# The Chinese Transition: The Role of Investment Frictions*

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**Abstract**

This paper proposes a parsimonious model to account for the main features of the Chinese transition from 1965 until recent years. We augment a standard neoclassical growth model with a distortion in the investment sector. We infer the level of distortion using the observed path of the capital-output ratio. The fit of the quantitative model to the data is good. We show that throughout the 1980s and early 1990s the model contains significant propagation effects after the distortion is removed that contributed towards higher GDP and consumption. We compute the social cost of our investment distortion in terms of the net present value of foregone consumption, and estimate large transitional losses. We estimate that the net present value in terms of consumption between 1965 and 1979 would have been 18% higher if distortions were removed immediately in 1965 instead of phased out over 15 years.

***JEL Classification: E17, E22, E65, H54, N15, O11, P23***

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# 1 Introduction

Over the last 50 years China's GDP per capita has increased from 5 % of the United States level to almost 30% in PPP-terms. This astonishing growth has captured the attention of many researchers for years. The focus has mostly been on the years since the policy reforms in 1978 and the growing privatization in the 1990s, where growth rates were the highest. Here, we take an even longer view and show how distortionary policies introduced by Mao Zedong initially depressed Chinese growth rates and how the dissolution of these policies in the 1970s aided and propagated growth way into the 1990s.

We use a model that features investment-specific technical change augmented with an investment distortion to explain transitional growth China for the period 1965-2016. We use exogenous variation in total factor productivity and capital prices (relative to consumption) and features of the transition to gauge the path of the investment distortion. In particular, we use information of the capital-output ratio path to calibrate the level and change over time in the distortion. The key intuition in identifying the distortion is that the capital-output ratio increased in a period of increasing total productivity growth after 1978. Moreover, we do not observe major changes in the relative price of capital. In the neoclassical growth model, an increase in the growth rate of productivity (given capital prices) implies a falling capital-output ratio. Through the lens of our model, an increase in the capital-output ratio will be interpreted as a reduction of distortions in the investment sector. We provide two types of evidence to validate our measure of distortion. First, there is historical evidence that coincides with the timing of our proposed distortion. In particular, it coincides with the unraveling of the effects of government led investment strategies that created inefficiencies in parts of the economy. In our model, the economy starts out distorted as a consequence of the investment policies during the years around the Great Leap Forward (GLF hereafter), where investments were made on the basis of social equity and military strategy rather than economic efficiency. The fall in our measure of distortions from 1965 to around 1980 coincides with a period of recovery during which the government backtracked by scaling back its ill-planned investment projects and placed greater emphasis on economic efficiency. We also check the ability of the quantitative model to match several moments of the Chinese transition that were not matched in the calibration exercise. We find that the model is consistent with the increasing path of output-per-capita, investment-output ratios, the relative size of the stocks of equipment and structures, as well as declining returns to capital and increasing wages.

Using our quantitative model we find that investment distortions are an important factor at explaining several features of the transition. In a counterfactual exercise, we shut down the investment distortion and find that it is particularly important to explain the dynamics of the capital-output ratio, the investment-output ratio, and the marginal return to capital. The distortion also plays a role years after it receded. In fact, we calibrate a model without an investment distortion for the period after trend growth shifted in 1978, and find that it cannot account for the paths of capital per worker, return to capital, or real wages. Finally, we compute the social cost of our investment distortion in terms of the net present value (NPV) of foregone consumption, and estimate large transitional losses. We estimate that the NPV in terms of consumption between 1965 and 1979 would have been 18% higher if distortions were removed immediately in 1965 instead of phased out over 15 years as in the benchmark.

**Related literature**. Our paper contributes to three different strands of the literature; papers on the Chinese transition, on transitional growth and the literature on aggregate resource misallocation. Most of the work on the Chinese transition has been done in the later period following the 1978 reforms. The main drivers behind post-reform growth have been identified as Total Factor Productivity (TFP) growth and structural transformation.

In explaining how China went from a poor country to a middle-income country, most papers focus on the period after 1978 and point to rapid growth in GDP and structural transformation from agriculture to manufacturing as the main drivers. Dekle and Vandenbroucke (2010) perform a growth-accounting exercise on the period from 1978-2003 and find private sector TFP to be the main driver. Brandt et al. (2008) and Dekle and Vandenbroucke (2012) study aggregate growth in the same period and focus on the transformation from agriculture to manufacturing as a driver of growth. Some studies have also been done on the pre-reform period. Cheremukhin et al. (2017) does a wedge-accounting exercise on the period between 1953 and 1978 and finds a relationship between the magnitude of different wedges and shifts in the political power of traditional versus reformist wings within the Communist Party. We believe it is useful to study the long lines of this transition in one unified model. Some work has been done on covering the entire period of Communist rule in China. Zhu (2012) covers both the pre and post-reform periods, but uses different analytical frameworks for the two periods. Chow and Li (2002) study the transition from 1952 to 2010 by estimating a Cobb-Douglas production function, but does not specify a full model with optimizing behavior. In addition to understanding investment frictions in China, we believe that using a consistent model for 50 years of Chinese growth is an important contribution to the literature.

The paper also contributes to the literature on transitional growth. King and Rebelo (1993) pointed out some unrealistic features of transitions in the neoclassical growth model, which have received a lot of attention since then. Our model is close to Greenwood et al. (1997) which features an investment-specific technological change captured by the price of capital. Chang and Hornstein (2015) later use this feature in a model of the economic transition of South Korea. Our model also touches on how frictions in the economy affect economic transitions, a topic previously discussed in, e.g. Buera and Shin (2013).

A key part of our understanding of the early part of China's transition is an investment friction implying resources were not used efficiently. This ties our paper to the literature on misallocation, where China has been a well-studied example. Much of this literature has focused on microlevel distortions; see, for instance, the work on distorted factor markets in Hsieh and Klenow (2009), Song et al. (2011) and Song and Wu (2015), but there is also significant evidence of sectoral and regional misallocation as in Brandt and Zhu (2010), Brandt et al.

(2013) and Tombe et al. (2015). Our friction is one between consumption goods and investment goods and is therefore more closely related to the last three papers. As an addition to our structural model, we also perform a wedge accounting exercise like that found in Chari et al. (2007).

The rest of the paper is organized as follows. Section 2 documents macroeconomic trends and historical facts motivating our paper. Section 3 formulates a growth model with investment specific technological growth in two types of capital and an investment friction. Section 4 contains different simulations of our model. Our baseline simulation fits the data well on untargeted moments, and counterfactual simulations show the importance of the investment friction in the transition. In Section 5 we discuss the implications for output and welfare of this investment friction. Section 6 concludes.

# 2 Stylized Facts about the Chinese Transition

In this section we present stylized facts about the Chinese transition from 1960 to 2015. We organize the discussion into two parts. First, we present the main aggregate trends in the economy with an emphasis on the periods before and after the 1978 reforms. We highlight the role of distortions and its impact on investment and TFP before the reforms. Moreover, we describe the trend in TFP growth after the reform period starting in 1978, and its macroeconomic consequences. Second, we discuss the main historic events that occur in China in the pre-reform period. We argue that these events are consistent with our modeling framework.

## 2.1 Macroeconomic Trends

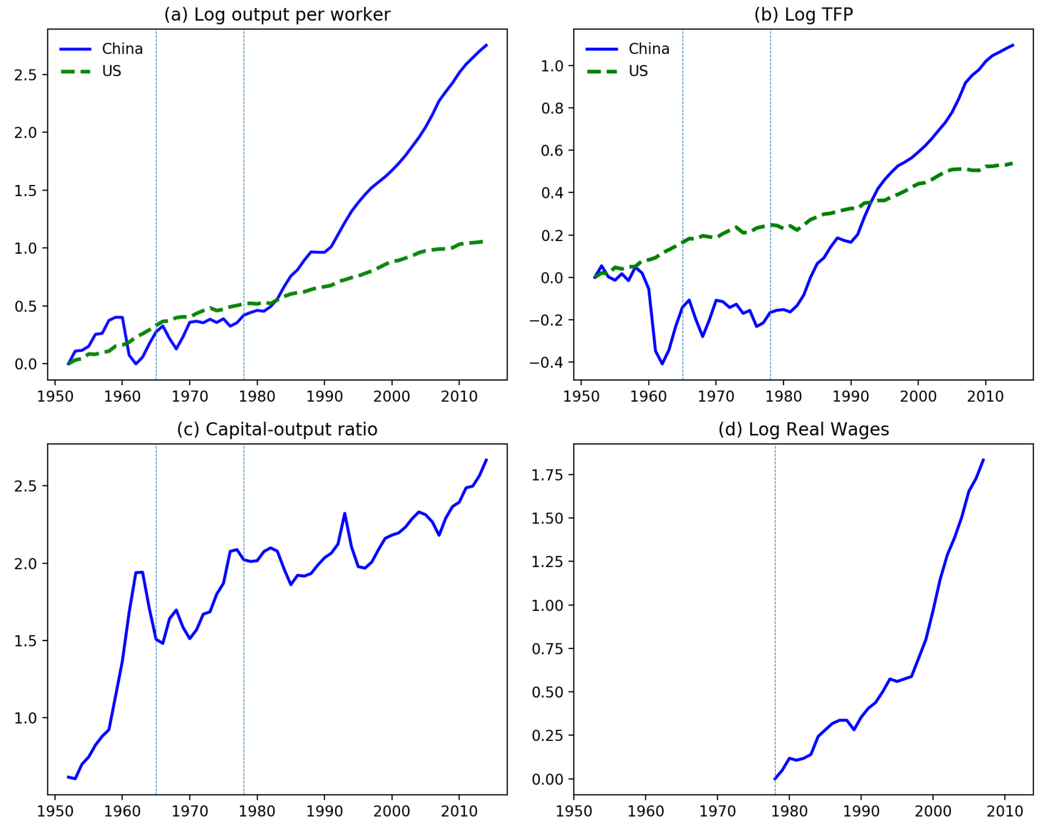
China has experienced sustained growth in output per worker for many years. In Figure 1, panel (a) we see that China grew at the same rate as the US from 1950 to around 1980 (roughly 3% per year on average), but from the early 80s the growth in output per worker change permanently to a rate of more than 8% per year. The first years of the People's Republic of China (PRC), from 1949 until 1965, were economically turbulent, which is visible from large fluctuations in output per worker. Panel (b) shows how TFP decreased by 37% from 1958 to 1962 following the failed policies of the GLF. In the period prior to the reforms, between 1965 and 1978 the TFP did not increase at all, whereas output per worker grew as the capital stock increased. Between 1978 and today the average annual TFP growth has been almost 3.5% a year and real output per worker has increased by a factor of 15. This rapid growth was accompanied by persistent increases in capital and investment-output ratios. In panel (c) we see that capital-output ratio increases rapidly from 1950 to 1980. It continues to increase also after 1980, but at a somewhat slower rate. It is puzzling to observe increasing capital output ratios in periods with sustained TFP growth. In a standard neoclassical model, an increase in TFP growth leads to a falling capital-output ratio in both transitions and on the balanced growth path. It could be consistent if the economy was at a lower than optimal level of capital and was "catching up", but this would be accompanied by very high interest rates and a gradually decreasing investment-output ratio, which was not the case. Another notable feature of the transition is the persistent growth in real wages. In panel (d) we plot the logarithm of real wages (normalized). The real wage grew fast throughout the period with a shift to an even higher growth rate around 1998.

## 2.2 Historical account

### Early years of the PRC and the Great Leap Forward

After more than 12 years of war and civil war, the PRC was established in October 1949. This marks the beginning of the socialist era in China, where most economic decisions were made top-down by the party in Beijing. The main focus of China's new leaders was to build a strong industry, a strategy labeled "Big push industrialization" by Naughton (2007). How and where these investments were to be made was debated within the Communist Party. Ma and

Figure 1: The Chinese transition




Notes: Panel (a) shows the log of output per worker and panel (b) shows the log of total factor productivity. Both are normalized by their 1952 values. Panel (c) shows the capital-output ratio for China and panel (d) shows real wages. For China the data comes from the National Bureau of Statistics of China (stats.gov.cn) and authors calculations, (see details below). US data are from Penn World Table (Feenstra et al. 2015).

Wei (1997) identify three objectives that stood in contrast to each other; social equity, military strategy, and economic efficiency. How the relative importance of each of these three objectives shifted throughout the following years, is important to understand the industrial policies of the PRC. The early years of the PRC (1949-1952) was a period of rebuilding and transition to a new economic system. Social equity and economic growth were key objectives. Land was redistributed from rich landowners to private households and factories were taken over by the government. There was still no large-scale production planning, but heavy industry was built in the northeast with Soviet aid as a testing ground for the command economy (Naughton, 2007, p. 65). The rebuilding effort was successful, and by 1952 both industrial and agricultural output had surpassed its pre-revolutionary levels.

