm makes an investment following the cutoff rule with respect to the productivity:

$$\begin{cases} V^A > V^{NA} & \textit{if } z > z^* \\ V^A \leq V^{NA} & \textit{if } z \leq z^* \end{cases}$$

As the bond price increases, the threshold rule z^* decreases:

$$\frac{\partial}{\partial Q} z^*(b; Q) < 0. \tag{28}$$

Proof. See Appendix B.4. ToDo ■

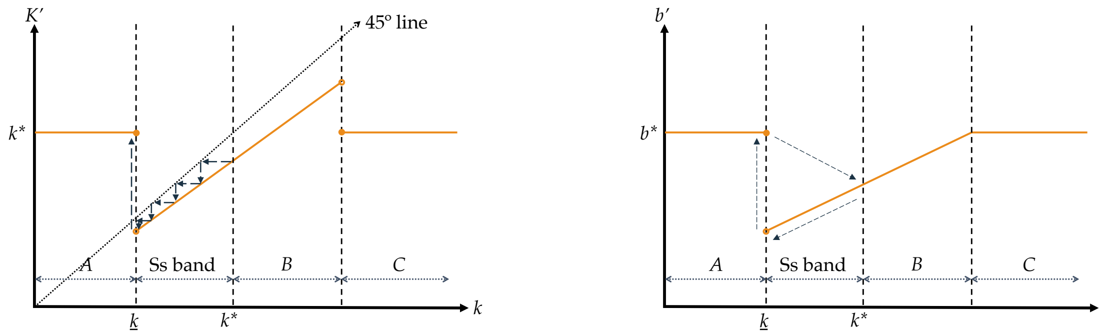
The impact that an increase in lumpiness has on spreads and the feedback effect that spreads have on the investment lumpiness gives rise to what we term lumpy investment spirals.

Overall, the mechanism will work as follows. In period one, the firm starts with a given probability of investment ψ^* and a given financing cost Q, which depends on the investment level. If the firm does not incur in large-scale investment in period 1, it will enter period two further away from the optimal capital level due to capital depreciation. As such, the optimal investment level is higher, and the firm becomes more lumpy. This translates into a higher financing cost, with $Q' > Q$ (Proposition 2), which feeds back to the investment threshold rule, with $z^{*'} > z^*$ (Proposition 3). If the firm does not invest in period 2, the situation it will face in period three will be even worse. This continues until the firm incurs a large-scale investment profile and gets out of the spiral. In appendix A.2 we present empirical evidence in support of the spiral.

The possibility of saving in case the firm does not incur in large scale investment will partially mute the mechanism aforementioned. However, for financially constrained firms, the savings will not be sufficiently large to escape the lumpy investment spiral. To illustrate this, we proceed to analyze the firms policy functions in the quantitative model.

Figure 4: Unconstrained firm's policy functions.

(a) Capital policy function (b) Bond policy function



Note. Both policy functions are plotted as a function of current capital

4.2 Illustration: Financial frictions and adjustment policies

The effect of the lumpy investment spiral will depend on if a firm is affected by the financial frictions or not. We define firms which are financial constrained as firms that cannot implement the optimal amount of capital.

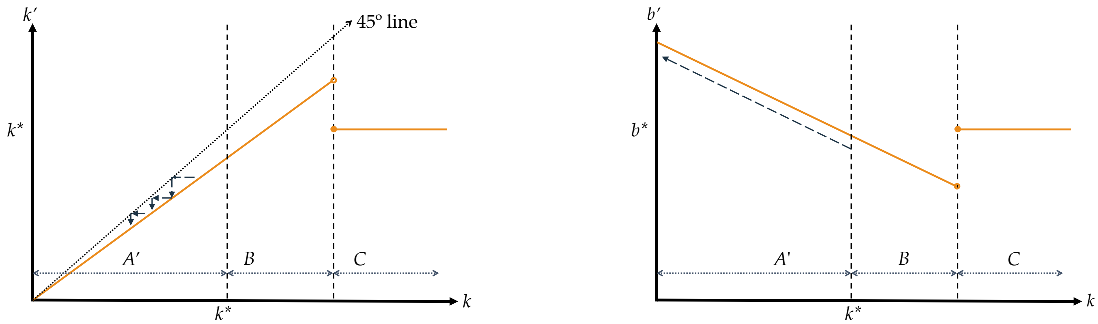
Unconstrained firms Figure 4 plots the debt and capital policy functions as a function of the firm's current capital. As this firm is unconstrained, the capital policy function is not affected by the financial friction. As such, the region where the firm is not adjusting capital is the (S,s) band, where the firm does not find it profitable to pay the adjustment cost and so capital next period will be the the undepreciated today's capital.

On the debt policy function, one thing stands out as well: there will be equally an (S,s) band. When the firm is in the capital (S,s) band, this will affect the debt policy function. As the firm is not investing, it doesn't need to issue new debt. When the firm does incur in a lumpy investment, it will equally incur in lumpy debt issuance. So, lumpy investment leads to lumpy debt issuance.

Constrained firms Figure 5 plots the capital and debt policy functions for a constrained firm. As this firm is constrained, it will not be able to pay the adjustment cost to invest in capital. As such, capital will continue to depreciate until the firm ends up leaving the market. As capital depreciates, the firm has less internal resources and needs more debt to guarantee it continues to have a non-negative dividend. As the firm issues more debt while internal resources are decreasing, it becomes more risky, and the spread will be increasing, which translates into even higher debt issuance just to guarantee a non-negative dividend. The firm will eventually default.

Figure 5: Constrained firm's policy functions.

(a) Capital policy function (b) Bond policy function



Note. Both policy functions are plotted as a function of current capital

5 Quantitative model

We start this section by presenting the model calibration. We then proceed to test the model fit to the data along three theoretical testable implications. Lastly, we assess the importance of the spiral at the macro level, by doing comparative statics on the steady state equilibrium and evaluating the propagation of an aggregate TFP shock with and without the spiral mechanism.

5.1 Solving and calibrating the model

Solution Method Given the nonconvexities due to the fixed adjustment cost we solve the firms problem using value function iteration together with Howard's improvement algorithm. In order to compute aggregate quantities we approximate the firm distribution over a fixed grid of capital, debt and productivity using the histogram method by Young (2010). The steady state solution is then given at the wage which is leading to a clearance of the goods market.

Steady state calibration Each period in the model represents one quarter. For most of the parameters we follow Ottonello & Winberry (2020). The set of these fixed parameters is documented in Table B1 in Appendix B.2. The discount factor, β, is set to yield an average annual real interest rate of 4%. The production parameters, α and γ, imply a labor share of 64% and capital share of 21%, respectively. Leisure preferences imply that households work approximately one third of their available time. The depreciation rate is set to 2.5%. The mean productivity level is normalized such that when transforming them into a log-normal distribution, the average productivity component equals one.

Table 1: Calibrated model fit

Moment	Data	Model
Mean investment spike rate	17.4%	22.0%
Mean investment rate	11.9%	34.0%
Mean default rate	3.0%	5.0%
Mean exit rate	8.7%	6.0%
Firms with positive debt	81.0%	93.0%

The remaining parameters, the adjustment cost ξ, fixed operational cost F_c, and the persistence and variance of the productivity shock, ρ_z and σ_z, are used to discipline the distributions of investment and debt as well as the default rate in the economy. The corresponding values of these parameters can be found in table B2 whereas table 1 presents the model fit to the data.