The whole of the 1950s was characterized by a heavy influence from the Soviet Union. Industrial plans, machinery, and command systems were copied from the Soviets. This gave rise to inefficient investment policies that would last well into the 1960s, motivating our modeled distortions. The first five year plan covered the years from 1953 to 1957 and brought extra high investment activity. Investment was gradually spread more equally across the nation and the share of investment located in the eastern region dropped from 51% in 1953 to 40% in 1956. The investment strategy was rooted in the objectives of social equity and military strategy, not economic rationale. This point is exemplified in a quote from Mao in 1956: "Without doubt, the greater part of the new industry should be located in the interior so that industry may gradually become evenly distributed; moreover, this will help our preparations against war ... " (Mao, 1977, p. 287).

The GLF consisted in a series of policies in line with previous "Big Push" strategies. Investment in industry surged and people moved in mass into the state sector. In 1958 alone, nearly 30 million workers were absorbed into state jobs and in the countryside people were drawn from agriculture into small-scale rural industry such as the infamous "backyard steel mills" (Naughton, 2007, p. 69). Figure 1, panel (c) shows the capital-output ratio almost doubled from 1958 to 1964. On an organizational level most of the remaining private incentives were removed during this period and rural labor were organized into communes, large scale organizations combining political and economical activities. The overall effect of these policies was loads of failed investments and huge drops in agricultural output. TFP in agriculture fell by 41% from its peak in 1958 to the trough in 1962; TFP in manufacturing fell in 1958 by 23% and again in 1961 by 26%. An important factor that affected TFP in both agriculture and non-agriculture was the worsening of incentives (Lardy, 1987). Figure 1, panel (b) shows that the aggregate TFP fell by approximately 40% in 1958.

Between 1964 and 1971 China carried out a massive investment program in the remote regions of southwestern and western China. This development program - called the Third Front - envisaged the creation of a huge self-sufficient industrial base area to serve as a strategic reserve in the event of China being drawn into war. This response to a perceived external threat is illustrated by a quote from Mao in January 1965: "We must pay close attention to Third Front construction: it is a way of buying time against the imperialists, against the revisionists.... In the construction of the Third Front, we have begun to build steel, armaments, machinery, chemicals, petroleum and railroad base areas, so that if war breaks out, we have nothing to fear." The first phase of the construction of the Third Front was focused on the southern Sichuan, Yunnan and Guizhou provinces. The objective was to create an entire industrial structure in the interior of the country. The dramatic effects on regional investment can be seen in Figure 2, panel (a).

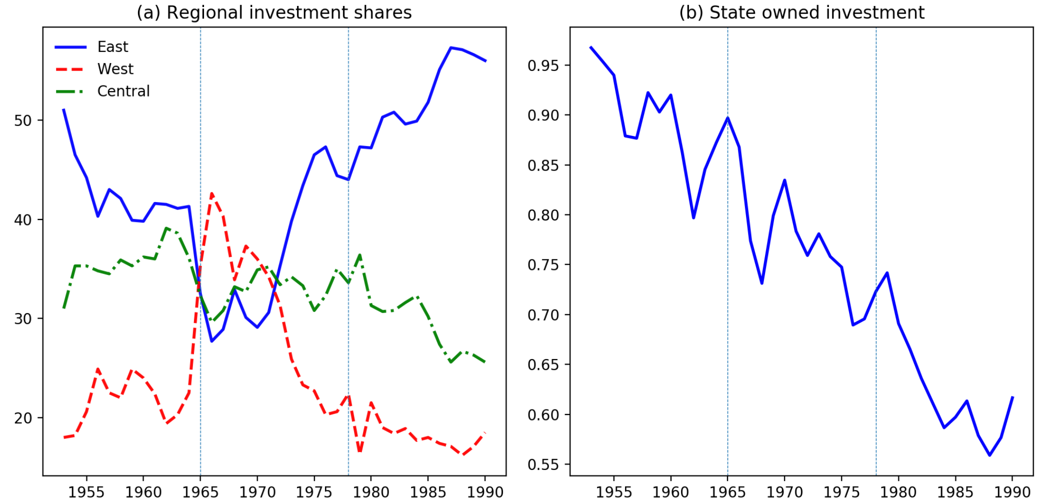
**The Cultural Revolution and the end of Maoist Model (1967-1978)**

The Cultural Revolution, defined narrowly as the period from 1967-1969, was an important political period in China, with Mao ridding himself of political opponents and attempting to revive the revolutionary spirit. Economically, on the other hand, the period was less eventful and investment policies were managed in an orderly fashion. After Mao had secured a tighter grip on his leadership he continued with his "Big Push" strategies. Once again, material incentives were criticized and bonuses eliminated and a new attempt was made to develop rural and urban industry simultaneously. This, together with a state of relative autarky and a highly militarized economy, is what Naughton (2007) call the Maoist model. In 1969, continuing the work on the Third Front, China mobilized for industrial construction in a way reminiscent of the GLF. A new surge of investment focused on the area at the intersection of Hubei, Henan and Shaanxi, and extending down through western Hunan and eastern Sichuan and Guizhou. The objective was to create an entire industrial structure, centering on railways, hydro- electric power and especially machine building in a relatively hard-to-reach part of the country. Investing in these remote areas was more expensive than in the populous coastal areas. Following the final push of the Third Front imbalances between agriculture and industry again forced a more moderate tone from the party. Zhou Enlai, the premier, was a leading figure in taking a more moderate route. More efficient investment along the coast was again resumed, and after Richard M. Nixon's visit in 1972, China also imported a substantial amount of industrial equipment from the US. The declining role of the central government throughout this period can be seen from figure 2, panel (b). Prior to 1990 there was little private enterprise, but some investment activity was decentralized and devolved to local government and other publicly owned entities. The reforms, however, was limited by the aging Mao, and economic policy-making was paralyzed between 1974 and the death of Mao in 1976 (Naughton, 2007, p. 77).

**Economic Reforms (1976-1990)**

After the death of Mao more moderate policies came into place. Still heavily plan-based policies, but with a stronger emphasis on economic efficiency and growth. The new leader in 1976, Hua Guofeng, believed that China could experience massive economic growth if leftist policies were scrapped and the focus shifted to economic efficiency and new economic institutions (Naughton, 2007, p. 78). The reign of Hua was relatively short and he was succeeded by the more pragmatic Deng Xiaoping in 1978.

Figure 2: Changing investment policies




Notes: Panel (a) shows regional investment shares are from Ma and Wei (1997). Panel (b) shows Central government investment activity as share of total from Holz (2013).

Deng is credited with much of the important reform work since 1978. The household responsibility system in agriculture was an important first step. The communal farming system was changed, and individual households given land to farm separately. This improved incentives in farming as households got to keep surplus produce after delivering a fixed amount to the government. Similar institutional changes were made in other areas of the economy. First state-owned enterprises were given some autonomy in production and investment, rather than following strict plans. Later firms were made financially independent, which meant that they could also retain profits after paying taxes to the state (Chow, 2004).

Another important policy decision in the period was the experiments with allowing FDI. The first special economic zone (SEZ) were introduced in 1979, which where places foreign companies could invest, and the where the Chinese leadership could learn from abroad. In the final stage of his leadership, Deng took his "southern tour", visiting SEZs and giving speeches that paved the way for further economic openness and private enterprise in China. “Development is the only hard truth,” Deng declared, “It doesn’t matter if policies are labeled socialist or capitalist, as long as they foster development.". This political push marked the start of an unprecedented flow of FDI and investment in private business in China that has taken it on the path to WTO membership and the mixed economy with expansive free markets we see today.

# 3 A Growth Model with Investment Frictions

To account for the Chinese transition, we present a simple model that features investment-specific technological change augmented with an investment distortion. Investments are subject to an overhead cost. The distortion motivated by the inefficiencies documented in the historical account described above. We aim to build a model that can capture trends in the aggregate data and also be in line with the historical facts. The model is similar to the neoclassical model, but is expanded on the production side. We include two types of capital, equipment capital, and structures, with differentiated prices to allow for trends in investment-specific technological changes both for aggregate investment and between the two types of capital. We also model an aggregate investment distortion to capture the effect of early inefficient investment policies.

## 3.1 A Growing Economy with Investment Distortion

**Households**: The economy is populated by a representative consumer of size $N_t$ with separable preferences over consumption and labor. Their discount factor is given by $\beta$. The consumer's problem is to maximize lifetime utility

$$\sum_{t=0}^{\infty} \beta^t N_t \left(\log(c_t) + \psi \log(1 - h_t)\right)$$

where $c_t$ is consumption, $h_t \in (0, 1)$ is hours worker per member, $\beta$ is the discount factor and $\psi$ is share of leisure in the utility function.

**Production**: Output is produced using labor, structures and equipment capital combined using an aggregate Cobb-Douglas production function:

$$Y_t = A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E} (h_t N_t)^{1-\alpha_S-\alpha_E}$$

where, $K_{E,t}$ is equipment capital and $K_{S,t}$ is structures and other capital. The parameter $A_t$ is the total factor productivity parameter.

Output produced can be used for consumption and investment, where the total cost of investment is capital expenditure, $p_{E,t}I_{E,t} + p_{S,t}I_{S,t}$, and an inefficiency loss parameter $\theta_t$ explained below. Hence, the resource constraint can be written as follows:

$$N_t c_t + \theta_t (p_{E,t} I_{E,t} + p_{S,t} I_{S,t}) = Y_t$$

where $\theta_t$ is a distortion in the investment sector, $p_S$ and $p_E$ are the prices of equipment and structures respectively, both in terms of the consumption good. There are two equations for capital accumulation for equipment and structures

$$K_{E,t+1} = (1-\delta_E) K_{E,t} + I_{E,t}$$

$$K_{S,t+1} = (1-\delta_S) K_{S,t} + I_{S,t}$$

Combining the last three equations we can write the aggregate resource constraint as

$$\begin{aligned} N_t c_t + \theta_t \left( p_{S,t} K_{S,t+1} + p_{E,t} K_{E,t+1} \right) &= A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E} (h_t N_t)^{1-\alpha_S-\alpha_E} \\ &\quad + \theta_t \left[ p_{S,t}(1-\delta_S) K_{S,t} + p_{E,t}(1-\delta_E) K_{E,t} \right] \end{aligned} \tag{1}$$

There are five exogenous variables in this economy, namely: Productivity, prices for both types of capital, population and investment distortions. Productivity grows at rate $\gamma_t - 1$, where $\gamma_{A,t} = A_t / A_{t-1}$. Population is growing at rate $n_t - 1$, where $n_t = N_t / N_{t-1}$. Similarly, the rate of growth for capital prices are $\gamma_{E,t} - 1$ and $\gamma_{S,t} - 1$ for equipment and structures respectively. Finally, investment distortion grows at rate $\gamma_{\theta,t} - 1$.

The investment distortion parameter $\theta_t$ represents the inefficiency in the investment policies of the Mao-era. As discussed in the previous section on the historical background, the GLF was characterized by massive investments in industry. This is not uncommon among countries in post-war episodes, but the Chinese experience was one of state-led investment that was especially inefficiently implemented. We could interpret $\theta_t$ as overhead cost over implemented investment expenditure. For example, if $\theta_t$ is 1.5 there is an additional cost of 50% of the investment expenditure that does not show up as productive capital. These costs may include planning, transportation, and installation of machinery or other capital equipment.

There is a point to be made about measurement regarding our definition of distortion. Notice that the way our investment distortion is modeled, it does not appear in investment expenditure, $p_{E,t}I_{E,t} + p_{S,t}I_{S,t}$, but as an additional dead-weight loss. Alternatively, one can consider the converse case where a certain amount of yuan is spent on expenditure, but only a fraction ends up as productive capital. This could be achieved through a production function for investment or by assuming that some investment goods "fell off the truck" along the way. One reason for not choosing this formulation of inefficiency is that we have a measure of the price of capital, and the price of capital should already reflect the difference between how many yuan is spent and how much productive capital you get. The second reason is that we believe that a lot of the excess costs created by the investment regime Mao implemented were of the sort that is not typically counted in measures of investment. A feature of this investment policy was to decentralize a lot of heavy industry (see Section 2.2) and this creates a lot of excess transportation and communication costs. An example of anecdotal evidence is the story of the backyard furnaces. To spur steel production, small blast furnaces were set up all over the countryside to a great cost. The way investment-data were collected also supports this approach. Data collection was done at the firm or commune level by adding up the cost of new machinery, etc. (Holz and Yue (2018)). The costs of planning, distributing and setting up these and other investment projects are unlikely to be included in this measure as there were no real price mechanism in place to allocate those costs. We will discuss the historical background for our distortion further when we calibrate the model in Section 4.

**Normalization**. Since the exogenous variables are trending, we need to transform the problem by detrending the endogenous variables. To this end, it is convenient to define the composite variable:

$$Z_t = A_{t-1}^{\frac{1}{1-\alpha_S-\alpha_E}} p_{E,t-1}^{\frac{-\alpha_E}{1-\alpha_S-\alpha_E}} p_{S,t-1}^{\frac{-\alpha_S}{1-\alpha_S-\alpha_E}}$$

which represents the trend component of consumption per capita. Now define the transformed variables:

$$\tilde{c}_t = \frac{c_t}{Z_t}, \tilde{K}_{S,t} = \frac{K_{S,t} p_{S,t-1}}{Z_t N_{t-1}}, \tilde{K}_{E,t} = \frac{K_{E,t} p_{E,t-1}}{Z_t N_{t-1}}$$

For a more compact notation we will be using growth rates of exogenous variables, $\gamma_{X,t} = \frac{X_t}{X_{t-1}}$ for any variable $X_t$.[1] . Using these new variables we can rewrite the resource constraint as (details in the appendix):

$$\begin{aligned} \tilde{c}_t + \theta_t \gamma_{Z,t+1} (\tilde{K}_{S,t+1} + \tilde{K}_{E,t+1}) &= \frac{\gamma_{A,t}}{\gamma_{N,t}^{(\alpha_E+\alpha_S)}} \tilde{K}_{S,t}^{\alpha_S} \tilde{K}_{E,t}^{\alpha_E} h_t^{1-\alpha_S-\alpha_E} \\ &+ \frac{\theta_t}{\gamma_{N,t}} \left[ \gamma_{S,t}(1-\delta_S)\tilde{K}_{S,t} + \gamma_{E,t}(1-\delta_E)\tilde{K}_{E,t} \right] \end{aligned} \tag{2}$$

The stationary social planner's problem is to choose $\{\tilde{c}_t, h_t, \tilde{K}_{S,t+1}, \tilde{K}_{E,t+1}\}$ in order to maximize lifetime utility subject to the budget constraint (2). The first-order conditions are as follows[2]:

$$\frac{\gamma_{A,t}}{\gamma_{n,t}^{(\alpha_E+\alpha_S)}} (1-\alpha_S-\alpha_E) \tilde{K}_{S,t}^{\alpha_S} \tilde{K}_{E,t}^{\alpha_E} h_t^{-\alpha_S-\alpha_E} \frac{1}{\tilde{c}_t} = \psi \frac{1}{1-h_t} \tag{3}$$

$$\theta_t \frac{\gamma_{Z,t+1}}{\gamma_{N,t+1}} \frac{1}{\beta} \frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \frac{\gamma_{A,t+1}}{\gamma_{N,t+1}^{(\alpha_E+\alpha_S)}} \alpha_S \tilde{K}_{S,t+1}^{\alpha_S-1} \tilde{K}_{E,t+1}^{\alpha_E} h_{t+1}^{1-\alpha_S-\alpha_E} + \frac{\theta_{t+1}}{\gamma_{N,t+1}} \gamma_{S,t+1}(1-\delta_S) \tag{4}$$

[1] Specifically, we use $\gamma_{A,t} = \frac{A_t}{A_{t-1}}$, $\gamma_{N,t} = \frac{N_t}{N_{t-1}}$, $\gamma_{E,t} = \frac{p_{E,t}}{p_{E,t-1}}$ and $\gamma_{S,t} = \frac{p_{S,t}}{p_{S,t-1}}$

[2] More details about the social planner's problem are provided in the appendix.

$$\theta_t \frac{\gamma_{Z,t+1}}{\gamma_{N,t+1}} \frac{1}{\beta} \frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \frac{\gamma_{A,t+1}}{\gamma_{N,t+1}^{(\alpha_E+\alpha_S)}} \alpha_E \tilde{K}_{S,t+1}^{\alpha_S} \tilde{K}_{E,t+1}^{\alpha_E-1} h_{t+1}^{1-\alpha_S-\alpha_E} + \frac{\theta_{t+1}}{\gamma_{N,t+1}} \gamma_{E,t+1}(1-\delta_E) \tag{5}$$

The Equations (2)-(5) characterize the dynamic properties of the model. To gain intuition about the role that the investment distortion parameter $\theta$ we present a simplified version of our model and solve for an analytical expression for the capital-output ratio.

## 3.2 Mapping investment distortion to capital-output ratio

A key feature of our quantitative framework is how to map distortion levels to observables. In particular, we identify the level of distortions by looking at the path of the capital-output ratio. To shed light on how this mapping works, we will work with a toy version of our model. Consider a simple growth model with a single capital good, and full depreciation. Assume also that the price of capital is equal to one. Given this assumption, we can write our simplified model as follows:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to:

$$c_t + \theta_t K_{t+1} = A_t K_t^{\alpha}$$

where as before $A_t$ is productivity and $\theta_t$ is the investment distortion parameter. Suppose the technology parameter $A_t$ grows at rate $\gamma_A - 1$, where $\gamma_{A,t} = \frac{A_t}{A_{t-1}}$. As before we normalize variables by dividing by $A_{t-1}^{\frac{1}{1-\alpha}}$, and let $\tilde{K}_t = K_t / A_{t-1}^{\frac{1}{1-\alpha}}$.

To solve the model we conjecture a closed form solution of the following form,

$$\tilde{K}_{t+1} = \frac{1}{\gamma_{A,t}^{1/(1-\alpha)} \theta_t} \alpha \beta \tilde{K}_t^{\alpha}$$

In the appendix we show that this policy function is indeed a solution using the first order conditions. Using the production function we can write the capital-output ratio as follows:

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{1}{\gamma_{A,t+1}\theta_t^{1-\alpha}} \left( \frac{\alpha\beta}{A_t^{\frac{\alpha}{1-\alpha}}} K_t^{\alpha} \right)^{1-\alpha}$$

This equation highlights how the investment distortion parameter affects the capital-output ratio. Observe that an increase in the growth of productivity (an increase in $\gamma_{A,t+1}$) decreases the capital-output ratio. As a result, to account for an increase in capital-output ratio (as we observe in the data) we need a decrease in $\theta$.

## 3.3 Balanced Growth Path

The balance growth path (BGP) of our model is characterized by a situation where consumption, investment, output and the capital stocks will grow at constant rates given by constant growth rates of the exogenous drivers. We also assume that the wedge is constant on the BGP. In particular, assume that we have growth in the population of $\gamma_N$, in productivity of $\gamma_A$, and that capital prices grow at constant rates $\gamma_S$ and $\gamma_E$. As a result, labor as a share of the population and the interest rate remain constant on the BGP. All tilde-variables will also be constant at the BGP. From the normalization, it is clear that consumption per worker will grow at rate $\gamma_Z = (\gamma_A \gamma_E^{-\alpha_E} \gamma_S^{-\alpha_S})^{\frac{1}{1-\alpha_E-\alpha_S}}$. This also is the growth rate of output per capita and wages. The stock of structures ($K_S$) and equipment ($K_E$) will grow at rate $(\frac{\gamma_Z\gamma_N}{\gamma_S})$ and $(\frac{\gamma_Z\gamma_N}{\gamma_E})$ respectively implying that a declining trend in capital prices ($\gamma_E, \gamma_S < 0$) increases the real capital stock. Investment expenditure, $p_E I_E + p_S I_S$, however grows at the same rate as total consumption ($\gamma_Z \gamma_N$). Also, as it is standard in growth models, factor income shares will stay constant along the BGP. One key feature relating to our investment friction is that zero TFP-growth and stable capital prices (like the period from 1965 to 1977) should imply zero growth in capital and output per capita, while in the data they grow slowly.

# 4 A Quantitative Analysis of China's Transition

In this section we propose a methodology to quantify the level of investment distortions over time and measure their implications for transitional growth in China from 1965 to 2015. We use our quantitative model to investigate how important the exogenous sources (technological growth, relative capital prices and investment distortions) are to explaining growth over the last five decades in China. Our calibration exercise uses information from the BGP and the transition to back out the level of distortion over time. It turns out that the moments from the BGP are not sufficient to identify the level of investment distortion. As a result, we use the structure of the model and infer $\theta_t$ using the dynamic of capital-output ratios over the transition.

We provide historical evidence that is consistent with economy-wide distortion in the investment sector. We also show that our model is consistent with the path of variables that were not calibrated. Finally, we perform several counterfactual experiments to quantify the relative contribution of different sources of growth.

## 4.1 Data

Our main data source is the National Bureau of Statistics of China (NBS). There has been a debate regarding the reliability of these data, but we land on the side of trusting the source. The quality of the data and some adjustments are discussed in detail in Holz (2013) and Holz (2014). Our source of capital data is Holz and Yue (2018); they are constructed using official investment statistics. Structure capital is an aggregate of structures and the category "others", the latter is relatively small and its price moves similarly to structures. Equipment data are a separate category. Our interest rate data are from the OECD databank.

## 4.2 Calibration

Parameters to be calibrated are the preference parameters $\beta$ and $\psi$, the production function shares for equipment and structures $\alpha_E$ and $\alpha_S$, the depreciation rates $\delta_E$ and $\delta_S$, the rate of growth of technology $\gamma_A$, population growth rate $\gamma_N$, and the growth rate for capital prices $\gamma_S$ and $\gamma_E$. Finally, we also need to calibrate a sequence of investment distortion parameters $\theta_{1,}...., \theta_T$. This makes the calibration non-standard as the initial (BGP) level of the wedge affects other calibrated parameters, meaning that the calibration depends on sequences of moments, not only BGP averages.

We take the exogenous growth rates $(\gamma_{N,t}, \gamma_{A,t}, \gamma_{S,t}, \gamma_{E,t})$ directly from the data. We set the depreciation rates to $\delta_E = 0.125$ and $\delta_S = 0.055$ consistent with values provided by Holz and Yue (2018). These are the same rates used to construct the capital data-series and are the only depreciation rates consistent with both investment and capital data. We set $\alpha_E + \alpha_S = 0.4$, consistent with an average labor share of 0.6.

To calibrate the rest of the parameters, we use information from the BGP and transition dynamics. This calibration is an iterative procedure over the initial level and the path of distortions. We need to guess on a series of wedges and then calculate calibrated parameter values and simulate the model to match capital-output ratios. The initial guess is then updated until we have a good match with the data. In particular, conditional on the initial value of distortions $\theta_0$ (where time 0 indicates the initial period in the BGP), we determine $\beta$, $\psi$, $\alpha_E$, and $\alpha_S$ using data moments on capital output ratios, labor supply and exogenous growth rates[3]. The restriction on the total capital share is used together with the following equations:

[3]From the budget constraint and the initial guess on the wedge we get the consumption share. We use this in the first order condition for labor, together with the assumption that $\alpha_E + \alpha_S = 0.4$, and a value for $h$ to solve for the preference parameter $\psi$. Next, we can use the optimality condition for $K_S$ and $K_E$ together with the restriction on the total capital share, to solve for a production function share, $\alpha_E$ and $\alpha_S$, and the discount factor $\beta$.

$$\frac{\tilde{c}_0}{\tilde{Y}_0} + \theta_0 \gamma_{Z,0} \left( \frac{\tilde{K}_{S,0}}{\tilde{Y}_0} + \frac{\tilde{K}_{E,0}}{\tilde{Y}_0} \right) = 1 + \frac{\theta_0}{\gamma_{N,0}} \left( (1-\delta_S)\gamma_{S,0} \frac{\tilde{K}_{S,0}}{\tilde{Y}_0} + (1-\delta_E)\gamma_{E,0} \frac{\tilde{K}_{E,0}}{\tilde{Y}_0} \right) \tag{6}$$

$$(1 - \alpha_S - \alpha_E) = \psi \frac{h_0}{1-h_0} \frac{\tilde{c}_0}{\tilde{Y}_0} \tag{7}$$

$$\frac{\gamma_{Z,0}}{\gamma_{N,0}} \frac{\theta_0}{\beta} = \alpha_S \frac{\tilde{Y}_0}{\tilde{K}_{S,0}} + \theta_0 (1-\delta_S) \frac{\gamma_{S,0}}{\gamma_{N,0}} \tag{8}$$

$$\frac{\gamma_{Z,0}}{\gamma_{N,0}} \frac{\theta_0}{\beta} = \alpha_E \frac{\tilde{Y}_0}{\tilde{K}_{E,0}} + \theta_0 (1-\delta_E) \frac{\gamma_{E,0}}{\gamma_{N,0}} \tag{9}$$

Next, we use these parameters and the guessed series of wedges to simulate a series of capital-output ratios. In particular, given a guess for the initial distortion $\theta_0$ and the rest of the parameters of the model, we can simulate the model using equations (2)-(5). In the forward iteration we use information for the exogenous drivers $\{\gamma_{N,t}, \gamma_{A,t}, \gamma_{S,t}, \gamma_{E,t}\}$ and a guess for the path of investment distortions $\theta_{1,....}, \theta_T$. We repeat this process, changing our guess of $\theta_{0,....}, \theta_T$, until the model generates capital output ratios $\left\{ K_j^{model} / Y_j^{model} \right\}_{j=0}^{T}$ are as close as possible to the data moments $\left\{ K_j^{data} / Y_j^{data} \right\}_{j=0}^{T}$. To reduce the dimensionality of the problem, we look for a linear decline in the distortion parameter. We thus need to find the size of the initial distiortion and the rate of decline until the distortion is phased out. We believe we lose little by this a simplification as the fit to data is good. An alternative way of backing out wedges is conducted in Section 5.3 and yields similar results.