5.2 Quantitative model fit

From the theoretical model and analyzing the firms policy functions we get three model predictions: 1) a positive correlation between firm lumpiness and spreads; 2) lumpy debt issuance; 3) flat(ter) investment hazard rates. We now proceed to test these three predictions in the data, as a validation to the model mechanisms.

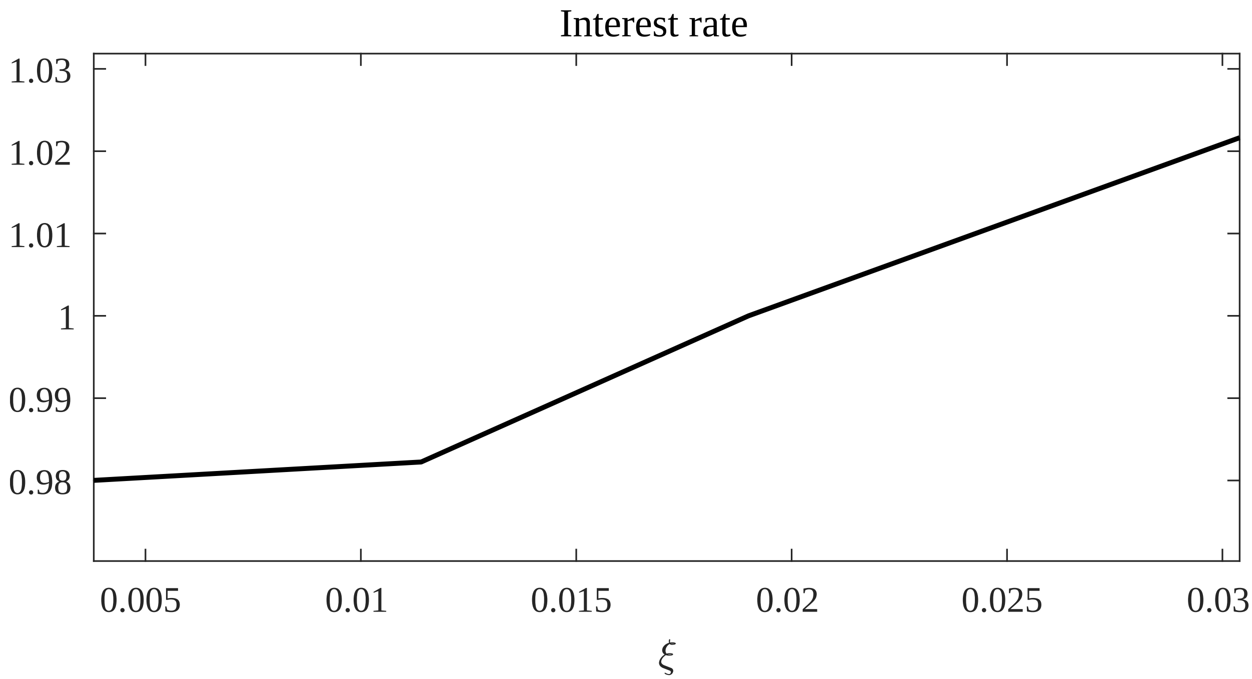
Prediction 1. The first prediction establishes a strong positive correlation between the interest rate spread and investment lumpiness. As we don't have heterogeneity of capital adjustment costs across firms, to check the model fit we do comparative statistics with respect to changes in ξ, which affects the investment lumpiness, and see how the average interest rate changes in the economy. [3]

Figure 6 illustrates how changes in ξ affect the average interest rate in the economy. The steady state level of ξ is 0.019. Whenever ξ is below the steady state level the average interest rate in the economy will decrease, with the opposite being true when ξ is above the steady state level, in line with the first stylized fact.[4] Additionally, this result illustrates that the theoretical mechanism is present in the quantitative model and strong enough to affect the average interest rate. This is comparable to stylized fact 1 in section

[3] To isolate the effects of the real friction on firm financing conditions, we keep the wage, risk-free rate and all other parameters fixed to the steady state level.

[4] To guarantee the differences in the average interest rate is coming from differences in the idiosyncratic risk of the firms and not movements in the distribution, we keep the distribution fixed for each value of ξ.

Figure 6: Average interest rate deviation from the steady state level.



Note. On the x-axis ξ, on the y-axis the the average interest rate deviation.

2.2, which establishes empirically the strong correlation between spreads and investment lumpiness.

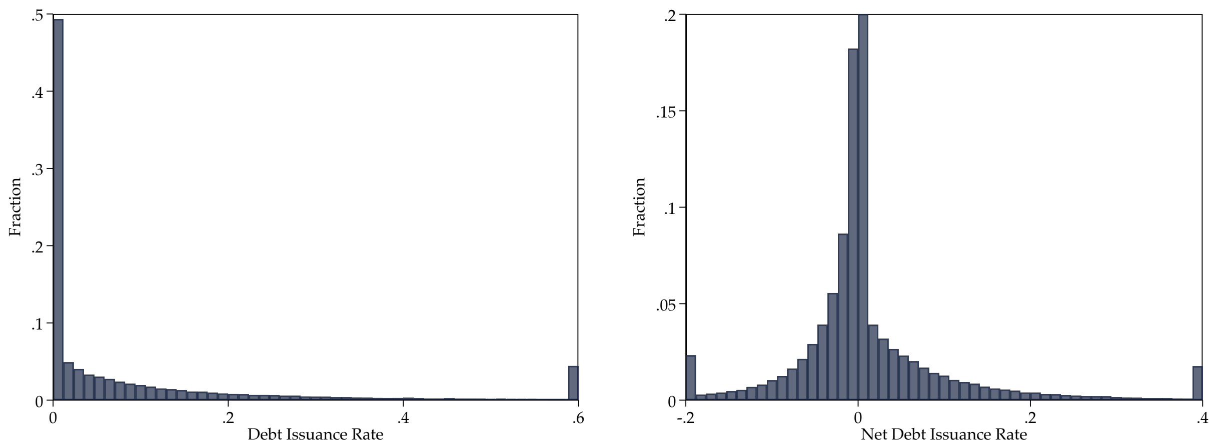
Prediction 2. The second model prediction regards debt issuance being lumpy. We empirically document that both debt and net debt issuance are lumpy. We define debt issuance rate by firm i at time t as total debt issued as a percentage of total assets. Net debt issuance rate is defined as total debt issued net of debt reimbursements as a percentage of total assets.

The distributions of debt and net debt issuance rate are reported in Figure 7. It is transparent that neither of the histograms follow a normal distribution, with a large mass at zero and fat tails. First, notice that around 50% and 40% of the observations entail a debt and net debt issuance rate of zero respectively. These episodes of inaction are complemented by periods of intensive debt issuance. We characterize spikes as issuance rate above 20%. Debt issuance rate exceeds 20% in about 12% of the observations. In terms of the net debt issuance rate, it exceeds 20% in 3% of the observations.

Figure 8 presents the model counterpart, to assess if, quantitatively, the model presents a similar pattern to the empirically observed one. Additionally, to establish the importance of the interaction between the real and financial frictions in explaining lumpy debt

Figure 7: Histograms of debt issuance in the data

(a) Debt Issuance (b) Net Debt Issuance



Note. On the left panel debt issuance rate histogram, the final bin includes any values above 0.6. On the right panel net debt issuance rate histogram, the first bin includes any values below 0.2 and the final bin includes any values above 0.4.

issuance, we compare how our benchmark model does without the capital adjustment cost. When we shut down the capital adjustment cost our model resembles the financial frictions heterogeneous firms model by Ottonello & Winberry (2020) (OW2020).

The distribution of the debt issuance rate (debt issued as percentage of total capital) for both models is quite distinct. While the model resembling OW2020 barely generates any firms in an inaction region, our model gets closer to the data with almost 40% of firms not issuing any debt in a given period.

Table 2 reports some of the main features of the two distributions, as well as the empirical counterparts. First, notice that in a model a la OW2020 the issuance rate is to high, as firms are constantly spiking and never in the inaction region. When adding a capital adjustment cost, the model starts generating firms in the inaction region and a smaller fraction of spikes, getting much closer to the data. This happens as firms mainly use debt to finance investment, and by having lumpy investment, debt issuance will consequently be lumpy as well. This is also reflected in the HHI values for debt and net debt issuance rates, that the baseline model approximates better than a model similar to OW2020.

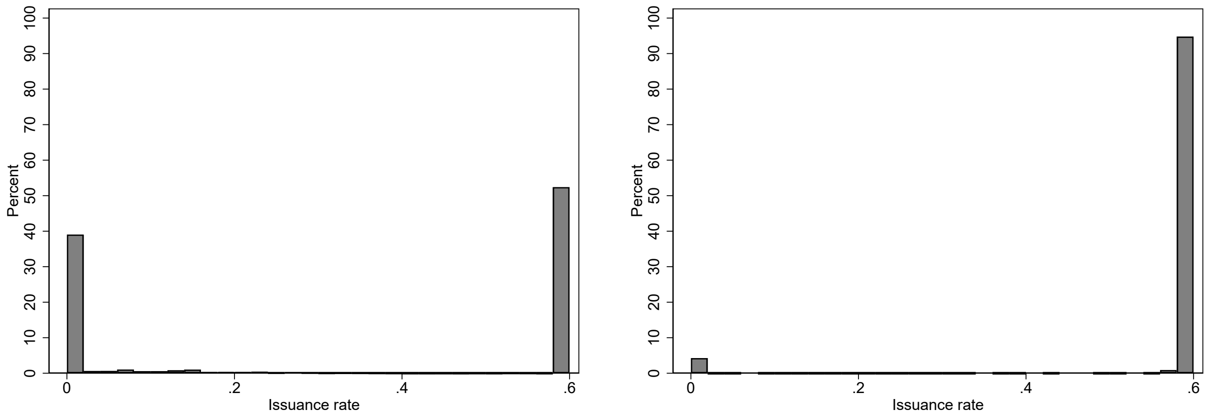
Prediction 3. The third model prediction regards flat(ter) investment hazard rates. To empirically estimate the hazard rates we use a Cox proportional hazard model controlling for firm's capital in $t-1$, sales, industry and year fixed effects.[5]

[5]For more details please see Appendix A.6.

Figure 8: Histograms of debt issuance in the model

(a) Benchmark model

(b) No real friction



Note. The left panel depicts histogram of bond issuance rate in the benchmark model. On the right panel the issuance rate without without real frictions, which resembles Ottonello & Winberry (2020).

Table 2: Debt issuance rate descriptive statistics.

	Data	Baseline model	OW2020
Net debt issuance HHI	0.398	0.408	0.550
	(0.297)	(0.369)	(0.426)
Debt issuance HHI	0.315	0.167	0.102
	(0.259)	(0.203)	(0.152)
Issuance rate	0.084	0.500	0.826
	(0.203)	(0.592)	(2.442)
Inaction rate	0.532	0.387	0.043
	(0.499)	(0.487)	(0.202)
Spikes	0.118	0.548	0.957
	(0.323)	(0.498)	(0.202)

Figure 9 shows the investment hazard rates are relatively flat and not increasing over time. This is in line with evidence by Nilsen & Schiantarelli (2003), but opposite to evidence by Cooper et al. (1999), Whited (2006) and Billett et al. (2011). However, we find that if we calculate the hazard rates for firms that spike within 9 years, similar to what Whited (2006) does, the hazard rates are increasing. It is only when including firms that go more than 9 years without a spike, that we find flat hazard rates, as shown in Figure A5 in appendix A.4.

In figure 10 we report the hazard rates in the benchmark model and in a model without the financial friction, which resembles the model by Khan & Thomas (2008) (KT2008).

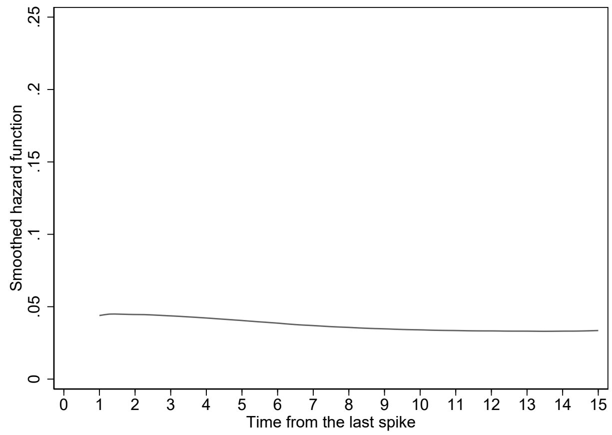
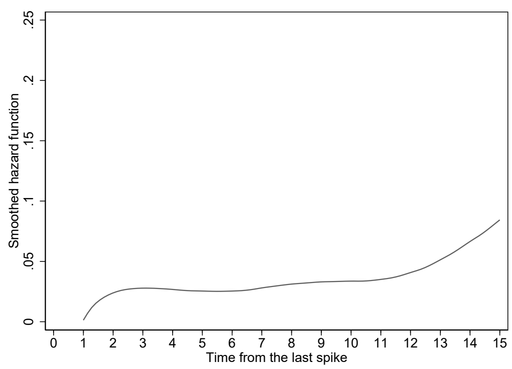
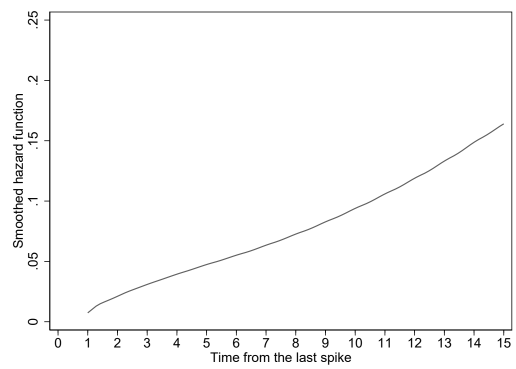


Figure 9: On the x-axis, time since last investment spike. On the y-axis, hazard rate.