Table 1: **Balanced Growth Calibration**

| Data moments | (1962-77) | Matched parameters | |
|---|---|---|---|
| $\frac{p_E K_E}{Y}$ | 0.244 | $\alpha_E$ | 0.032 |
| $\frac{p_S K_S}{Y}$ | 1.492 | $\alpha_S$ | 0.368 |
| $h = \frac{L}{N}$ | 0.426 | $\psi$ | 1.592 |
| **Parameters from data** | (1962-77) | **Calibrated parameters** | |
| $\gamma_A$ | 0.013 | $\beta$ | 0.990 |
| $\gamma_N$ | 0.041 | $\theta_0$ | 1.700 |
| $\gamma_S$ | 0.003 | | |
| $\gamma_E$ | 0.003 | | |

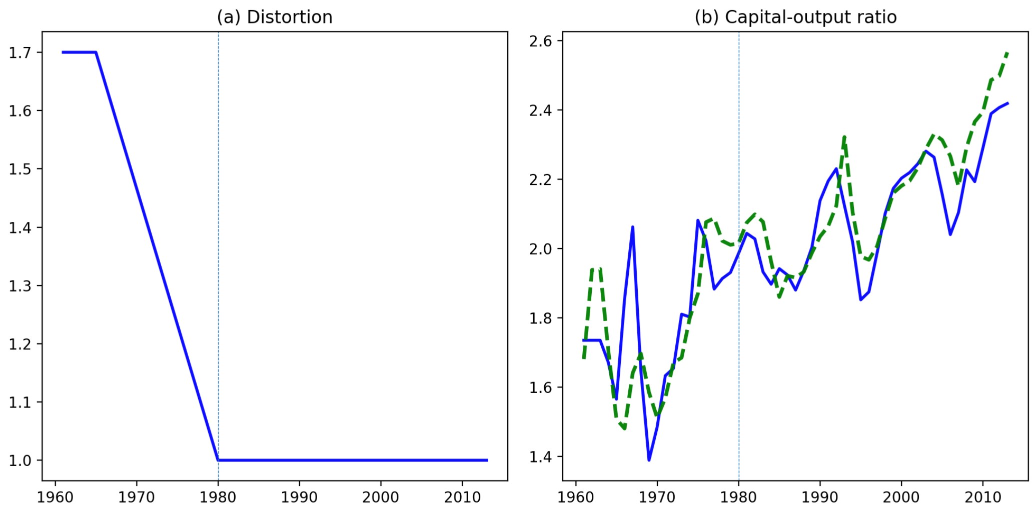
Notes: Data moments to the left, sources: Holz and Yue (2018) and CNBS. Calibrated moments to the right.

In Table 1 we show the BGP moments computed for the period 1962-1977. We have chosen this period because of relatively constant growth in the output per capita and no growth in total factor productivity or relative prices of capital. The moments are averages for the period. The growth rates for capital prices were close to zero, while productivity increased by 1% per year. The calibrated parameters all lie within a reasonable range. If we compare our calibration with Greenwood et al. (1997)'s, who calibrates a similar model to the US, our discount factor is higher. As we calibrate taking into account the entire path of capital-output ratios, this high $\beta$ reflects the high savings rate in China in the latter part of the time series. We also get lower input elasticity on equipment, $\alpha_E$, and higher input elasticity on structures, $\alpha_S$, than in their calibration. This reflects a higher structure-to-equipment ratio in China than in the US, especially in the early period.

In Figure 3 we plot the results of our calibrated investment distortion $\theta_t$. Panel (a) shows how the calibrated distortion is high in 1965, declining gradually until 1980. It is worth mentioning that what matters is the relative change in $\theta_t$ and not the levels. Therefore, we do not claim that after 1980 the Chinese economy is not distorted, but rather that the investment sector is less distorted in 1980 relative to 1965 and that this distortion has remained relatively unchanged since 1980. In panel (b) we show the that our calibration matches relatively well the path of capital-output ratios.

We now present two types of evidence to validate the proposed investment distortion. First, we will provide an historical account that matches the timing of an investment distortion and the subsequent gradual removal. Then, we use our calibrated model and show that the model indeed matches well other features of the transition that we did not match in the calibration exercise.

Figure 3: Calibration for the path of distortions $\theta_t$




Notes: Panel (a) shows the calibrated distortion. Panel (b) shows the Capital-output ratio in the model simulation and data.

### Relating the distortion to historic facts

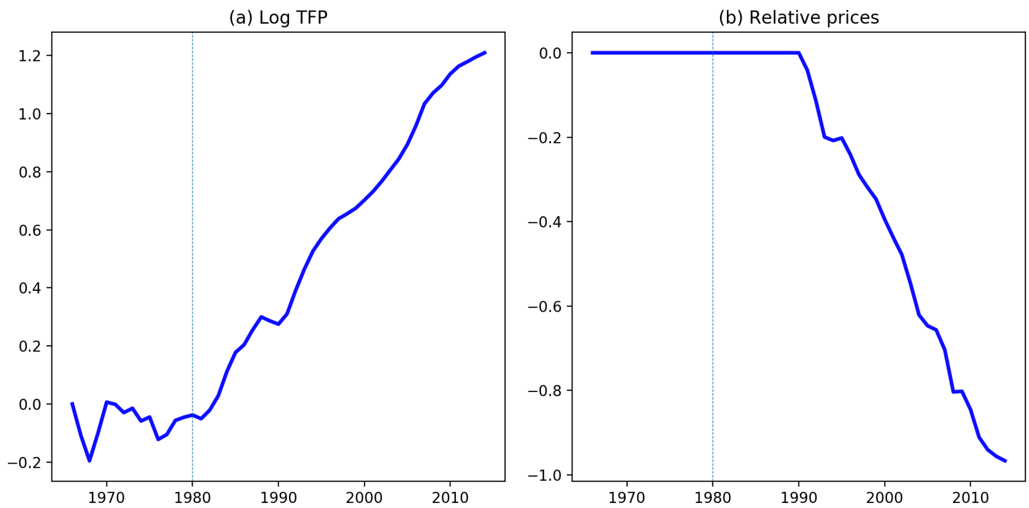
The calibrated distortion is initially (pre-1965) high and drops rapidly until the economy is undistorted from 1980. This fits well our reading of the history of the PRC. As discussed in Section 2.2, three main objectives determined investment policies throughout the first four decades of communist rule: social equity, military strategy and economic efficiency. We argue that the first two of these objectives increased the cost of capital investment, while in periods where economic efficiency was pursued, investment costs were reduced. The increase in costs was mainly due to geographical location. Most of the capital goods were initially imported from the Soviet Union, and transportation costs could be high. In the pre-1965 period, the economic efficiency objective was almost completely absent. Mao tried to rebuild the economy through a "Big Push" strategy where the placement of factories was not chosen on the basis of economic efficiency but for social equity and later military strategy reasons. Combined with the very high share of central government controlled investments, this made for high costs. During the GLF, a large fraction of investments were made in rural communes, often in the form of small-scale industry like the backyard furnaces, which also increased investment costs.

In Figure 2, panel (a), we show gross fixed asset formation in three large regions in China as shares of the total. The distribution between them is almost equal in the early 1950 and today, but with large swings in between. The eastern region has always been the economic powerhouse of China and, with its geography and proximity to the sea, is the region with the lowest transportation costs. We view a high share of investments in the east as consistent with economically rational policies. There are other reasons why this pattern might change over time, but the first drop in eastern investment in the early 1950s coincide well with the focus on social equity in the PRC's investment policy. When the share drops even further in the mid 1960s, it is a direct consequence of the Third Front, moving industry to less accessible locations inland in case of an invasion. We start our calibration with an initially high distortion pre-1965 and interpret that as a consequence of the policies of social equity, military strategy, and the failed small-scale policy of the GLF. The calibration suggests that these investment distortions were dismantled in the 15-year period until 1980. The policies of the GLF were already starting to reverse in 1965, and we can see that the central government's share of investment decreases throughout the period from panel (b) in Figure 2. Panel (a) also shows that the regional distortions caused by the Third Front is completely reversed within this 15-year period.

## 4.3 Fit of the Model

We now show how the model can account for different aspects of the transition. We use three exogenous variables as drivers in the model. We use the proposed distortion as shown in Figure 3, panel (a). In addition, we use path TFP growth and relative prices from the data. In Figure 4, panel (a) we show the path of log TFP ($\log(A_t)$) and in panel (b) relative prices ($p_{E,t}/p_{S,t}$). The measurement of prices for equipment capital and structure capital individually started in 1990. Relative prices are assumed to be constant prior to that. This assumption is supported by the fact that the stock of equipment capital relative to structure capital was stable prior to 1990, but has increased considerably since the relative price started to fall.

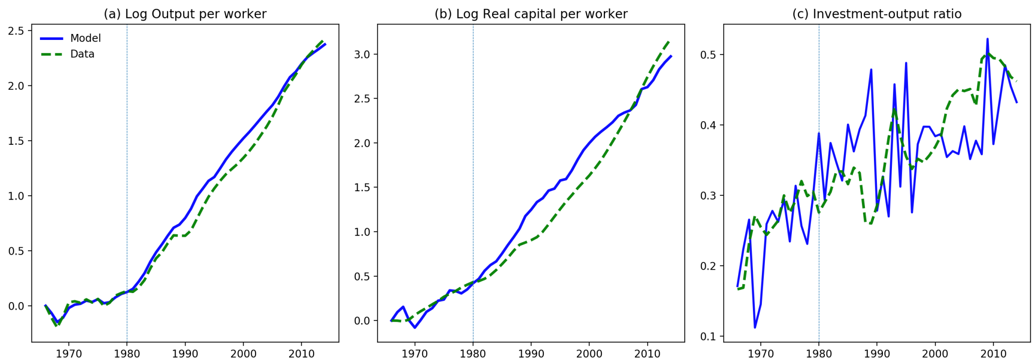
Figure 4: Exogenous inputs: TFP and prices




Notes: The two exogenous drivers of the transition in our model: (a) TFP moves as in the data, calculation consistent with the production function used in our model. (b) The price of equipment capital relative to structure capital falls dramatically after detailed measurement starts in 1990.

We solve and simulate the model and compare the simulated path with those in the data. Figure 5 compares the paths of output per worker, capital per worker, and the investment-output ratio using our quantitative model and the same moments in the data. The model performs reasonably well in explaining the path of these variables for most of the transition. In particular, in panel (a) we see that the fit of output per capita is relatively good, but the model seems to slightly overestimate its level from 1985 on. In panel (b) we observe the model tracks well the level of capital per worker with a similar slight overestimation from 1985 to 2005. Panel (c) shows the investment-output ratio, the model tracks the overall trend in data well, but our simulated moment is more volatile than in the data. There are no adjustment costs or similar rigidities in our model, which makes investments react quickly to changes in prices or growth rates, which change quite a lot in this fast-growing period in China. Overall, the fit of these nontargeted moments to the data seems reasonable.