(a) Benchmark model



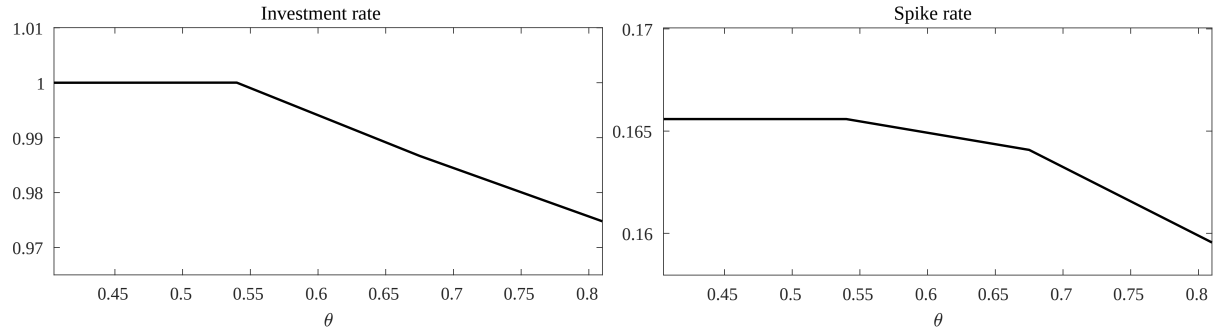
(b) KT2008

Figure 10: On the left panel the investment hazard rates in the benchmark model. On the right panel, the investment hazard rates in a model without financial frictions, similar to Khan & Thomas (2008).

Without the presence of the financial frictions the hazard rates are strongly increasing, opposite to what we find in the data. The presence of financial frictions is crucial to the lumpy investment spiral mechanism, which explains the flatter hazard rates. The firms that are pulled into the spiral, as time goes by, become less and less likely to invest as their capital depreciates and the financial friction gets tighter.

5.3 Steady state implications

In this section we first present evidence that the mechanisms identified in Section 4.1 are present in the quantitative model and survive aggregation. The first part of the lumpy



(a) Investment rate

(b) Spike rate

Figure 11: On the left panel, the average investment rate (normalized). On the x-axis θ, on the y-axis the firms investment rate, normalized to 1 in the steady state. On the right panel, the spike rate, conditional on adjusting firm. On the x-axis θ, on the y-axis the percentage of firms that incur in investment spikes $\frac{i}{k} > 0.2$, conditional on firms investing.

investment spiral was already shown to be present in the aggregate when presenting the model fit to the first stylized fact. We now change the recovery rate parameter θ, which affects the interest rate, to establish the second part of the spiral, how financial frictions affect investment lumpiness, and the impacts of the spiral on capital misallocation.

Changing θ. To isolate the effects of the financial friction on firm lumpiness, we keep the wage and all other parameters fixed at the steady state level. From the theoretical results in sections 4.1 we would anticipate that an increase in θ, which alleviate the financial friction, would decrease firm lumpiness.

In figure 11 we can see the effects of changes in θ on both the average investment rate (left panel) and the spike rate (right panel).[6] An increase in θ leads to both a decrease on the investment and spike rates, translating into a less lumpy economy. As θ increases, the financial friction weakens, which brings the economy closer to the first best allocation, with firms closer to their optimal amount of capital, consequently decreasing the lumpiness in the economy.

Lastly, we look at the implication of the presence of the lumpy investment spiral on capital misallocation. To establish this we check how capital misallocation changes when θ varies when lumpy investment is present vs a model without capital adjustment costs, this is when $\xi = 0$. Results are presented in figure 12 and illustrate that the gains from increasing θ on capital misallocation are superior in the presence of real frictions, estab-

[6]Investment rate defined as $\frac{i}{k}$ is normalized to one for the steady state level of θ. Spike rate is defined as the percentage of firms having an investment rate above 20%.

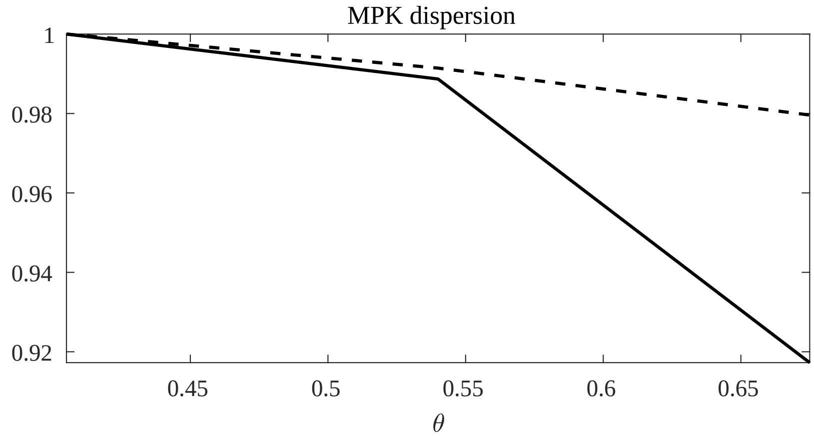


Figure 12: MPK dispersion in our benchmark model (solid line) and when $\xi = 0$ (dashed line). On the x-axis θ, on the y-axis the standard deviation of MPK.

lishing how the lumpy investment spiral contributes further to capital misallocation.

5.4 Dynamic implications

Lastly we assess the quantitative importance of the spiral for the propagation of TFP shocks. Initially at the steady state, we shock the economy with one standard deviation negative TFP shock, with a persistence of 0.9. To assess the importance of the spiral for the propagation of the shock, we compare the benchmark model response to the case where the bond pricing schedule Q is fixed to the steady state and so, does not react to the shock, and also to both situations, with Q fixed and fluctuating, under no real friction - i.e. ξ set to zero.

Figure 13 plots the investment impulse response function to the shock. In the benchmark model case, the persistence of the recession is greatly amplified, compared to the scenario when Q is fixed. While with a fixed Q the economy bottoms in the quarter following the shock, when Q reacts to the shock the economy bottoms 5 quarters after the shock and takes longer to recover to the steady state. Additionally, the overall peak of the recession is 50% larger than when Q is fixed.

The spiral is the leading force behind the amplification. As firms become more risky following the negative TFP shock, the bond price drops, driving up the cost of capital. Due to the spiral, firms will postpone their investment which will lead to an even higher spread. This prolonged effect explains why the peak of the recession happens four quarters after the shock, opposite to the scenario when Q is fixed, in which case the peak of the recession happens immediately after the shock.

Additionally, this postponing of investment will cause investment to be lower for

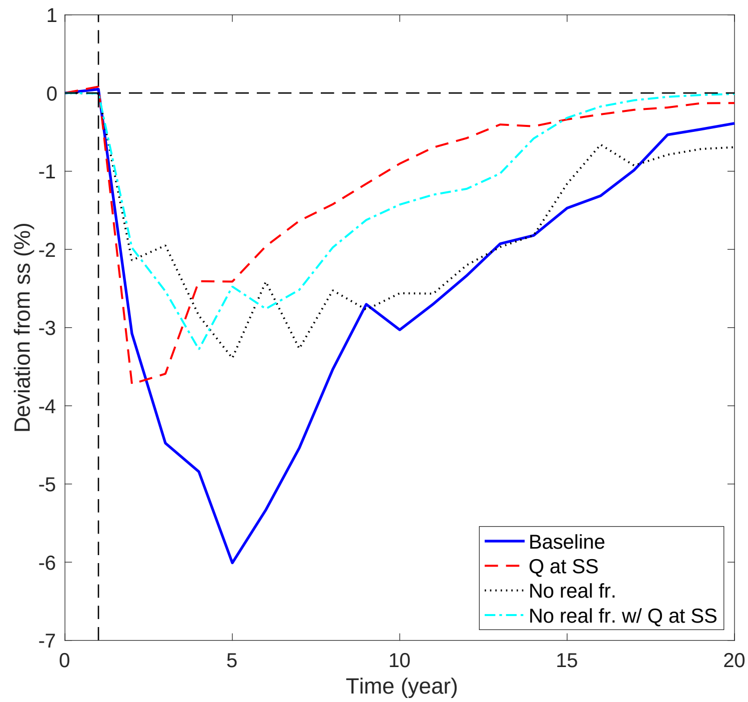


Figure 13: Investment IRF to a one standard deviation TFP shock. Blue line plots the IRF for the benchmark model. Red dashed line plots the IRF assuming Q is fixed at the steady state level. Black dashed line plots the IRF assuming $\xi = 0$, but with Q responding to the shock. Pale blue dashed line plots the IRF assuming $\xi = 0$ and Q fixed at the steady state value.

longer, which equally explains why the overall drop is 50% larger than in the scenario when Q is fixed.

Lastly, we compare to a model with no real friction. In this case, the overall investment response and persistence are not as strong. Additionally, keeping Q fixed at the steady state value further highlights the effects of the spiral, as the persistence effect is much more muted in this case due to the Q effect. So what matters for the persistence is really the interaction between the two frictions.

6 Conclusion

This paper documents two empirical facts: i) strong positive correlation between interest rate spreads and investment lumpiness; ii) strong correlation between investment and debt issuance spikes. We then proceed to show that a heterogeneous firms general equilibrium model with both a fixed capital adjustment cost and financial frictions can match these facts. If not accounting for both the real and financial frictions, the model cannot match the stylized facts.

Two mechanisms generated by the presence of real and financial frictions are key for the model to match the three stylized facts. First, firms with a lumpier investment profile invest less often but larger amounts when active. As such, when active, firms become riskier as investment profitability is reduced, paying, on average, a higher interest rate spread. Second, the higher spread will in turn affect the firm's investment decision. As the interest rate spread increases, the firm postpones investment decisions, leading to a lumpier investment profile. This interaction between the two mechanisms creates what we term lumpy investment spirals - lumpier firms pay higher spreads which in turn causes investment to become even lumpier, leading to higher spreads.

These two mechanisms are not only important at the micro level, but equally at the macro level, with a lumpier economy having higher average interest rate spreads and higher spreads leading to a lumpier economy. Additionally, the gains for capital allocation from loosening the financial friction are amplified by the presence of the real friction.

We conclude the paper by assessing the importance of the spiral for the propagation of TFP shocks. We show that the presence of the spiral can amplify an aggregate investment drop by 50% more and the effects become much more persistent.

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A Data Appendix

A.1 Data construction

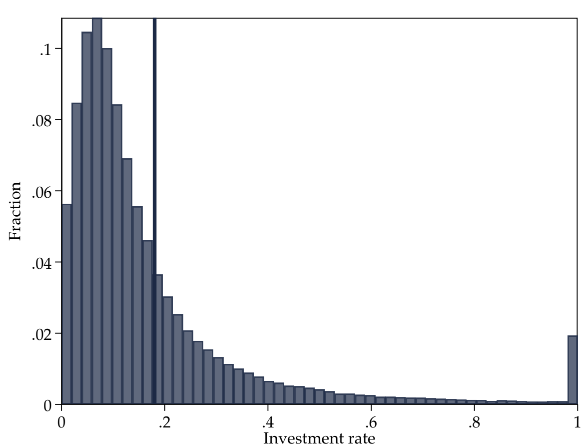
We start with the annual Compustat sample from 1974 to 2018. We do the following cleaning steps

- Drop financial firms (SIC 6000-6799)
- Drop utilities firms (SIC 4900-4999)
- Drop nonoperating firms (SIC 9995)
- Drop industrial conglomerates (SIC 9997)
- Keep only firms incorporated in the US

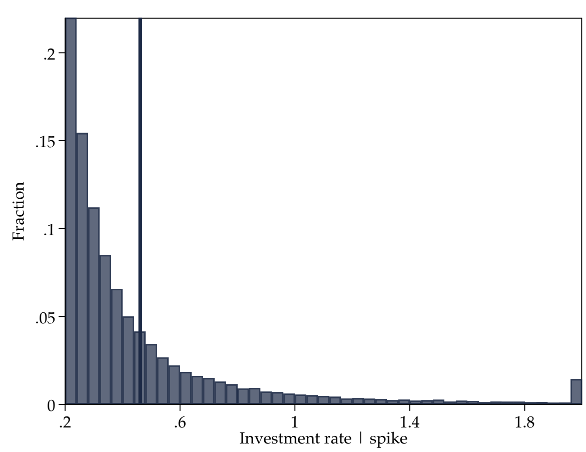
Figure A1 show the histograms for investment rate and the investment rate conditional on a debt spike. Figure A2 shows the histograms of age and size (defined as log real assets) in our sample.

Figure A1: Histograms of investment rates

(a) Investment rate



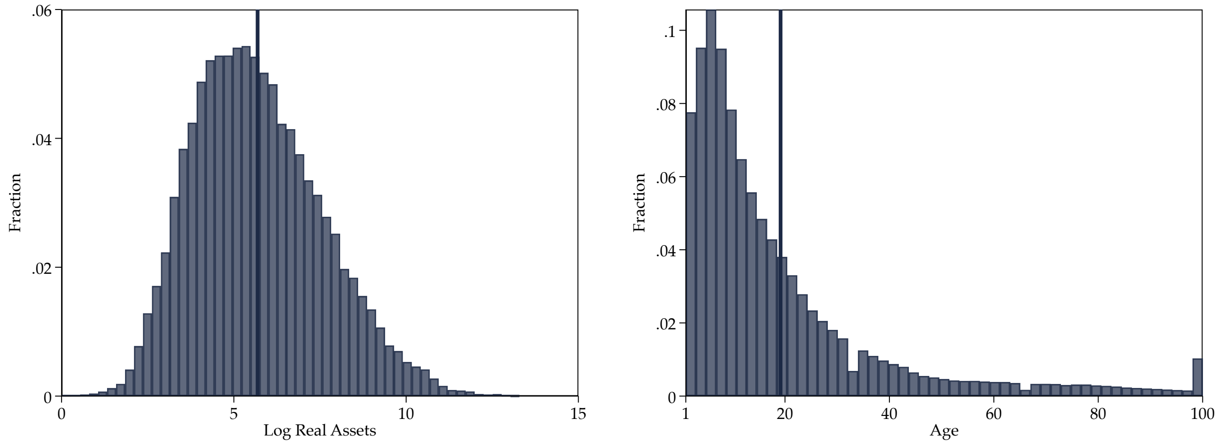
(b) Investment rate conditional on spike



Note. The left panel depicts histogram of investment rates in the data, together with the sample mean. The last bin includes any values above 1. The right panel depicts the histogram of investment rates conditional on having a spike in the data, together with the sample mean. The last bin includes any values above 2.