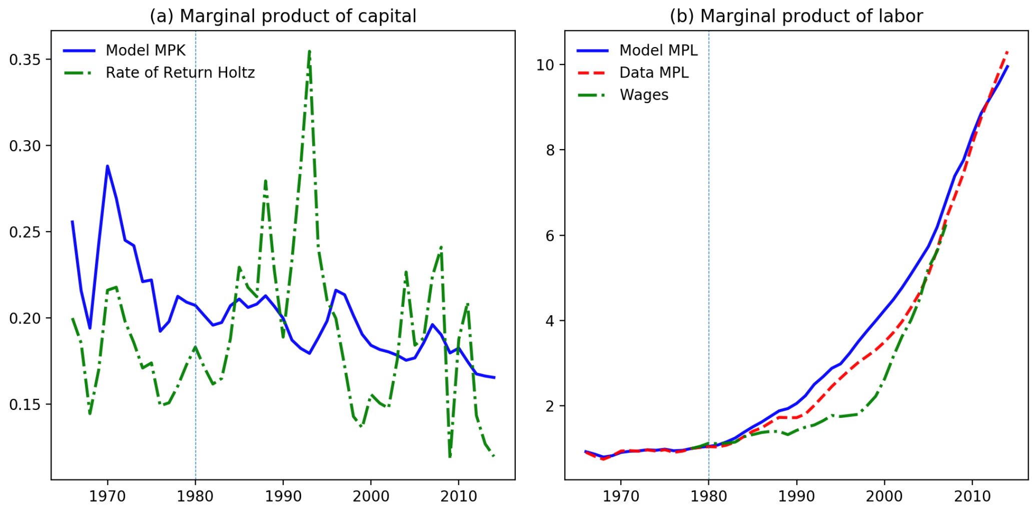
Figure 5: Fit of the simulated paths of macro-variables




Notes: Output per worker in panel (a) and capital per worker in panel (b) is in log form and normalized by the value in the first year. Output per worker: $Y/hN$ (output in real terms), Real cap per worker : $(K_s + K_e)/hN$, Investment-Output : $(p_e I_e + p_s I_s)/Y$ and $I_i = K_{i,t} + 1 - (1 - \delta_i)K_{i,t}$

In Figure 6 we compare the marginal product of capital and labor in the data with those of the calibrated model. In panel (a) we observe a relatively good fit for the marginal product of capital. The data counterpart of the marginal product of capital is calculated using the capital estimates from Holz and Yue (2018), and the interest rate is from the OECD. In particular, the model's marginal product of capital tracks reasonably well the level and the fall of the interest rate after 1985. Panel (b) shows the evolution of the marginal product of labor in the model compared with the same moment from the data (normalized to 1 in 1965). The simulated value follows the data reasonably well. We also add a measure of urban wages and match the average growth rate from 1978 to 2005 well. The path is still quite different, as urban wages were growing very slowly until the end of the 1990s and then picked up pace. This is consistent with the story of Song et al. (2011) that wages lagged behind productivity growth until the private sector started to gain a larger market share by the end of the 1990s.

Figure 6: Simulated marginal products



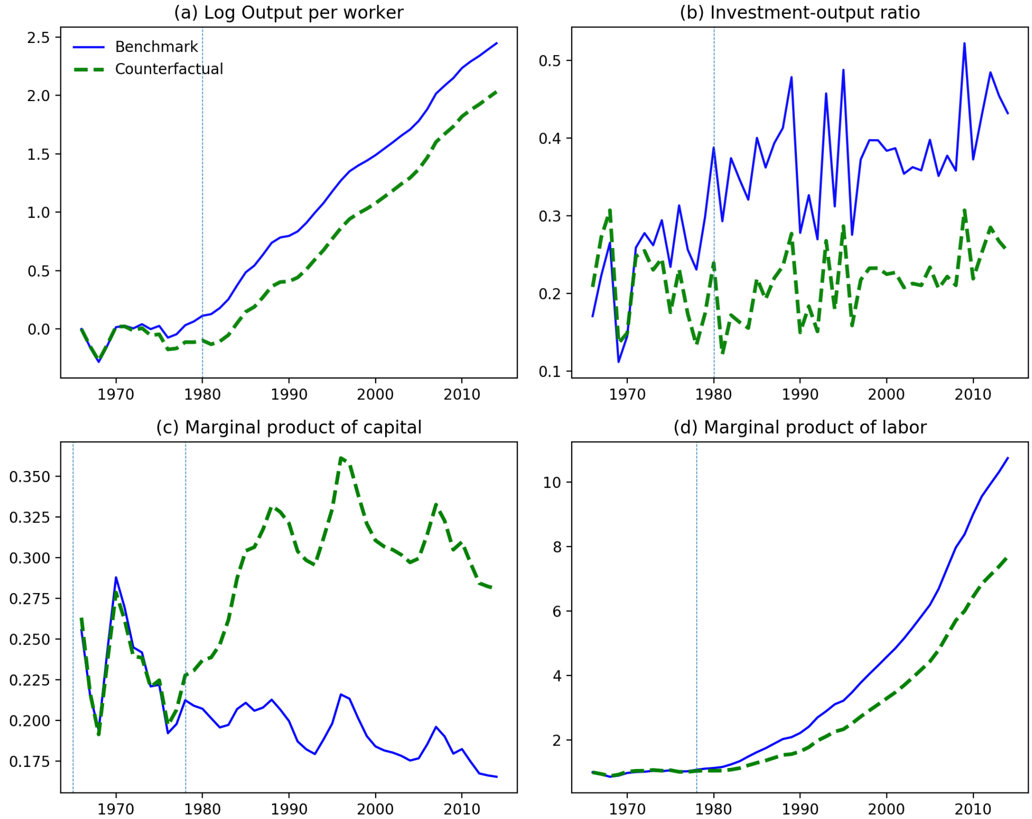

Notes: Panel (a) plots the marginal product of capital calculated by our simulated data against the real rate of return calculated in Holz and Yue (2018). Marginal products: $MPK_i = \alpha_i Y/K_i$. MPK is weighted average of the two (weighted by stock size) . Panel (b) plots the marginal product of labor calculated from simulated moments and data moments against wages from Yang et al. (2010). $MPL = (1 - \alpha_S - \alpha_E)Y/hN$.

## 4.4 Counterfactual

To better understand the mechanics of how the reduction in investment distortions drives output growth and other variables, we perform a counterfactual experiment. We keep the distortion at its initial level, and let the other exogenous variables move as in the benchmark model [4]. Figure 7 compares the simulation derived from the counterfactual experiment with the benchmark simulation and data. In panel (a) we show that output per worker shows a similar trend as in the benchmark but lies permanently at a lower level mainly due to lower capital accumulation. In 2015, output per worker is 30% lower in the case where the distortion is never reduced. In panel (b), we show that without any reduction in the distortion the investment-output ratio remains relatively stable around 20%, while in the data it is increasing to more than 40%. Panel (c) and (d) show how the marginal products are affected. The return to capital is higher than in the benchmark because less capital is deployed. In contrast, the return to labor is lower than the benchmark for the same reason.

[4] We also compare the counterfactual scenario where we keep the TFP level constant at its initial value. We find that TFP growth is the main driver behind the growth of Chinese output per worker after 1985. Shutting down TFP growth leads to virtually no growth in output per worker in the last 25 years. There is still some growth before that, as the wedge is reduced and the economy settles down with a new and higher level of capital around 1990. The investment-output ratio is not really affected by changes in TFP. As expected, TFP growth also has large effects on marginal products. The return to labor is almost constant for the entire simulation period, if TFP does not grow and the return to capital is pushed almost down to zero in later periods.

Figure 7: Constant distortion counterfactual




Notes: Figures show simulated data the distortion is kept at its initial value, plotted against our benchmark. All variables calculated as in Figures 5 and 6.

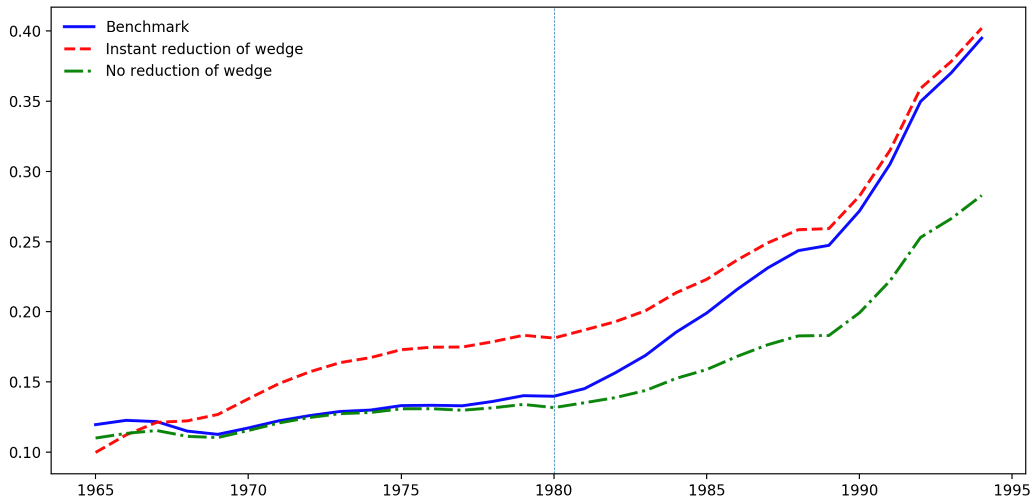
# 5 Implications

In the previous sections, we introduced an investment distortion to explain the Chinese transition after 1965. In this section, we extend the analysis in three directions. First, we ask how large of a welfare loss do these distortions imply. Then, we explore whether a simple neoclassical model only augmented with separate capital prices can rationalize part of the transition, especially after 1978. Finally, we compare our results with an alternative method to measure wedges in the spirit of Chari et al. (2007).

## 5.1 Social Cost of Inefficient Investment

We compute the social cost of the investment distortion as the net present value (NPV) of forgone consumption in 1965 terms using the calibrated discount factor. Figure 8 shows the path of consumption in China from 1965 to 1994 under three alternative counterfactuals. The choice of 1994 is quite arbitrary but is motivated by the fact that it takes roughly 30 years for all the relevant variables to settle on a new BGP in the simulations. The solid (blue) line shows the path of consumption in our benchmark scenario. The dash-dotted (green) line shows the counterfactual scenario where the distortion is never reduced. We see that the simulation settles to a level that is roughly 40% lower than the benchmark. In other words, both consumption per capita and output per worker is roughly 40% higher in 1995 than it would have been if the distortion were still in place. The dashed (red) line shows the effect of removing the entire distortion at once, so the distortion becomes one right after 1965. Recall that on the BGP, output per capita is given by $\tilde{Y} = \gamma_A \gamma_N^{-\alpha_E - \alpha_S} \tilde{K}_S^{\alpha_S} \tilde{K}_E^{\alpha_E} h^{1-\alpha_E-\alpha_S}$. The variable $\theta$ distorts both capital stocks, while $h$ is unaffected. The total distortion to both output and consumption per capita is given by $\theta^{\frac{-\alpha_E - \alpha_S}{1-\alpha_S-\alpha_E}}$ compared with no distortion. With the parameter values used in our benchmark specification and an initial distortion of $\theta = 1.7$ output and consumption per capita is 42.4% higher in the undistorted balanced growth path (details in the appendix).

Figure 8: Counterfactual consumption paths




Notes: Counterfactual paths of consumption per capita. The green dotted line is the case where distortions are never reduced. The solid blue line is our benchmark simulation. The red line is where the distortion is cut immediately rather than reduced gradually.

Table 2 shows the cost of distortions in discounted consumption terms. We compare the NPV of the counterfactual consumption streams showed in Figure 8 to the benchmark, given by $\frac{NPV(C_{C,t})}{NPV(C_{B,t})}$. Each row represents different counterfactuals and the columns different time periods for which NPVs are calculated. For the NPV calculations we discount by pure time preference, $\beta$, as the interest rate path will be different for different consumption paths[5]. The first row shows the NPV calculation for different periods when we remove the distortion in 1965. The NPV of consumption between 1965 and 1979 would have been 18% higher if distortions were removed immediately in 1965 instead of phased out over 15 years as in the benchmark. This represents the ratio between the discounted area under the red dashed line and the blue solid line. This gain is smaller the longer time period we look at because the difference is smaller the further into the simulation we get. The second scenario represents the NPV loss of never-reducing distortions, implying that

[5]Using consumption based discounting is problematic with different streams. The appendix shows that the results are essentially the same using the implied interest rate path from the benchmark consumption stream to discount all series.)

they are on the same level today as they were in 1965. Over the first 15 years that loss is very modest, only 3%, but it increases to 25% if you take the NPV all the way up to 2014. This modest gain in the early period mainly reflects the fact that in the benchmark investments become gradually more attractive, so even though GDP grows faster than in the counterfactual, more and more resources are channeled towards investment. Only after 1980, consumption in the benchmark starts to pull away from the counterfactual and at the end of the period the level difference in both output and consumption is roughly 42%, which corresponds to our BGP calculations. The magnitude of this effect is comparable to that in Hsieh and Klenow (2009) who find that reducing misallocation frictions in China in the 1990s to the same level as in the US would increase GDP by 30-50%.

Table 2: **NPV welfare implications**

| NPV cons. relative to benchmark | 1965-1979 | 1965-1994 | 1965-2014 |
|---|---|---|---|
| Instant removal of distortions in 1965 | 1.18 | 1.12 | 1.04 |
| No reduction of distortions | 0.97 | 0.84 | 0.75 |

Notes: NPV of consumption relative to the benchmark for different counterfactuals and time periods.

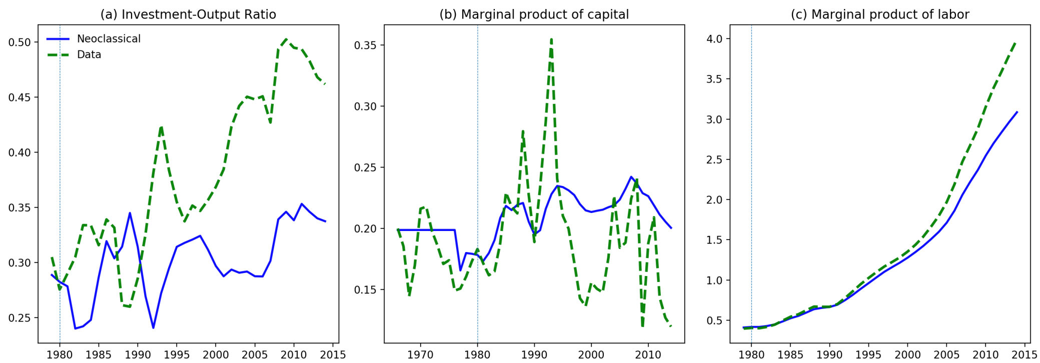
## 5.2 Can the Neoclassical Model Explain the Transition after 1978?

We have seen that our quantitative model is capable of explaining several features of the transition from 1965 to 2015. Since the calibrated investment distortion ends around 1980, a natural question is to check whether our approach adds anything to explaining the transition from 1978 onward. The answer depends on how important the propagation of the distortion in the model is over time. To answer this question, we calibrate a neoclassical model without investment distortions for the period 1975-1985. Since there is no distortion this is a standard calibration exercise based only on average moments between

1975 and 1985. The main difference between the calibrations is that the discount factor is now lower, 0.96 compared with 0.99 in the full model earlier in this article.

In Figure 9 we plot the simulated path using the calibrated neoclassical model. In panel (a) we compare the investment-output ratio simulated with the observed counterpart. We see that the neoclassical model cannot explain the rate of growth of investment-output especially after 1990. This implies that there is quite a lot of propagation in our benchmark model after removing the distortion to explain the continued increase in investment-output rates. In panel (b) we show that the model generated marginal product of capital misses the level and the trend of the data. Finally, in panel (c) we see that the neoclassical model tracks the trend in the marginal product of labor until 2000 with some smaller differences after that. The main difference between our benchmark and the neoclassical is thus the investment rates.

Figure 9: Neoclassical model (calibrated without distortions)




Notes: Simulation starting from 1978 capital-level without any distortions. Variables calculated as above.

## 5.3 Comparison With Wedge Accounting

In this paper we infer the distortion using information on the BGP and the path of the capital-output ratio. Alternatively, we could use information from the first-order conditions in the spirit of the wedge accounting exercise in Chari et al. (2007). We allow separate equipment- and structure-capital wedges, and we also add a labor wedge. Allowing for more than one wedge provides a more robust test than just trying to back out the one we use in our model.

The wedges on capital are placed as an additional cost over investment expenditure as above, while the wedge on labor can be interpreted as either a preference shifter or as a tax on labor earnings. The first-order equations with wedges $\theta_{L,t}$,$\theta_{E,t}$ and $\theta_{S,t}$ are:

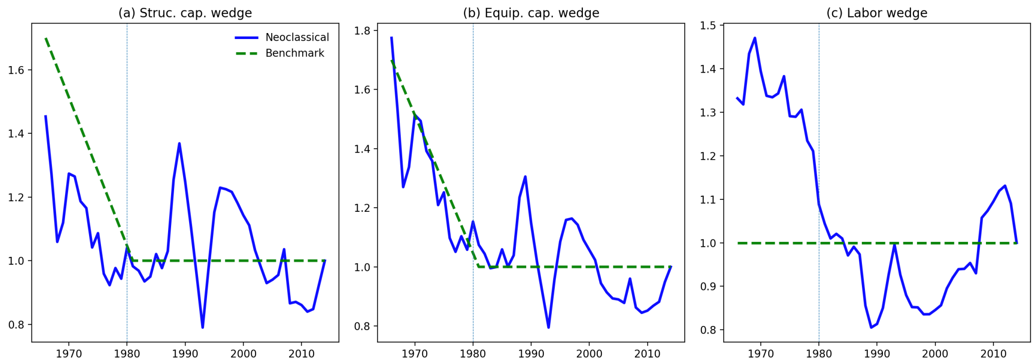
$$\frac{1}{c_t}(1-\alpha_E-\alpha_E)A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E}(h_t N_t)^{-\alpha_S-\alpha_E} = \frac{\theta_{L,t}\psi}{1-h_t} \tag{10}$$

$$\frac{c_{t+1}}{c_t} = \frac{1}{\theta_{E,t}}\frac{\beta}{p_{E,t}}\left[\theta_{E,t+1}p_{E,t+1}(1-\delta_E)+\alpha_E A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E-1}(h_t N_t)^{1-\alpha_S-\alpha_E}\right] \tag{11}$$

$$\frac{c_{t+1}}{c_t} = \frac{1}{\theta_{S,t}}\frac{\beta}{p_{S,t}}\left[\theta_{S,t+1}p_{S,t+1}(1-\delta_S)+\alpha_S A_t K_{S,t}^{\alpha_S-1} K_{E,t}^{\alpha_E}(h_t N_t)^{1-\alpha_S-\alpha_E}\right] \tag{12}$$

Taking these to the data, we can back out the wedges that make the three first-order equations hold exactly each period. For labor we do it period by period, but the dynamic structure of capital with $t$ and $t+1$ variables in each first order equation needs a different procedure. This dynamic structure implies that the vector of wedges contains one more element than the number of time periods we have. We solve this by assuming that $\theta_{E,T+1} = \theta_{S,T+1} = 1$ and iterate backwards from there. This implies that the capital wedges displayed are relative to the level of the wedges in 2016. The result is shown together with our calibrated wedge in Figure 10. In panel (a) we see that the wedge on equipment capital fits very well with our capital wedge, while the wedge on structures shows a less clear trend than the calibrated aggregate trend, this suggests we could have provided a more accurate model by including separate wedges for structures and equipment capital. There is also evidence of a declining trend in labor. While this is certainly plausible given the historical labor market conditions in China, this is not a point we want to press in this article. A more detailed description of demographics and the labor market would be needed, the transition from agricultural work to industry is particularly important in this regard.

Figure 10: Wedge Analysis




Notes: Estimated wedges plotted against distortions proposed in our main exercise. Wedges are calculated assuming last period is undistorted. All levels are thus relative to 2015.

# 6 Conclusion

In this paper, we have built a model consistent with the entire Chinese transition from the 1960s until today. Previous literature has generally focused on either the pre-1978-reform period or the post-reform period. A key part of our model reconciling these two periods is the investment friction present in the early period of the People's Republic of China. We identify this friction by exploiting the entire path of capital-output ratios in the data together with a careful reading of historical facts.

We conjecture that investment decisions in the 1960s were not made on the basis of economic efficiency and that this is visible in the data through low investment rates. When policy gradually shifted towards making efficient investment decisions through the end of the 1960s and 1970s, investment rates gradually rose. After calibrating this distortion our model does a remarkably good job in explaining the evolution of key variables in the Chinese economy from 1965 to 2016 and we show that it outperforms the neoclassical growth model. We run counterfactual simulations to assess the importance of the main drivers in the model and find that while TFP was the main driver of the extraordinary growth China experienced, the reduction in investment frictions has left China's GDP more than 40% higher today than if they remained high. We also show that even though the distortion is completely phased out in 1980 significant propagation effects still contributed towards higher GDP and consumption throughout the 1980s and early 1990s.

There is much debate over distortions in the Chinese economy today. This paper suggests that the distortions in Mao's China were perhaps more severe, and more importantly, that important progress in reducing distortions has been made in the past, even before 1978.

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# For Online Publication

## A Welfare comparison with consumption based NPVs

This table replicates Table 2 using consumption based discounting rather than simply the pure time preference. Compared with the main specification, the difference is that the future is discounted a bit more after consumption started growing fast in the 1980s. The first column, in thus is essentially unchanged. In the second column all numbers are a bit higher than in the main specification, but only by a few percentage points. The last column is where the difference is greatest, especially for the counterfactual where the distortion never existed; discounting the future more means that the initially higher consumption level is given more weight.

Table 3: **Welfare implications**

| NPV consump. relative to benchmark | 1965-1979 | 1965-1994 | 1965-2014 |
|---|---|---|---|
| Instant removal of distortions in 1965 | 1.17 | 1.15 | 1.10 |
| No reduction of distortions | 0.97 | 0.89 | 0.82 |

Notes: NPV of consumption relative to the benchmark for different counterfactuals and time periods using consumption based discounting.

## B Solution algorithm

We solve the model numerically using a version of a forward shooting algorithm, see Judd (1998). This system has a steady state characterized by constant normalized variables as long as the exogenous trends are stable (Constant growth in $Z_t$, $A_t$, $N_t$, $p_{S,t}$, $p_{Et}$ and constant level of $\theta_t$. The solver tries to find the saddle path leading to this steady state. In our main simulations, we use data points for these exogenous drivers for the simulation period. We run the model for T-periods with exogenous drivers and let the exogenous series grow at constant rates for an after period of at least 50 years to let the model settle in the new steady state.

The system consists of four endogenous dynamic variables, $K_{E,t}$, $K_{S,t}$, $C_t$, and $h_t$ which are determined by a vector of first-order equations and budget constraints. To explain the algorithm, we first present the setup of the model with generic functions. We use the first-order conditions for capital for each period $t(F_{3,t}$ and $F_{4,t})$, the period $t$ budget constraint($F_{1,t}$) and the period $t+1$ first-order condition with respect to labor($F_{2,t+1}$):

$$F_{1,t}(\tilde{c}_t, \tilde{K}_{E,t}, \tilde{K}_{E,t+1}, \tilde{K}_{S,t}, \tilde{K}_{S,t+1}, h_t) = 0 \text{ I: (BC)}$$

$$F_{2,t+1}(\tilde{c}_{t+1}, \tilde{K}_{E,t+1}, \tilde{K}_{S,t+1}, h_{t+1}) = 0 \text{ II: } (h_{t+1})$$

$$F_{3,t}(\tilde{c}_t, \tilde{c}_{t+1}, \tilde{K}_{E,t+1}, \tilde{K}_{S,t+1}, h_{t+1}) = 0 \text{ III: } (K_{E,t+1})$$

$$F_{4,t}(\tilde{c}_t, \tilde{c}_{t+1}, \tilde{K}_{E,t+1}, \tilde{K}_{S,t+1}, h_{t+1}) = 0 \text{ IV: } (K_{S,t+1})$$

The steps of the algorithm is as follows:

1. Calibrate parameters and exogenous inputs.
2. Set initial values for the two capital stocks, $\tilde{K}_{E,0}$ and $\tilde{K}_{S,0}$.
3. Guess on initial value for consumption, $\tilde{c}_0$.
   - Bounds: $0 \leq \tilde{c}_0 \leq Y(\tilde{K}_{S,0}, \tilde{K}_{S,0}, h = 1)$
4. Calculate $h_0$ from $F_{2,0} = 0$ given initial capital and the guess on consumption.
5. Iterate over time periods from 1 to $T + \tau$ where $T$ is the period with exogenous drivers and $\tau$ is the number of periods needed for the system to settle down.
   - Given $\tilde{K}_{E,0}$, $\tilde{K}_{S,0}$,$\tilde{c}_0$ and $h_0$ the system above is solved by a non-linear solver (fsolve in MATLAB) for the period 1-variables. Iterate on this procedure.
   - Stop iterating when new steady state is reached.
6. Evaluate solution:
   - If consumption explodes or endpoint is too high, start from 3. with a higher initial guess of consumption.
   - If consumption implodes or endpoint is too low, start from 3. with a lower initial guess of consumption.
   - If endpoint is at the new BGP, we are done.