Figure A2: Histograms of size and age

(a) Log real assets (b) Age



Note. The left panel depicts histogram of log real assets, together with the sample mean. The right panel depicts the histogram of age, together with the sample mean. The last bin includes any values above 100.

A.2 Mechanism validation

Lastly, we go back to the data used in Section 2 to validate the theoretical mechanisms identified in Section 4.1. As we will look at monetary policy shocks, we will use quarterly data instead of annual, as we did in Section 2.

Prediction 1: Increases in interest rate spread leads to higher lumpiness To validate the first direction of the spiral, from spread to lumpiness, we use monetary policy shocks to get exogenous variations in the spread charged to firms. We use high frequency monetary policy shocks, following the methodology proposed by Gürkaynak et al. (2021). Then, following Ottonello & Winberry (2020), we aggregate the shocks at quarterly level in two different ways: 1) simple sum; 2) weighted sum, using as weigh the number of days since the shock until the end of the quarter. The reasoning behind it is that firms have more time to respond to the shocks if they happen earlier on in the quarter than closer to the end of the quarter. For the baseline results we consider the simple sum of monetary policy shocks.

If we would use monetary policy shocks directly as an instrument for distance to default - the proxy for external finance spread used here - we would have no variation in the shock across firms or industries. To have some variation across the cross-sectional dimension we use the average spread at the industry level - defined at the three digits NAICS - and its response to the monetary policy shock. This is, we first run the following specification for one industry j at the time

$$D2D_{ijt} = \lambda_j \text{mon shock}_t + \alpha_i + \gamma_j + \epsilon_{ijt} \quad \forall j \tag{29}$$

where α_i and γ_j are firm and sector fixed effects. The coefficient of interest here is the λ_j, which captures the spread elasticity to monetary policy shocks for industry j. We then use the interaction between λ_j and the monetary policy shock as instrument to distance to default at the firm level. The main specification of interest is

$$y_{ijt} = \beta \widehat{D2D_{ijt-1}} + \Gamma_h X_{ijt-1} + Z_t + \alpha_i + \gamma_j + \epsilon_{ijt} \tag{30}$$

where α_i and γ_j are firm and sector fixed effects, X_{ijt-1} are firm controls such as total

Table A1: Spikes and distance to default

	(1)
$\widehat{D2D_{ijt-1}}$	0.021
	(0.006)
Firm FE	Yes
Sector FE	Yes
Firm controls	Yes
Instrument	Mon. Pol. shock*Ind. Elast.

assets, revenues and Z_{t-1} are aggregate controls. y_{ijt} takes the form of three lumpiness measures: i) dummy for investment spikes, equal to one when the investment to capital ratio is above 20%, similar to other papers in the literature such as Cooper & Haltiwanger (2006) or Gourio & Kashyap (2007), and zero otherwise; ii) investment ratio, conditional on an investment spike, and zero otherwise; iii) inaction duration, which we calculate has the number of consecutive periods in which the firm does not make an investment spike. The $\widehat{D2D_{ijt-1}}$ is the exogenous variation in distance to default given by

$$D2D_{ijt} = \kappa \lambda_j \times \text{mon shock}_t + \Gamma_h X_{ijt-1} + \theta_h Z_t + \alpha_i + \gamma_j + \epsilon_{ijt} \tag{31}$$

Results for the lumpy measure spike are presented in Table A1 and indicate that and exogenous decrease in the spread - equivalent to an increase in distance to default - lead to a higher probability of an investment spike. Tables A8 and A9 in Appendix A.3 show that results hold when using the other lumpy measures of investment ratio, conditional on a spike, and inaction duration. Additionally, results are robust to using the weighted monetary policy shocks, as presented in Table A10.

Lastly, we test if we consider a spike to happen when the investment ratio is above

10%. As we are using quarterly data, 20% spikes occur in less than 5% of the observations. With a 10% spike threshold we have spikes occurring in close to 20% of the observations, a value in line with Cooper & Haltiwanger (2006). Results in Table A11 in Appendix A.3 are robust to the different spike definition.

All this empirical evidence is in line with the theoretical prediction that an increase in the interest rate spread leads to a more lumpy investment profile, as it causes firms to postpone investment decisions and making investment more concentrated.

Prediction 2: From lumpiness to interest rate spread The final step to empirically validate the lumpy investment spiral is to show how investment lumpiness affects the spread. To do this we analyse behaviour of external finance proxies after investment spike. We run the following local projection

$$\begin{aligned}\text{D2default}_{it+h} = \beta_h \text{Spike}_{it} + \gamma \text{D2default}_{it} + \Gamma_h X_{it-1} \\ + \theta_h Z_t + \alpha_i + \gamma_j + \epsilon_{ijt} \quad \forall h = 0, \ldots, 12 \end{aligned} \tag{32}$$

where β_h is the coefficient of interest. Including D2default_{it} on the RHS guarantees that we measure the effect of an investment spike which does not move distance to default contemporaneously. However, this is still not a causal statement, and just indicates the correlation between spikes and future changes in the spread.

The response of distance to default to a spike in Figure A3, follows the suggested pattern from the theoretical model. An increase in lumpiness leads to a firm becoming riskier and consequently drives the spread up (or distance to default down). Figures A6 and A7 in the Appendix show that results are robust to using different proxies for external finance spread and in line with the theoretical predictions.

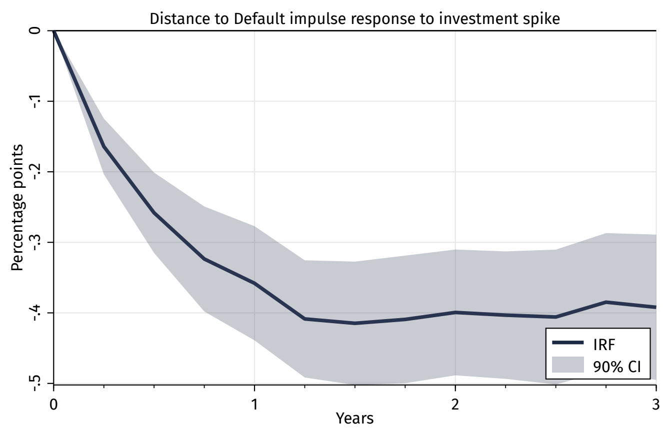


Figure A3: IRF of distance to default to a spike.

A.3 Additional tables

Table A2: HHI coefficient of quarterly firm level investment and proxies for financing costs

Liquidity	-0.058			
	(0.010)			
Distance to default		-0.003		
		(0.000)		
Leverage			0.042	
			(0.008)	
Interest Expenses				0.601
				(0.184)
Observations	6511	5862	6511	6402

Correlations are conditional on sector fixed effects and a set of firm-level observables averaged over time

Table A3: HHI coefficient of annual firm level investment and proxies for financing costs

Liquidity	-0.077				
	(0.011)				
Distance to default		-0.004			
		(0.000)			
Leverage			0.048		
			(0.009)		
Interest Expenses				0.218	
				(0.044)	
Age					-0.001
					(0.000)
Observations	5342	4826	5342	5333	5342

Table A4: Coefficient of variation of quarterly firm level investment and proxies for financing costs

Liquidity	0.084				
	(0.020)				
Distance to default		-0.017			
		(0.002)			
Leverage			0.031		
			(0.025)		
Interest Expenses				0.712	
				(0.256)	
Age					-0.001
					(0.000)
Observations	6504	5857	6504	6395	6504

Table A5: Coefficient of variation of annual firm level investment and proxies for financing costs

Liquidity	0.059				
	(0.027)				
Distance to default		-0.011			
		(0.002)			
Leverage			0.048		
			(0.027)		
Interest Expenses				0.561	
				(0.168)	
Age					-0.000
					(0.000)
Observations	5340	4826	5340	5331	5340

Table A6: Gini coefficient of quarterly firm level investment and proxies for financing costs

Liquidity	0.041				
	(0.007)				
Distance to default		-0.006			
		(0.000)			
Leverage			-0.004		
			(0.008)		
Interest Expenses				0.090	
				(0.161)	
Age					-0.001
					(0.000)
Observations	6510	5862	6510	6401	6510

Table A7: Gini coefficient of annual firm level investment and proxies for financing costs

Liquidity	0.021				
	(0.010)				
Distance to default		-0.005			
		(0.001)			
Leverage			0.017		
			(0.009)		
Interest Expenses				0.221	
				(0.057)	
Age					-0.000
					(0.000)
Observations	5342	4826	5342	5333	5342

Table A8: Investment rate conditional on a spike and distance to default

	(1)
$\widehat{D2D}_{ijt-1}$	0.010
	(0.002)
Firm FE	Yes
Sector FE	Yes
Firm controls	Yes
Instrument	Mon. Pol. shock*Ind. Elast.

Table A9: Inaction duration and distance to default

	(1)
$\widehat{D2D}_{ijt-1}$	-0.504
	(0.182)
Firm FE	Yes
Sector FE	Yes
Firm controls	Yes

Table A10: Spikes and distance to default, with weight monetary policy shocks

	(1)
$\widehat{D2D}_{ijt-1}$	0.015
	(0.006)
Firm FE	Yes
Sector FE	Yes
Firm controls	Yes
Instrument	Mon. Pol. shock*Ind. Elast.

Table A11: Spikes (investment rate above 10%) and distance to default

	(1)
$\widehat{D2D}_{ijt-1}$	0.022
	(0.010)
Firm FE	Yes
Sector FE	Yes
Firm controls	Yes
Instrument	Mon. Pol. shock*Ind. Elast.

A.4 Additional figures

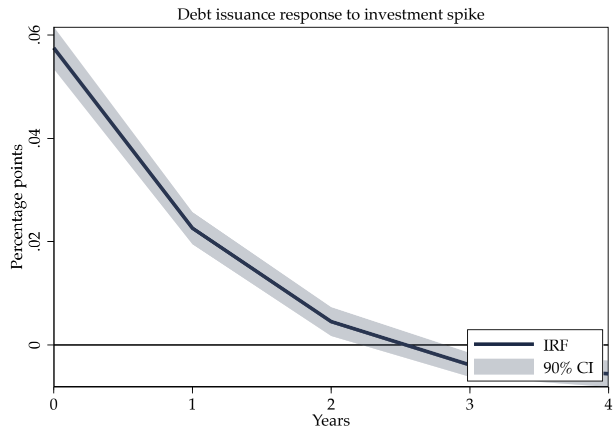
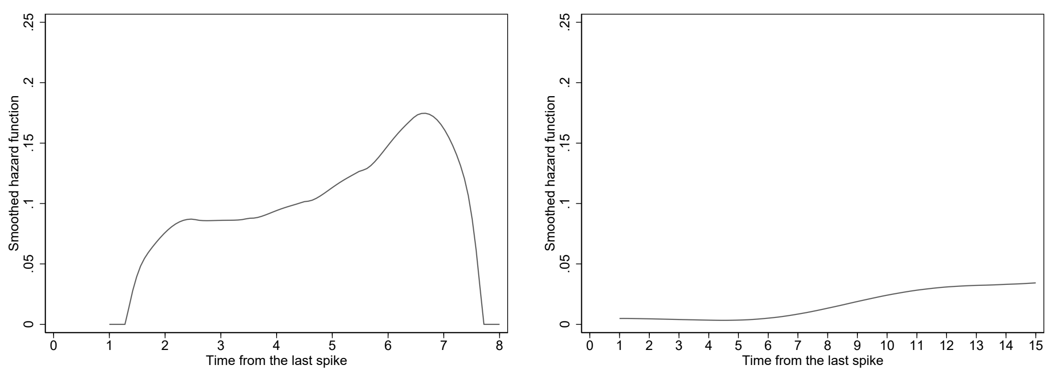


Figure A4: Response of net debt issuances following an investment spike.



(a) Duration < 7 years

(b) Duration ≥ 7 years

Figure A5: On the left panel hazard rates by firms that spike within 7 years of the last adjustment. On the right panel, firms that spike after 7 years since the last adjustment.

A.5 Lumpiness measures

The second measure we consider is a coefficient of variation for all positive investment

$$\text{CV}_i = \frac{\sigma(\frac{I_{i,t}}{K_{it-1}} | I_{i,t} > 0)}{\mathbf{E}(\frac{I_{i,t}}{K_{it-1}} | I_{i,t} > 0)} \tag{33}$$

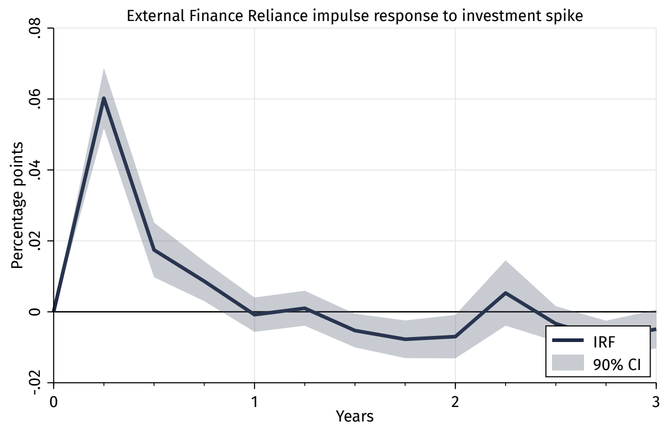


Figure A6: IRF of interest rate expenses to an investment spike.

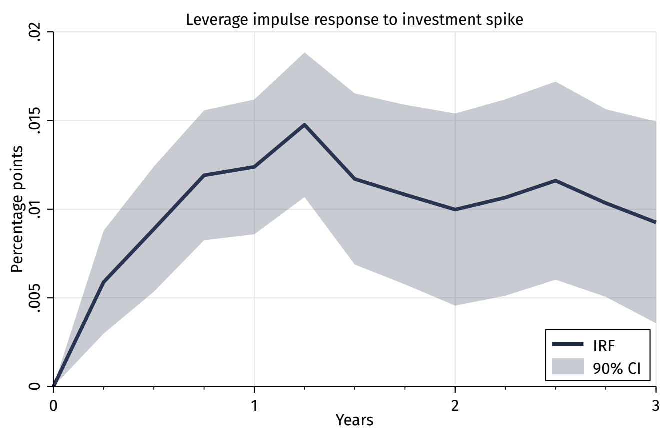


Figure A7: IRF of leverage to an investment spike.

it is the standard deviation of the firms investment normalized by the mean positive investment rate.