# C Detailed derivations

This subsection contains detailed derivations used in the main text.

## C.1 Normalization of the resource constraint

Recall the resource constraint:

$$N_t c_t + \theta_t \left( p_{S,t} K_{S,t+1} + p_{E,t} K_{E,t+1} \right) = A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E} (h_t N_t)^{1-\alpha_S-\alpha_E} \\ + \theta_t \left[ p_{S,t}(1-\delta_S) K_{S,t} + p_{E,t}(1-\delta_E) K_{E,t} \right]$$

Divide the budget constraint (1) by $N_t Z_t$ to:

$$\frac{N_t c_t}{N_t Z_t} + \theta_t \left( \frac{p_{S,t} K_{S,t+1}}{N_t Z_t} + \frac{p_{E,t} K_{E,t+1}}{N_t Z_t} \right) = \frac{A_t K_{S,t}^{\alpha_S} K_{E,t}^{\alpha_E} (h_t N_t)^{1-\alpha_S-\alpha_E}}{N_t Z_t} + \\ \theta_t \left[ \frac{p_{S,t}(1-\delta_S) K_{S,t}}{N_t Z_t} + \frac{p_{E,t}(1-\delta_E) K_{E,t}}{N_t Z_t} \right] \tag{13}$$

Consider the following terms:

$$\frac{p_{S,t} K_{S,t+1}}{N_t Z_t} = \frac{Z_{t+1}}{Z_t} \frac{p_{S,t} K_{S,t+1}}{N_t Z_{t+1}} = \gamma_{Z,t} \tilde{K}_{S,t+1}$$

$$\begin{aligned} \frac{p_{S,t}(1-\delta_S) K_{S,t}}{N_t Z_t} &= \frac{p_{S,t-1}(1-\delta_S) K_{S,t}}{N_{t-1} Z_t} \left( \frac{N_{t-1}}{N_t} \right) \left( \frac{p_{S,t}}{p_{S,t-1}} \right) \\ &= (1-\delta_S) \tilde{K}_{S,t} \frac{\gamma_{S,t}}{\gamma_{N,t}} \end{aligned}$$

and the production function:

$$\begin{aligned} \frac{Y_t}{N_t Z_t} &= \frac{A_t}{N_t Z_t} \left( \frac{K_{S,t} p_{S,t-1}}{Z_t N_{t-1}} \right)^{\alpha_S} \left( \frac{Z_t N_{t-1}}{p_{S,t-1}} \right)^{\alpha_S} \left( \frac{K_{E,t} p_{E,t-1}}{Z_t N_{t-1}} \right)^{\alpha_E} \left( \frac{Z_t N_{t-1}}{p_{E,t-1}} \right)^{\alpha_E} (h_t N_t)^{1-\alpha_S-\alpha_E} \\ &= \frac{A_t}{N_t Z_t} \tilde{K}_{S,t}^{\alpha_S} \tilde{K}_{E,t}^{\alpha_E} h_t^{1-\alpha_S-\alpha_E} \left( \frac{Z_t N_{t-1}}{p_{S,t-1}} \right)^{\alpha_S} \left( \frac{Z_t N_{t-1}}{p_{E,t-1}} \right)^{\alpha_E} N_t^{1-\alpha_S-\alpha_E} \end{aligned}$$

Now using the definition of $Z_t$, we have:

$$Z_t^{1-\alpha_S-\alpha_E} = A_{t-1} p_{E,t-1}^{-\alpha_E} p_{S,t-1}^{-\alpha_S}$$

which simplifies to:

$$\begin{aligned}
\frac{Y_t}{N_t Z_t} &= \frac{A_t}{A_{t-1}N_t}\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{1-\alpha_S-\alpha_E}\left(N_{t-1}\right)^{\alpha_S+\alpha_E}{N_t}^{1-\alpha_S-\alpha_E} \\
&= \frac{A_t}{A_{t-1}}\left(\frac{N_{t-1}}{N_t}\right)^{\alpha_S+\alpha_E}\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{1-\alpha_S-\alpha_E} \\
&= \frac{\gamma_{A,t}}{\gamma_{N,t}^{(\alpha_E+\alpha_S)}}\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{1-\alpha_S-\alpha_E}
\end{aligned}$$

Hence, the resource constraint becomes:

$$\begin{aligned}
\frac{c_t}{Z_t} + \theta_t\left(\gamma_{Z,t}\tilde{K}_{S,t+1} + \gamma_{Z,t}\tilde{K}_{E,t+1}\right) = &\frac{A_t K_{S,t}^{\alpha_S}K_{E,t}^{\alpha_E}(h_t N_t)^{1-\alpha_S-\alpha_E}}{N_t Z_t} + \\
&\theta_t\left[(1-\delta_S)\tilde{K}_{S,t}\frac{\gamma_{S,t}}{\gamma_{N,t}} + (1-\delta_E)\tilde{K}_{E,t}\frac{\gamma_{E,t}}{\gamma_{N,t}}\right]
\end{aligned}$$

Using the growth rate of the composite variable:

$$\gamma_{Z,t} = \frac{Z_t}{Z_{t-1}} = \left(\gamma_{A,t-1}\gamma_{E,t-1}^{-\alpha_E}\gamma_{S,t-1}^{-\alpha_S}\right)^{\frac{1}{1-\alpha_E-\alpha_S}}$$

or as in equation (2):

$$\begin{aligned}
\tilde{c}_t + \theta_t\gamma_{Z,t+1}(\tilde{K}_{S,t+1} + \tilde{K}_{E,t+1}) = &\frac{\gamma_{A,t}}{\gamma_{N,t}^{(\alpha_E+\alpha_S)}}\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{1-\alpha_S-\alpha_E} \\
&+\frac{\theta_t}{\gamma_{N,t}}\left[\gamma_{S,t}(1-\delta_S)\tilde{K}_{S,t} + \gamma_{E,t}(1-\delta_E)\tilde{K}_{E,t}\right]
\end{aligned}$$

## C.2 Optimality conditions

$$\max_{\{\tilde{c}_t,h_t\}_0^\infty}\sum_{t=0}^{\infty}\beta^t N_t\left(\log(\tilde{c}_t) + \psi\log(1-h_t)\right)$$

subject to:

$$\begin{aligned}
\tilde{c}_t + \theta_t\gamma_{Z,t+1}(\tilde{K}_{S,t+1} + \tilde{K}_{E,t+1}) = &\frac{\gamma_A}{\gamma_{N,t}^{(\alpha_E+\alpha_S)}}\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{1-\alpha_S-\alpha_E} \\
&+\frac{\theta_t}{\gamma_{N,t}}\left[\gamma_{S,t}(1-\delta_S)\tilde{K}_{S,t} + \gamma_{E,t}(1-\delta_E)\tilde{K}_{E,t}\right]
\end{aligned} \tag{14}$$

**First order conditions**

Foc. wrt.. labor:

$$\frac{\gamma_{A,t}}{\gamma_{N,t}^{(\alpha_E+\alpha_S)}}(1-\alpha_S-\alpha_E)\tilde{K}_{S,t}^{\alpha_S}\tilde{K}_{E,t}^{\alpha_E}h_t^{-\alpha_S-\alpha_E}\frac{1}{\tilde{c}_t} = \psi\frac{1}{1-h_t} \tag{15}$$

wrt. $K_S$:

$$\theta_t\frac{\gamma_{Z,t+1}}{\gamma_{N,t+1}}\frac{1}{\beta}\frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \frac{\gamma_{A,t+1}}{\gamma_{N,t+1}^{(\alpha_E+\alpha_S)}}\alpha_S\tilde{K}_{S,t+1}^{\alpha_S-1}\tilde{K}_{E,t+1}^{\alpha_E}h_{t+1}^{1-\alpha_S-\alpha_E} + \frac{\theta_{t+1}}{\gamma_{N,t+1}}\gamma_{S,t+1}(1-\delta_S) \tag{16}$$

wrt. $K_E$:

$$\theta_t\frac{\gamma_{Z,t+1}}{\gamma_{N,t+1}}\frac{1}{\beta}\frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \frac{\gamma_{A,t+1}}{\gamma_{N,t+1}^{(\alpha_E+\alpha_S)}}\alpha_E\tilde{K}_{S,t+1}^{\alpha_S}\tilde{K}_{E,t+1}^{\alpha_E-1}h_{t+1}^{1-\alpha_S-\alpha_E} + \frac{\theta_{t+1}}{\gamma_{N,t+1}}\gamma_{E,t+1}(1-\delta_E) \tag{17}$$

## C.3 Closed form derivations

**Normalize the resource constraint**.

$$c_t + \theta_t K_{t+1} = A_t K_t^{\alpha}$$

Divide the resource constraint by $Z_t$ to:

$$\frac{c_t}{A_{t-1}^{\frac{1}{1-\alpha}}} + \theta_t\frac{K_{t+1}}{A_{t-1}^{\frac{1}{1-\alpha}}} = \frac{A_t K_t^{\alpha}}{A_{t-1}^{\frac{1}{1-\alpha}}} \tag{18}$$

where

$$Z_t = A_{t-1}^{\frac{1}{1-\alpha}}$$

$$\gamma_Z = \gamma_A^{\frac{1}{1-\alpha}}$$

which becomes:

$$
\begin{aligned}
\frac{c_t}{A_{t-1}^{\frac{1}{1-\alpha}}} + \theta_t \frac{K_{t+1}}{A_t^{\frac{1}{1-\alpha}}} \frac{A_t^{\frac{1}{1-\alpha}}}{A_{t-1}^{\frac{1}{1-\alpha}}} &= \frac{A_t K_t^{\alpha} h_t{}^{1-\alpha}}{A_{t-1}^{\frac{1}{1-\alpha}}} \\
&= \frac{A_t}{A_{t-1}^{\frac{1}{1-\alpha}}} \left( \frac{K_t}{A_{t-1}^{\frac{1}{1-\alpha}}} \right)^{\alpha} A_{t-1}^{\frac{\alpha}{1-\alpha}} \\
&= \frac{A_t}{A_{t-1}} \left( \frac{K_t}{Z_t} \right)^{\alpha}
\end{aligned}
$$

which simplifies to:

$$
\tilde{c}_t + \gamma_{A,t}^{\frac{1}{1-\alpha}} \theta_t \tilde{K}_{t+1} = \gamma_{A,t} \tilde{K}_t^{\alpha} h_t^{1-\alpha} \tag{19}
$$

All variables are normalized by Z, no $\theta$ in capital normalization.

**Optimality conditions**

$$
\max_{\{\tilde{c}_t\}_0^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(\tilde{c}_t)
$$

subject to:

$$
\tilde{c}_t + \gamma_{A,t}^{\frac{1}{1-\alpha}} \theta_t \tilde{K}_{t+1} = \gamma_{A,t} \tilde{K}_t^{\alpha} \tag{20}
$$

The first order condition is:

$$
\gamma_{A,t}^{\frac{1}{1-\alpha}} \frac{\theta_t}{\tilde{c}_t} = \beta \frac{1}{\tilde{c}_{t+1}} \left[ \gamma_{A,t+1} \alpha \tilde{K}_{t+1}^{\alpha-1} \right]
$$

**Guess and verify**

Use the guess:

$$
\tilde{K}_{t+1} = (\gamma_{A,t})^{-\frac{\alpha}{1-\alpha}} \theta_t^{-1} \alpha \beta \tilde{K}_t^{\alpha}
$$

and the resource constraint to solve for consumption:

$$
\begin{aligned}
\tilde{c}_t &= \gamma_{A,t} \tilde{K}_t^{\alpha} - \gamma_{A,t}^{\frac{1}{1-\alpha}} \theta_t \tilde{K}_{t+1} \\
\tilde{c}_t &= (1 - \alpha\beta) \tilde{Y}_t
\end{aligned}
$$

The Euler equation

$$\gamma_{A,t}^{\frac{1}{1-\alpha}} \frac{\theta_t}{\tilde{c}_t} = \beta \frac{1}{\tilde{c}_{t+1}} \left[ \gamma_{A,t+1} \alpha \tilde{K}_{t+1}^{\alpha-1} \right]$$

$$\gamma_{A,t}^{\frac{1}{1-\alpha}} \frac{\theta_t}{(1-\alpha\beta)\tilde{Y}_t} = \beta \frac{1}{(1-\alpha\beta)\tilde{Y}_{t+1}} \left[ \alpha \frac{\tilde{Y}_{t+1}}{\tilde{K}_{t+1}} \right]$$

$$\gamma_{A,t}^{\frac{1}{1-\alpha}} \frac{\theta_t}{\tilde{Y}_t} = \beta\alpha \frac{1}{\tilde{K}_{t+1}}$$

$$\tilde{K}_{t+1} = \frac{\alpha\beta}{\gamma_{A,t}^{\frac{1}{1-\alpha}} \theta_t} \tilde{Y}_t$$

$$\tilde{K}_{t+1} = \frac{\gamma_{A,t}\alpha\beta}{\gamma_{A,t}^{\frac{1}{1-\alpha}} \theta_t} \tilde{K}_t$$

which simplifies to:

$$\tilde{K}_{t+1} = \frac{1}{\gamma_{A,t}^{\alpha/(1-\alpha)} \theta_t} \alpha\beta \tilde{K}_t^{\alpha}$$

which is the same as the guess.

The capital output ratio is

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{1}{\gamma_{A,t+1}\theta_t^{1-\alpha}} \left( \frac{\alpha\beta}{A_t^{\frac{\alpha}{1-\alpha}}} K_t^{\alpha} \right)^{1-\alpha}$$

And we can discuss how high growth rates in productivity (high $\gamma_Z$) reduces the capital-output ratio unless it is counteracted by continuing reduction in $\theta$.

- An increase in $\gamma_{A,t+1}$ decreases the K/Y.
- To account for an increase in K/Y we need a decrease in $\theta$ (level).

## C.4 Balanced growth path (BGP)

Here we provide an analytical solution of the BGP. Given the growth rates above the BGP will be given where the tilde-variables are the same over time.$\gamma_A$,$\gamma_E$,$\gamma_S$ and $\gamma_N$ are assumed constant. The level of the distortion is assumed to be constant on the

BGP. Per capita consumption growth will be given by:

$$\gamma_Z = (\gamma_A \gamma_E^{-\alpha_E} \gamma_S^{-\alpha_S})^{\frac{1}{1-\alpha_E-\alpha_S}}$$

Starting off with the first order conditions for the capital stocks at the BGP:

$$\theta \frac{\gamma_Z}{\gamma_N} \frac{1}{\beta} \frac{\tilde{c}}{\tilde{c}} = \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_S \tilde{K}_S^{\alpha_S-1} \tilde{K}_E^{\alpha_E} h^{1-\alpha_S-\alpha_E} + \frac{\theta}{\gamma_N} \gamma_S (1-\delta_S) \tag{21}$$

$$\theta \frac{\gamma_Z}{\gamma_N} \frac{1}{\beta} \frac{\tilde{c}}{\tilde{c}} = \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_E \tilde{K}_S^{\alpha_S} \tilde{K}_E^{\alpha_E-1} h^{1-\alpha_S-\alpha_E} + \frac{\theta}{\gamma_N} \gamma_E (1-\delta_E) \tag{22}$$

where consumption drops out. Also moved $h$ into the capital expressions.

$$\theta \frac{\gamma_Z}{\gamma_N} \frac{1}{\beta} = \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_S (\frac{\tilde{K}_S}{h})^{\alpha_S-1} (\frac{\tilde{K}_E}{h})^{\alpha_E} + \frac{\theta}{\gamma_N} \gamma_S (1-\delta_S) \tag{23}$$

$$\theta \frac{\gamma_Z}{\gamma_N} \frac{1}{\beta} = \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_E (\frac{\tilde{K}_S}{h})^{\alpha_S} (\frac{\tilde{K}_E}{h})^{\alpha_E-1} + \frac{\theta}{\gamma_N} \gamma_E (1-\delta_E) \tag{24}$$

It's convenient to work with stocks per worker in the derivation as both are homogeneous of degree 1 in labor. Define $k_s = \frac{\tilde{K}_S}{h}$ and $k_s = \frac{\tilde{K}_S}{h}$.

$$k_S^{\alpha_S-1} k_E^{\alpha_E} \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_S = \frac{\theta(\gamma_Z - \beta\gamma_S(1-\delta_S))}{\beta\gamma_N} \tag{25}$$

$$k_S^{\alpha_S} k_E^{\alpha_E-1} \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} \alpha_E = \frac{\theta(\gamma_Z - \beta\gamma_E(1-\delta_E))}{\beta\gamma_N} \tag{26}$$

This solves to:

$$k_S = \left(B_S^{1-\alpha_E} B_E^{\alpha_E}\right)^{\frac{1}{1-\alpha_S-\alpha_E}}$$

$$k_E = \left(B_S^{\alpha_S} B_E^{1-\alpha_S}\right)^{\frac{1}{1-\alpha_S-\alpha_E}}$$

with

$$B_E \equiv \frac{\beta\alpha_E\gamma_A\gamma_N^{1-\alpha_E-\alpha_S}}{\theta(\gamma_Z - \beta(1-\delta_E)\gamma_E)}$$

and

$$B_S \equiv \frac{\beta\alpha_S\gamma_A\gamma_N^{1-\alpha_E-\alpha_S}}{\theta(\gamma_Z - \beta(1-\delta_S)\gamma_S)}$$

Now we have the solution for the capital stocks in per worker terms. To find consumption and labor supply per capita we use the stationary budget constraint and f.o.c wrt. labor. The budget constraint is:

$$\begin{aligned}\tilde{c} + \theta\gamma_Z(hk_S + hk_E) &= \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} k_S^{\alpha_S} k_E^{\alpha_E} h \\ &+ \frac{\theta}{\gamma_N}\left[\gamma_S(1-\delta_S)hk_S + \gamma_E(1-\delta_E)hk_E\right]\end{aligned} \tag{27}$$

We can find consumption per worker as:

$$\begin{aligned}\frac{\tilde{c}}{h} &= \frac{\gamma_A}{\gamma_N^{(\alpha_E+\alpha_S)}} k_S^{\alpha_S} k_E^{\alpha_E} - \theta\gamma_Z(k_S + k_E) \\ &+ \frac{\theta}{\gamma_N}\left[\gamma_S(1-\delta_S)k_S + \gamma_E(1-\delta_E)k_E\right]\end{aligned} \tag{28}$$

consumption per capita will be given by $\tilde{c} = Qh$ where $Q$ is given by:

$$Q = \gamma_A \gamma_N^{-\alpha_E-\alpha_S} k_S^{\alpha_S} k_E^{\alpha_E} + \theta\left(\frac{\gamma_S}{\gamma_N}(1-\delta_S) - \gamma_Z\right)k_S + \theta\left(\frac{\gamma_E}{\gamma_N}(1-\delta_E) - \gamma_Z\right)k_E$$

And foc wrt. labor (note that $h$ drops out on the left hand side):

$$\frac{\gamma_A}{\gamma_n^{(\alpha_E+\alpha_S)}}(1-\alpha_S-\alpha_E)(hk_S)^{\alpha_S}(hk_E)^{\alpha_E} h^{-\alpha_S-\alpha_E}\frac{1}{\tilde{c}} = \psi\frac{1}{1-h} \tag{29}$$

From the foc. wrt. labor:

$$\gamma_A \gamma_n^{-(\alpha_E+\alpha_S)}(1-\alpha_S-\alpha_E)k_S^{\alpha_S}k_E^{\alpha_E} = \psi\frac{\tilde{c}}{1-h} \tag{30}$$

insert for $\tilde{c} = Qh$:

$$\gamma_A \gamma_n^{-(\alpha_E+\alpha_S)}(1-\alpha_S-\alpha_E)k_S^{\alpha_S}k_E^{\alpha_E} = \psi\frac{Qh}{1-h} \tag{31}$$

labor is determined by:

$$h = \frac{(1-\alpha_S-\alpha_E)k_S^{\alpha_S}k_E^{\alpha_E}\gamma_A\gamma_N^{-\alpha_E-\alpha_S}}{\psi Q + (1-\alpha_S-\alpha_E)k_S^{\alpha_S}k_E^{\alpha_E}\gamma_A\gamma_N^{-\alpha_E-\alpha_S}}$$

Plug that back into the expression for consumption:

$$\tilde{c} = Qh$$

And we have a solution for consumption and labor supply per capita ($\tilde{c}$ and $h$) and the capital stocks per worker ($k_S$ and $k_E$). Multiplying these last equations with the number of workers per capita we have the per capita capital stocks as:

$$\tilde{K}_E = \tilde{k}_E h$$

$$\tilde{K}_S = \tilde{k}_S h$$

Output will be:

$$\tilde{Y} = \frac{Y_t}{Z_t N_{t-1}} = \gamma_A \gamma_N^{-\alpha_E - \alpha_S} \tilde{K}_S^{\alpha_S} \tilde{K}_E^{\alpha_E} h^{1-\alpha_E-\alpha_S}$$

Real investment will be:

$$\tilde{I}_E = (\gamma_Z - \gamma_E \gamma_N^{-\alpha_E - \alpha_S}(1-\delta_E))\tilde{K}_E$$

$$\tilde{I}_S = (\gamma_Z - \gamma_S \gamma_N^{-\alpha_E - \alpha_S}(1-\delta_S))\tilde{K}_S$$

Observe that $k_S$ and $k_E$ are constant, implying that the labor share $h$ is also constant on the BGP. Further, this implies that normalized consumption ($\tilde{c}$), output ($\tilde{Y}$), and investment ($\tilde{I}_E$ and $\tilde{I}_S$) are constant and their nominal values will grow by the rate of their normalization values. To show that the rate of interest is constant we can start with the first order equation with respect to structure capital of the consumer problem without the normalization. It could be arranged such that the left hand side showed how the rate of interest would equal the consumption discount factor $1 + r_t = \frac{c_{t+1}}{c_t \beta}$. The right hand side would then show:

$$1 + r_t = \frac{\alpha_S}{\theta_t} \frac{A_{t+1}}{p_{S,t}} K_{s,t+1}^{\alpha_S - 1} K_{E,t+1}^{\alpha_E} (h_{t+1} N_{t+1})^{1-\alpha_S-\alpha_E} + \frac{\theta_{t+1}}{\theta_t} \frac{p_{S,t+1}}{p_{S,t}} (1 - \delta_s)$$

In the steady state the distortion is constant, $\theta$, labor is constant, $h$, price growth is given by$\gamma_{p_S}$productivity growth is given by $\gamma_A$, population growth is $\gamma_N$ and the capital stocks grow by $\frac{\gamma_Z \gamma_N}{\gamma_{p_S}}$ and $\frac{\gamma_Z \gamma_N}{\gamma_{p_E}}$ respectively. The one period ahead interest rate is thus given by:

$$1+r_{t+1}=\frac{\alpha_S}{\theta}\frac{A_{t+1}\gamma_A}{p_{S,t}\gamma_S}K_{s,t+1}^{\alpha_S-1}(\frac{\gamma_Z\gamma_N}{\gamma_S})^{\alpha_S-1}K_{E,t+1}^{\alpha_E}(\frac{\gamma_Z\gamma_N}{\gamma_E})^{\alpha_E}(hN_{t+1})^{1-\alpha_S-\alpha_E}\gamma_N^{1-\alpha_S-\alpha_E}+\gamma_S(1-\delta_s)$$

$$1+r_{t+1}=\frac{\gamma_A}{\gamma_S}(\frac{\gamma_Z\gamma_N}{\gamma_S})^{\alpha_S-1}(\frac{\gamma_Z\gamma_N}{\gamma_E})^{\alpha_E}\gamma_N^{1-\alpha_S-\alpha_E}(1+r_t-\gamma_S(1-\delta_s))+\gamma_S(1-\delta_s)$$

Collect terms:

$$1+r_{t+1}=\gamma_A\gamma_Z^{\alpha_S+\alpha_E-1}\gamma_S^{-\alpha_S}\gamma_E^{-\alpha_E}(1+r_t-\gamma_S(1-\delta_s))+\gamma_S(1-\delta_s)$$

and insert $\gamma_Z=(\gamma_A\gamma_E^{-\alpha_E}\gamma_S^{-\alpha_S})^{\frac{1}{1-\alpha_E-\alpha_S}}$:

$$1+r_{t+1}=\gamma_A(\gamma_A\gamma_E^{-\alpha_E}\gamma_S^{-\alpha_S})^{\frac{-(1-\alpha_S-\alpha_E)}{1-\alpha_S-\alpha_E}}\gamma_S^{-\alpha_S}\gamma_E^{-\alpha_E}(1+r_t-\gamma_S(1-\delta_s))+\gamma_S(1-\delta_s)$$

$$1+r_{t+1}=(1+r_t-\gamma_S(1-\delta_s))+\gamma_S(1-\delta_s)$$

$$1+r_{t+1}=1+r_t$$