A.6 Hazard rates methodology

B Model appendix

B.1 Numerical Calibration

B.2 Numerical Solution Algorithm

1. Guess $V^{old}(z, \xi, k, b)$ for a given wage w^{old}

2. Define

$$\begin{aligned} W(x, \xi, k', b') \equiv & \beta \mathbb{E}[\pi_d (1 - \chi^{Exit}(z', k', b')) x(z', k', b') + \\ & (1 - \pi_d)(1 - \chi^{Cont.}(z', \xi', k', b')) V(z', \xi', k', b') | \xi, z] \end{aligned} \tag{34}$$

3. Based on that we can compute

   (a) $\tilde{V^a}(z, \xi, k, b, k', b') = x(z, k, b) + qb' - k' - \xi w + W(z, \xi, k', b')$.
   Set this value to $-\infty$ if constraint is violated

   (b) $\tilde{V^{na}}(z, \xi, k, b, b^{na}) = x(z, k, b) + qb^{na} - (1 - \delta)k + W(z, \xi, (1 - \delta)k, b^{na})$.
   Set this value to $-\infty$ if constraint is violated

   (c) Find $V^a(z, \xi, k, b) = \max_{k', b'} \left\{ \tilde{V}^a(.) \right\}$

   (d) Find $V^{na}(z, \xi, k, b) = \max_{b^{na}} \left\{ \tilde{V}^{na}(.) \right\}$

4. Compute $V^{new}(z, \xi, k, b) = \max \{V^a(.), V^{na}(.)\}$

5. Compare V^{new} and V^{old}. If sufficiently close exit. Otherwise update V^{old} and go back to the beginning.

6. Calculate the distribution using the histogram method proposed by Young (2010).

7. Calculate the new equilibrium wage w^{new}. If sufficiently close from w^{old}, stop. Otherwise update the wage and go back to the beginning.

Table B1: Fixed parameters values

Parameter	Description	Value	Source
β	Discount factor	0.99	O&W (2020)
η	Labor coefficient	2.40	O&W (2020)
α	Returns on capital	0.21	O&W (2020)
γ	Returns on labor	0.64	O&W (2020)
δ	Depreciation rate	0.025	O&W (2020)
π_d	Exogenous probability of exit	0.01	O&W (2020)
θ	Recovery rate	0.54	O&W (2020)
μ_w	Productivity process (mean)	0	Normalized
k_0	Initial capital	0.18	O&W (2020)
b_0	Initial debt	0	O&W (2020)

Notes. O&W (2020) is short for Ottonello & Winberry (2020).

Table B2: Fitted parameters values

Parameter	Description	Value	Source
ξ	Adjustment cost	0.02	Calibrated
σ_z	Std. dev.: productivity shock	0.11	Calibrated
ρ_z	Persistence of productivity shock	0.90	Calibrated
F_c	Fixed operational cost	0.07	Calibrated

B.3 Theoretical model

The bond market We propose a simple theory on external financing given the firm's endogenous default decision. Denote the borrowing amount as b, the liquidation value as x, and the risk-free return as R. x is a random variable to be realized in the future period.[7] All the information is symmetric between the firm and funding providers at the bond market. Assume the firm defaults whenever the borrowing amount is larger than the liquidation value $b > x$.

When the firm defaults, the future payoff of the bond is $\max\{x, 0\}$. It is worth noting that the debt holder does not carry the liquidation value if it is realized to be negative. If the firm does not default $(b \leq x)$, the future payoff of the bond is b. In summary, the bank's payoff is as follows:

$$\min\{b, \max\{x, 0\}\} \tag{35}$$

The bond pricing Q is determined by the bundle of (b, x, R):

$$Q = Q(b, x, R) > R \tag{36}$$

Due to the risky nature of the investment project, the bond interest rate $1/Q$ is greater than R. We assume the bond market is competitive. Therefore, the bond price is determined at the level where the funding provider's expected profit is zero. Specifically,

$$\begin{aligned} Q &= \arg_q \left\{ -\frac{1}{q} b + \frac{1}{R} \mathbb{E} \min\left\{ b, \max\{y, 0\} \right\} = 0 \right\} \\ &= \frac{R}{\mathbb{E} \min\left\{ 1, \frac{\max\{x,0\}}{b} \right\}} \end{aligned} \tag{37}$$

Interest rate spreads and investment lumpiness We consider a simple firm-level extensive-margin investment problem as follows:[8]

$$V(z, b; Q, R) = \max\{V^A(z, b; Q, R), V^{NA}(z, b; Q, R)\} \tag{38}$$

[7] Under the conventional Markov productivity assumptions, future x realization depends on the current fundamentals of the firm such as firm-level productivity.

[8] To sharpen the theoretical point, we intentionally assume that the size of the investment project is fixed at I. In the full model, the firm endogenously determines the size of investment I.

where

$$V^A(z,b,\xi;Q,R) = \frac{1}{R}\mathbb{E}_z \max\left\{z' I^\alpha + b - Q(I+\xi), 0\right\}$$
$$V^{NA}(z,b;Q,R) = \frac{b}{R}$$

V^A is the value function when a firm invests. The investment entails a fixed adjustment cost $\xi > 0$. Q is the price of investment, which a firm takes as given. I is the predetermined size of investment; b is the existing liquidity of a firm, and the negative b corresponds to the net debt of a firm. If a firm defaults, the payoff is 0. z and z' are the current and future productivity, which follows a Markov chain $z' \sim \Gamma(z'|z)$ that captures the persistence of the productivity process.

We define $z^* = z^*(b;Q)$ such that

$$V^A(z^*, b; Q, R) = V^{NA}(z^*, b; Q, R) \tag{39}$$

B.4 Proofs

Proposition 1 Proof.
Define a function G as follows:

$$\begin{aligned} G(b;x) &= \mathbb{E}\min\left\{1, \frac{\max\{x,0\}}{b}\right\} \\ &= \frac{1}{b}\mathbb{E}\min\left\{b, \max\{x,0\}\right\} \\ &= \frac{1}{b}\int_0^b x d\Phi(x) + (1 - \Phi(b)) \end{aligned} \tag{40}$$

From the bond price characterization,

$$Q = \frac{R}{\mathbb{E}\min\left\{1, \frac{\max\{x,0\}}{b}\right\}} = \frac{R}{G(b,x)} \tag{41}$$

Then,

$$\frac{\partial}{\partial b}Q = -\frac{R}{G(b,x)^2}\frac{\partial}{\partial b}G(b;x). \tag{42}$$

Therefore, depending on the sign of $\frac{\partial}{\partial b}G$, the sign of $\frac{\partial}{\partial b}Q$ is determined.

$$\begin{aligned} \frac{\partial}{\partial b}G(b;x) &= -\frac{1}{b^2}\int_0^b x d\Phi(x) \\ &= -\frac{1}{b^2}\mathbb{E}\left[x|\text{Default}\right] < 0. \end{aligned} \tag{43}$$

Therefore,

$$\frac{\partial}{\partial b}Q > 0. \tag{44}$$

■

Proposition 2 Proof.

ToDo ■

Proposition 3 Proof.

ToDo ■